人工林の直径分布を表現するワイブルパラメータに関する一考察

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https://doi.org/10.15017/10895

出版情報：九州大学農学部演習林報告. 72, pp.1-10, 1995-03-30. Kyushu University Forests

九大演報 (Bull. Kyushu Univ. For.), 72： 1 -10, 1995

A Study on the Weibull Parameters Expressing the Diameter Distribution of Plantations*

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Abstract

A problem in the expression of diameter distribution of plantations using only the Weibull parameters *a, b* and *c* was examined. First, it was shown that the scatter of diameters could not be evaluated exactly using only parameter *b*. Therefore, to solve this, a simple calculating method of a standard deviation and variation coefficient of diameters using the Weibull parameters *b* and *c* was devised. From examples taken from application, it became clear that differences in the scatter of diameter distributions, where parameter *b* was nearly the same and *c* was greatly different, could be evaluated precisely by comparison of standard deviation and variation coefficient calculated using the above method. Furthermore, when comparing of some diameter distributions in which parameter *b* was different, it was shown that difference in the scatter of diameters must be evaluated using standard deviation and variation coefficient.

From the viewpoint of forest management technology, it is very important to ascertain the scatter of diameters with regard to plantations. Therefore, the simple method of calculating standard deviation and variation coefficient of diameters proposed in this study should be utilized effectively in study of forest management technology of plantations using the Weibull distribution.

Key words : plantations ; diameter distribution ; Weibull distribution ; Weibull parameters.

1. Introduction

The Weibull distribution is a probability density function with three parameters *a*, *b* and *c*, and well fits the diameter distribution of plantations (Nishizawa *et al.*, 1976a ; Abe, 1980 ; Kakihara, 1982). Parameter *a* is minimum stand diameter. *b* is the value for cumulative distribution of diameters at the 63% point, showing spread of distribution. *c* expresses the shape of distribution. Therefore, the outline of diameter distribution can be understood from the values of the Weibull parameters. Thus, studies clarifying the way in which the Weibull parameters change with age or by species, cultivars and thinning

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have been proposed (Iehara, 1987 ; Kakihara, 1982 ; Kakihara and Kinashi, 1984, 1986b; Matsumura *et al.*, 1991 ; Yamamoto, 1988). However, in such studies, characteristics of a, b and c were analyzed, respectively. As a result, the scatter of diameters could not be evaluated exactly. In this study, this defect in past studies was clearly identified and a solution was proposed.

2. A problem in expression of diameter distribution using the Weibull distribution

In the Weibull distribution expressing diameter distribution shown Eq. (1), variance ($V(x)$), mean diameter ($E(x)$) and variation coefficient (CV) are obtained using Eqs. (2)~(4) (Nishizawa *et al.*, 1976b ; Nishizawa, 1978).

$$f(x_i) = (c/b)(x_i/b)^{c-1} \exp\{-(x_i/b)^c\} \quad (1)$$

$$V(x) = b^2\{\Gamma(1+2/c) - \Gamma^2(1+1/c)\}$$
$$= b^2(\Gamma_2 - \Gamma_1^2) \quad (2)$$

$$E(x) = a + b\,\Gamma_1 \quad (3)$$

$$CV = \sqrt{V(x)}/E(x) \quad (4)$$

where $x_i = d_i - a$, d_i stand for each diameter, a, b and c are parameters.

Parameter a shows that no trees have a diameter less than a. Therefore, a has no relation to the degree of scatter. As shown in Eqs. (2)~(4), standard deviation ($\sqrt{V(x)}$) and variation coefficient are closely connected with parameters b and c. This indicates that if b is the same the scatter of diameters differs corresponding to c, as shown in Fig.1. Therefore, the scatter of diameters becomes larger generally when parameter b increases, but the difference in the scatter of diameter distributions, in which parameter b is nearly the same, and c is greatly different, can not be evaluated using only b (See Fig. 1). This is a problem in the expression of diameter distribution using only the Weibull parameters.

