Modeling and Analysis of Artificial Auction Markets Consisting of Multi-agents using Genetic Programming for Learning and its Applications

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Modeling and Analysis of Artificial Auction Markets Consisting of Multi-agents using Genetic Programming for Learning and its Applications

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1 Introduction

Auction mechanisms have been attracting increasing attentions in recent years motivated by selling systems over the Internet as well as conventional ones [1, 3]. Most of theoretical researches in auction theory assume that bidders will be making competitive bids, and these bidders are symmetrical in size and are risk neutral. However, many real auctions involve various complicated trades such as an oligopoly of asymmetric bidders who repeatedly meet and bid for the same commodity[4, 5]. Then, theoretical analyses become very hard to find the different rules which govern the auction process and affect the market price. Then, simulation studies are expected to bring us novel results.

In this paper, we show modeling and analysis of artificial auction markets consisting of multi-agents who learn from past experiences based on the Genetic Programming (GP) and its applications [6]. By assuming multi-agents as bidders who learn from past results of auctions based on the GP, we can analyze the capability to learn successful auctions by agents, and the change of profit of agents in various conditions of auctions in a range of multi-unit, multi-period auction settings.

The GP method has been successfully applied to the estimation of chaotic dynamics using the observed time series, and a direct control method for chaotic dynamics is proposed based on the GP [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Moreover, the GP method has been widely used to emulate the agents’ behavior in various markets such as the stock market[11, 12]. In this paper, we utilize the GP method to model agents’ behavior in auction markets.

In this paper, we consider two types of auctions, namely, the sealed-bid auction and English auction. Even though the systems of two auctions are different, we can apply the same GP procedure to model learning of agents. Each agent has a pool of individuals represented in tree structures to predict future bid price using the past result of auctions. The fitness of individuals are defined by using the successful bid depending on their private preference, and agents improve their individuals based on the fitness to get higher return in coming auctions.
In the simulation studies, the effects of parameters such as the private evaluation function are discussed. The change of return of bidders is examined depending on the weight between the average profit obtained by successful bids and the number of successful bids. As a result, we can see the relation between the weight and the structure of bidding such as profit of bidders. Then, we extend the system to auctions including multi-sellers, and we observe an increase in amplitude of the contract price.

In the following discussion, in Section 2, we describe the system configuration. Section 3 and Section 4 show the overview of agents’ behavior in the sealed-bid and English auctions, respectively. In Section 5, we treat the simulation studies of two types of auctions by showing main results. Section 6 shows the extension of the system to multi-seller cases.

2 System configuration

2.1 Two types of auction models

We assume well-known two types of auction models in the following [1, 2, 3]. The first one is the sealed-bid auction where bidders can exhibit price for successful bid (bid price) only once, and they cannot know prices of other bidders. The auction model is employed in many bidding of construction of public utilities. In the scheme, a bidder who exhibits the highest bid price can win the bid.

The other type of auction is the English auction where the auction is carried out in real time (sometime the auction model is called as online auction), and bidder can know current highest price of bidding, and they can exhibit bid price repeatedly. Usually, a bidder exhibiting the highest bid price can win the bid, but it is also assumed that a bidder exhibiting second highest bid price can win the bid (called the Vickrey auction).

Originally, there are three types of agents in the artificial auction market, namely, bidders who wish to suppress the bid price as low as possible, sellers who wish higher bid price, and the auctioneer who manages the auction. For simplicity, we assume that the seller and the auctioneer is the same agent in the following. We also assume at the first stage that only one seller agent shows only one kind of item (commodity) in the market, and is traded by many bidders (agents).

2.2 Agents’ learning using the GP

In the following, we assume that agents learn from past results of auctions to find appropriate bid price (actions) for future auction based on the GP. The GP is an extension of the conventional GA in which each individual in the population (pool of individuals) is a computer program composed of the arithmetic operations, standard mathematical expressions and variables [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

In the GP, the system equations (action rules of the auction) are represented in the tree structure (called individuals). In the parse tree, non-terminal nodes are taken from some well-defined functions such as binomial operation $+, -, \times, \div$, and if-then etc.
Terminal nodes consist of arguments chosen from set of constant and variable such as \( CP(t - 1) \) which is the time lag of contract price of auction. We calculate a profit and a rate of successful bids of the auction obtained by an action of individual, and use it to define the fitness of the individual.

By using the measure of fitness to evaluate each individual, we apply the GP to the population to derive better description for future auctions promising higher profit or rate of successful bids. By selecting a pair of individuals having higher fitness, the crossover operation is applied to generate new individuals. Two subtrees from a pair of individuals are extracted and swapped each other.