3. Method

Standard deviation and variation coefficient are the fundamental scale for evaluating the scatter. Therefore, to solve the above problem, it is necessary to evaluate the scatter using standard deviation and variation coefficient in place of parameter b. Thus, a simple calculating method for standard deviation and variation coefficient using parameters b and c was devised. Next, using standard deviation and variation coefficient obtained using the above method, it could be demonstrated that the difference in the scatter of diameter distributions, in which parameter b is nearly the same, and c is greatly different, could be evaluated exactly. Furthermore, in the case of the comparison of diameter distributions in which parameter b is different, it could be shown that the difference in the scatter of diameters could not be evaluated exactly using only parameter b.

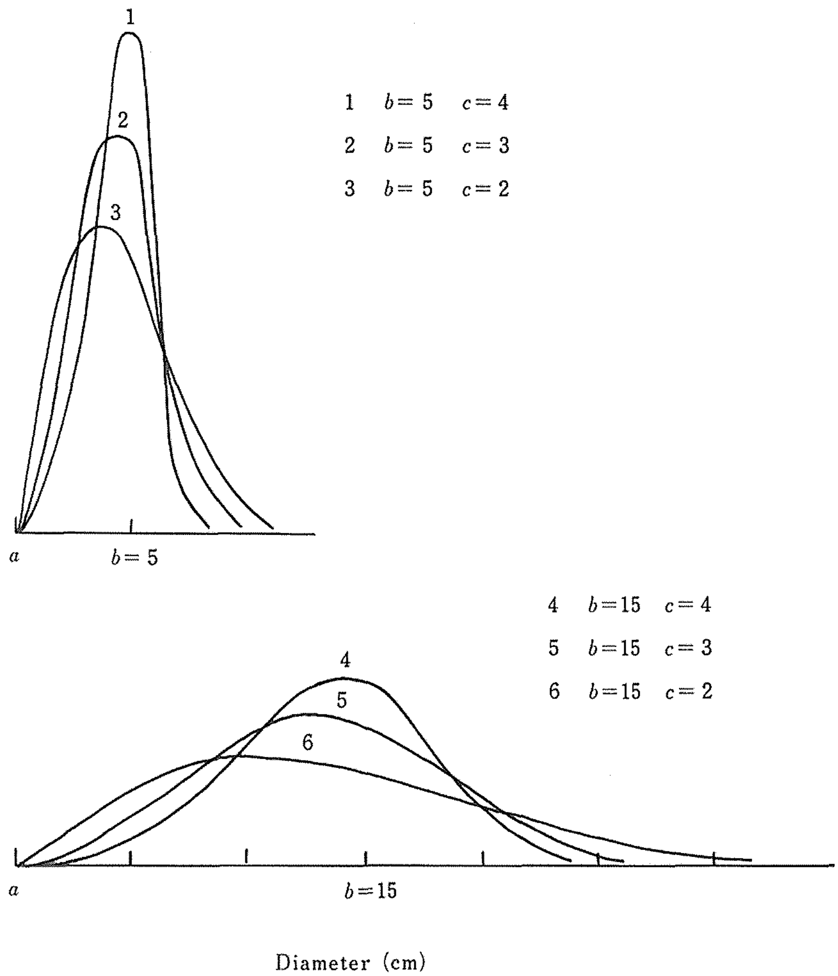

Fig. 1 Diameter distributions corresponding to parameters b and c.

Data obtained from the experimental forests established by the Kyushu University Forests were used in this study.

4. Results

4.1. A simple calculating method for standard deviation and variation coefficient using the Weibull parameters b and c

As indicated by Eqs. (2) and (3), the calculation of standard deviation and mean diameter is very complicated. Consequently, it is necessary to devise a simple calculating method. Thus, the Table of Γ_1 and $(\Gamma_2-\Gamma_1^2)^{1/2}$ corresponding to parameter c was constructed (See Table 1).

Table 1 was used in the following way. Where $a=10$, $b=10$ and $c=3$, the values of Γ_1 and $(\Gamma_2-\Gamma_1^2)^{1/2}$ corresponding to $c=3$ are 0.89298 and 0.32456 from Table 1, respectively. Hence, the following values were obtained.

$$\sqrt{V(x)} = 10\times 0.32456 = 3.2456$$
$$E(x) = 10+10\times 0.89298 = 18.9298$$
$$CV = 3.2456/18.9298 = 0.17$$

As demonstrated in the above example, standard deviation and variation coefficient could be calculated very easily using Table 1.

Table 1 Γ_1 and $(\Gamma_2-\Gamma_1^2)^{1/2}$ corresponding to parameter c.

c	Γ_1	$(\Gamma_2-\Gamma_1^2)^{1/2}$	c	Γ_1	$(\Gamma_2-\Gamma_1^2)^{1/2}$
1.00	1.00000	1.00000	3.00	0.89298	0.32456
1.05	.98079	.93440	3.05	.89364	.32001
1.10	.96491	.87827	3.10	.89430	.31562
1.15	.95170	.82971	3.15	.89498	.31134
1.20	.94066	.78724	3.20	.89565	.30722
1.25	.93138	.74977	3.25	.89633	.30320
1.30	.92358	.71643	3.30	.89702	.29927
1.35	.91699	.68657	3.35	.89770	.29549
1.40	.91142	.65965	3.40	.89838	.29181
1.45	.90672	.63522	3.45	.89907	.28821
1.50	.90275	.61293	3.50	.89975	.28473
1.55	.89939	.59253	3.55	.90043	.28133
1.60	.89657	.57373	3.60	.90111	.27801
1.65	.89241	.55924	3.65	.90178	.27480
1.70	.89224	.54025	3.70	.90245	.27165
1.75	.89602	.51596	3.75	.90312	.26858
1.80	.88929	.51122	3.80	.90379	.26556
1.85	.88821	.49811	3.85	.90445	.26264
1.90	.88736	.48580	3.90	.90510	.25982
1.95	.88671	.47419	3.95	.90576	.25700
2.00	.88623	.46325	4.00	.90640	.25430
2.05	.88589	.45291	4.05	.90704	.25235
2.10	.88569	.44311	4.10	.90768	.24901
2.15	.88561	.43379	4.15	.90831	.24648
2.20	.88562	.42496	4.20	.90894	.24396
2.25	.88573	.41652	4.25	.90956	.24154
2.30	.88591	.40849	4.30	.91017	.23917
2.35	.88617	.40049	4.35	.91078	.23681
2.40	.88648	.39346	4.40	.91138	.23454
2.45	.88685	.38642	4.45	.91198	.23230
2.50	.88726	.37967	4.50	.91257	.23010
2.55	.88772	.37318	4.55	.91316	.22792
2.60	.88821	.36697	4.60	.91374	.22581
2.65	.88873	.36097	4.65	.91431	.22376
2.70	.88928	.35521	4.70	.91488	.22173
2.75	.88986	.34963	4.75	.91544	.21975
2.80	.89045	.34427	4.80	.91600	.21754
2.85	.89106	.33909	5.00	.91817	.21030
2.90	.89169	.33408	6.00	.92772	.17977
2.95	.89233	.32924	7.00	.93544	.15717

4.2. Examples of application

4.2.1. Difference in the scatter of diameters between 2 stands

Table 2 shows the Weibull parameters and stand factors in 2 young Sugi (*Cryptomeria japonica* D. Don) experimental forests (plot 1 and 2) (Kakihara and Kinashi, 1986a). In terms of only parameter b, the scatter of diameters in plot 1 is considered to be a little smaller than in plot 2 (See Table 2). However, as shown in Table 3, the scatter of the former is larger than of the latter. This can be confirmed easily from the fact that standard deviations and variation coefficients of plot 1 are larger than of plot 2 (See Table 2). This is due to difference of parameter c in both plots. Therefore, when some diameter distributions, in which parameter b is nearly the same and c is greatly different, are compared, it is necessary to calculate standard deviation and variation coefficient using Table 1.