To keep the crossover operation always producing syntactically and semantically valid programs, we look for the nodes which can be a subtree in the crossover operation and check for no violation. The crossover operation creates new offsprings by exchanging sub-trees between two parents.

Besides the crossover operations, we use the mutation operations. The goal of the mutation operation is the reintroduction of some diversity in an population. Two types of mutation operation in GP is utilized to replace a part of the tree by another element. (Global mutation :G-mutation)

Generate a individual \( I \), and select a subtree which satisfies the consistency of representation. Then, select at random a subtree in the individual \( J \) in the pool, and replace the subtree by the subtree of the individual \( I \). After the mutation, we only retain the modified individual \( J \) in the pool.

(Local mutation: L-mutation)

Select at random a locus in a parse tree \( J \) to which the mutation is applied, we replace the place by another value (a primitive function or a variable).

We iteratively perform the following steps until the termination criterion has been satisfied.

(Step 1)

Generate an initial population of random composition of possible functions and terminals for the problem at hand. The random tree must be syntactically correct program.

(Step 2)

Evaluate each individual (evaluation of system equation or action program) in the population, then, assign it a fitness value. Then, sort the individuals according to the fitness \( S_i \).

(Step 3)

Select a pair of individuals chosen with a probability \( p_i \) based on the fitness. The probability \( p_i \) is defined for \( i \)th individual as follows.

\[
p_i = \frac{(S_i - S_{\min})}{\sum_{i}^{N}(S_i - S_{\min})},
\]

where \( S_{\min} \) is the minimum value of \( S_i \), and \( N \) is the population size.

(Step 4)

Then, create new individuals (offspring) from the selected pair by genetically recombining randomly chosen parts of two existing individuals using the crossover operation.
applied at a randomly chosen crossover point.

Iterate the procedure several times, and we gather sufficient number of new offsprings necessary for the replacement of individuals.

(Step 5)
At a certain probability, we apply the mutation operations for the pool of individuals.
If the result designation is obtained by the GP (the maximum value of the fitness become larger than the prescribed value), then terminate the algorithm, otherwise go to Step 2.

2.3 GP learning of agents
Each agent has a pool of individuals each of which corresponds to the estimation of appropriate bidding value (or action) for the next time auction. The individuals are represented by using arithmetic operators, comparative operators, and the observation of past auction results. Since the ability (called fitness) of each individual can be evaluated after the bidding is realized (ended), agents can improve the estimation of individuals by applying the GP operations (crossover and mutation) to the pool of individuals.

It is assumed that the first $N_1$ times of auctions are used for learning for agents, and no commodity is delivered to bidder, and seller gets no money. In this learning period, each agent try to improve bidding value definition functions or bidding actions by using the GP procedure. Then, in successive $N_2$ times of bidding, agents action is determind by individuals of GP. After $N_2$ times of auctions, the profit and the rate of successful bids of each agent are determined.

3 Model of sealed-bid auction

3.1 Behavior of agents
We concisely summarize the way of agents’ learning in sealed-bid auctions. It is assumed that each bidder agent has its own pool of functions (individuals) for deciding bid price. To simplify the simulation in reasoning of agents, we restricted ourselves to the cases where the functions can be represented in binary tree structures. But, the restriction has no serious effect on generalization of the method.

For example, an agent has following function including if-then rules.

if (vi>72) then 1.2 P1-0.1 else 0.9 MAX

Fig.1 shows the corresponding tree structure of function. In this case, the agent exhibits bid price as 1.2 P1-0.1 if the condition $v_i > 72$ is satisfied, otherwise exhibits bid price as 0.9 MAX. We also show general form of functions in Fig.2 using symbols R, M and T which mean the root node, intermediate node and terminal node, respectively. The function includes various terminal symbols as well as constants. At first, we introduce the private evaluation $v_i$ for $i$th agent as the terminal symbol. Each bidder agent $i$ reacts to the commodity offered by seller, and assigns a value $v_i$ representing the private
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Figure 1: Example of function evaluation (preference) of the commodity. The function to determine bid price at time $t$ can have also the symbols $P_1 = CP(t - 1)$ where $CP(t - 1)$ is the contract price in previous auction. The function has also symbols $AV$, $MAX$ and $MIN$ defined by taking the average, maximum and minimum of successful bid prices in previous $t_1, t_2, t_3$ time periods of auctions, respectively.

$$AV = \frac{\sum_{j=1}^{t_1} CP(t - j)}{t_1},$$

$$MAX = \max[CP(t - t_2 - 1), CP(t - t_2), ..., CP(t - 1)],$$

$$MIN = \min[CP(t - t_3 - 1), CP(t - t_3), ..., CP(t - 1)].$$

The nodes of tree structure are composed of arithmetic operators $+, -, \times, \div$, and comparative operators $=, \neq, <, >, \geq, \leq$.