Table 2 Comparison of the Weibull parameters and stand factors between 2 plots.
(Species : Sugi, Age : 28 years, Area : 0.06 ha)

Plot	a	b	c	s (cm)	$\bar{d}$ (cm)	cv
1	13	7.1	2.0	3.3	19.3	0.17
2	13	7.7	3.0	2.5	19.9	0.13

a, b and c : Weibull parameters, s : standard deviation,
$\bar{d}$: mean diameter, cv : variation coefficient

Table 3 Comparison of number of trees for each diameter class between 2 plots.

Plot	DBH (cm)								
	14	16	18	20	22	24	26	28	Total
1	6	12	16	15	10	6	3	1	69
2	1	8	18	23	17	7	1		75

4.2.2. Change in the scatter of diameters by thinning

Tables 4 and 5 show the results of thinning in Kuromatu (*Pinus thunbergii* Parl.) experimental forest (Kakihara and Kinashi, 1985). In this forest, the thinning method for large diameter trees (Kakihara, 1987) was carried out. Parameter b changed from 3.4 to 3.3 as a result of thinning. However, since parameter c increased from 2.0 to 3.6, the scatter of diameters decreased greatly as shown in Table 5. This can be confirmed from the fact that standard deviatation and variation coefficient decreased greatly from 1.6 to 0.9 and from 0.29 to 0.16 due to thinning, respectively. The above results indicate that the change in the scatter of diameters could not be evaluated using only parameter b.

Table 4 Change in the Weibull parameters and stand factors by thinning.
(Species : Kuromatsu, Age : 13 years, Area : 0.04 ha)

	a	b	c	s (cm)	$\bar{d}$ (cm)	cv
Before thinning	2.5	3.4	2.0	1.6	5.5	0.29
After thinning	2.5	3.3	3.6	0.9	5.5	0.16

See below Table 2

Table 5 Change in number of trees for each diameter class by thinning.

DBH (cm)	3	4	5	6	7	8	9	10	11	12	Total
N(1)	16	38	41	38	25	11	4	2		1	176
N(2)	15	25	5	1	13	10	4	2		1	76
N(3)	1	13	36	37	12	1					100

N(1) : Number of trees before thinning
N(2) : Number of thinned trees
N(3) : Number of trees after thinning

4.2.3. Change in diameter distribution with age

Stand composition at ages 19 and 41 years in the experimental forest for Karamatsu (*Larix leptolepis* Goldon) (kakihara, 1979) is shown in Tables 6 and 7. In this forest, the normal thinning method (Kakihara, 1987) was carried out when trees were 24 and 27 years of age. Parameter b decreased from 9.1 to 8.8 after 22 years. Considering only b, it is considered that the scatter of diameters decreased. However, since parameter c decreased from 3.3 to 1.9, the scatter of diameters increased (See Table 7). This is confirmed from the fact that standard deviation increased from 2.7 to 4.3 with increase in age. However, since diameter growth in this experimental forest over 22 years was very good, variation coefficient showed almost no change (See Table 6). This example indicates that the difference of diameter distributions, in which parameter b is nearly the same and c is greatly different, must be evaluated using standard deviation and variation coefficient in place of parameter b.

4.2.4. Comparison of the scatter of diameters between 2 stands in which parameter b is different

Table 8 shows the Weibull parameters and stand factors in 2 Sugi (*Cryptomeria japonoca* D. Don) experimental forests (plot 3 and 4) (Kakihara and Kinashi, 1986a). Parameter b in plot 4 is larger than in plot 3. Therefore, considering only b, the scatter of diameters in plot 4 is considered to be larger than in plot 3. However, as shown in Table 9, the range of diameters in both plots is the same. This is due to the difference in parameter c in both plots (See Table 8). Standard deviation in plot 4 is a little smaller than in plot 3 and the variation coefficient in both plots is nearly the same (See Table 8).

Table 6 Change in the Weibull parameters and stand factors with age. (Species : Karamatsu, Area : 0.08 ha)

Age (Years)	a	b	c	s (cm)	$\bar{d}$ (cm)	cv
19	7	9.1	3.3	2.7	15.2	0.18
41	17	8.8	1.9	4.3	24.8	0.17

See below Table 2

Table 7 Change in number of trees for each diameter class with age.

Age : 19 years											
DBH (cm)	8	10	12	14	16	18	20	22			Total
Number of trees	1	6	18	42	34	24	9	2			136

Age : 41 years											
DBH (cm)	18	20	22	24	26	28	30	32	34	36	Total
Number of trees	2	10	12	8	13	3	5	3	2	1	59

Table 8 Comparison of the Weibull parameters and stand factors between 2 stands in which parameter b is different. (Species : Sugi, Age : 28 years, Area : 0.06 ha)

Plot	a	b	c	s (cm)	$\bar{d}$ (cm)	cv
3	23	6.5	1.6	3.7	28.8	0.13
4	13	11.2	3.6	3.1	23.0	0.14

See below Table 2

Table 9 Comparison of number of trees for each diameter class between 2 stands in which parameter b is different.

Plot 3									
DBH (cm)	24	26	28	30	32	34	36	38	Total
Number of trees	5	8	7	6	4	3	1	1	35

Plot 4									
DBH (cm)	16	18	20	22	24	26	28	30	Total
Number of trees	2	6	13	19	19	13	6	2	80

Therefore, in the case of comparison of diameter distributions in which parameter b is different, it is necessary to evaluate the difference in the scatter of diameters using standard deviation and variation coefficient.

5. Discussion

Since the Weibull distribution well fits the diameter distribution of plantations, it is possible to explain the characteristics of diameter distribution using the Weibull parameters a, b and c. The fact that characteristics of the diameter distribution can be explained without utilizing figures or tables is one of the merits of the Weibull distribution. However, the difference in the scatter of diameter distributions, in which parameter b is nearly the same and c is greatly different, could not be completely understood using only parameter b. To solve this problem, a simple calculating method of standard deviation and variation coefficient using parameters b and c was devised and this method was shown to be very effective.

As shown in Fig. 1, the scatter of diameters increases with an increase in parameter b. However, this is a general tendency. Since standard deviation of the Weibull distribution is closely connected with parameters b and c, in the case of the comparison of some diameter distributions in which the values of parameter b are different, it is dangerous to evaluate the difference in the scatter using only b. For a more precise evaluation, as is obvious from Tables 8 and 9, the difference in the scatter of diameters must be examined using standard deviation and variation coefficient.

The scatter of the diameters in plantations with homogeneous diameters is small and with varing diameters large. Thus, from the viewpoint of forest management technology, it is better to ascertain the degree of scatter in diameters than the minimum stand diameter (parameter a) or the shape of the distribution (parameter c). Therefore, a simple calculating method of standard deviation and variation coefficient of diameters proposed in this study should be utilized effectively in study of forest management technology of plantations using the Weibull distribution in the future.

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* The titles are tentative translation from Japanese title by the author of this paper.

(*Received 31 October 1994* ; *Accepted 27 December 1994*)

人工林の直径分布を表現するワイブルパラメータに関する一考察

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要 約

人工林の直径分布をワイブルパラメータ a，b，c のみで表現するさい，直径のバラツキが，尺度のパラメータ b のみで正確に判断できない欠点があった．本研究は，この欠点を解決するために，先ず，パラメータ b，c を用いて直径の標準偏差，変動係数を簡単に計算する方法を提案した．次に，本方法により計算した標準偏差，変動係数を比較すれば，パラメータ b がほぼ同じで c が著しく異なる直径分布のバラツキの違いが正確に判断できることを，三つの応用例を示すことにより明らかにした．さらに，パラメータ b が異なる直径分布を比較するさいにも，一つの応用例から，直径のバラツキの差は，標準偏差，変動係数を用いて検討しなければならないことを指摘した．森林施業の立場からみると，直径のバラツキの程度を知ることは重要である．したがって，今回提案されたパラメータ b，c を用いた直径の標準偏差，変動係数の簡単な計算法は，今後のワイブル分布を用いた人工林の施業技術の研究に有効に利用できるものと考えられる．

キーワード：人工林，直径分布，ワイブル分布，ワイブルパラメータ