### 3.2 Fitness of individuals

The ability of individuals corresponding to the functions is defined as the fitness in the GP. As the first ability measure, we use following value based on the average profit of successful bids.

$$p_{r_{ik}} = \frac{\sum_j [CP(j) - v_i(j)]}{N_{ik}^w},$$

where $v_i(j)$ is the $i$th agent's private evaluation of bidding in $j$th auction, and $N_{ik}^w$ is the number of successful bid obtained by using $k$th individual in the pool of GP. Then, the numerator of equation (5) corresponds to average profit obtained by successful biddings.

We also employ the second evaluation measure for fitness in $k$th individual for $i$th agent as follows.

$$r_{ik} = \frac{sb_{ik}}{N_{ik}^w},$$

where $sb_{ik}$ means the number of successful bid, and $N_{ik}^w$ means the number of time where the agent uses the $k$th individual.
Finally, by changing the weight $\omega_i$ between $pr_{ik}$ and $r_{ik}$, we have aggregated fitness measure for $k$th individual as follows.

$$ s_{ik} = \omega_i (pr_{ik} - \min_j pr_{ij})/R_{pr} + (1 - \omega_i)(r_{ik} - \min_j r_{ij})/R_r, $$

where $R_{pr}, R_r$ mean the ranges of two measures to normalize the fitness.

### 3.3 GP learning

To improve initial set of individuals, we apply following procedure.

A) Select private evaluation $v_i$. Evaluate $AV,\text{MAX,MIN}$ in equations (2),(3),(4). At initial stage, agents have no result of auction, then we assign random numbers in place of these equations.

B) Select one ($k$th) individual from the pool with the probability

$$ P_{ik}^s = s_{ik}/\sum_{j=1}^n s_{ij}. $$

Since under changing environments such as auction market, it is not relevant to use fixed function (prediction) for bid price, then agents are allowed to select an individual in proportion to the fitness values.

C) Seller determines the successful bid $CP(t)$ at time $t$ by observing bid prices given by bidders.

D) Re-evaluate fitness of individuals using current $CP(t)$ and equations (5),(6),(7).

E) Iterate procedures from A) through D) for sufficient times, and then apply following GP.

F) Apply the GP (crossover and mutation operations). Select a pair of individual with probabilities proportional to equation (8), and then exchange portions of tree structure (two subtrees) which are selected at random. An example of crossover operation is shown in Fig.2. (T:Terminal node, M:Middle node, R:Root node).

In this example, a terminal node of Parent A and a middle node of Parent B are exchanged. We have two offsprings, and to keep the size of pool unchanged, two individuals having lower fitness are replaced by two offsprings.

Besides crossover operations, we use mutation operations with a certain probability by replacing a portion of the tree by another symbol (the Local and Global mutations).

### 4 Model of English auction

#### 4.1 Behavior of agents

Different from sealed-bid auctions, in English auctions agents can exhibit bid price repeatedly at multiple times. It is also assumed that agents are allowed to wait for another
exhibition of bid price in the auction as well as to join the auction. Then, the agents’ behaviors are described by programs rather than functions. We also use tree structures to represent these programs, but their terminal nodes include action part of rules, and on their intermediate nodes if-then-else type rules are placed. As the result of rules, agents take one of two actions, namely, ”wait” (no action) and ”join” (exhibit bid price).

In case of ”join”, the agent must determine the bid price. Then, we assume that the agent use one of following two methods for decision.

1. incremental price
   By adding price increment inc to the present price s by several times, then $s + m \times inc$ will be the bid price.

2. random selection of multiple
   Assuming set $[b_1, b_2, ..., b_l] = [1.1, 1.2, ..., 2.0]$, then the agents select one of these numbers to obtain bid price as $s \times b_i$.

The if-then-else type rules treated here are the same as used in the sealed-bid auction, but in place of terminal node we use ”wait” or ”join”.

4.2 Interpretation of individuals

The interpretation of trees (individuals) is slightly complicated. For example, in a tree structure in Fig.3, we start if-node at the root node. If the current bid price $s$ is 80, and the condition is true, then we go to left branch and meet ”div” node which means we go further to left branch. Then, we meet if-node, and depending on the condition, we choose whether left branch of right branch. These two branches are denoting ”join” showing the bid prices, and the action taken by the agent is terminated in this step. In this example, if $s < v_i$, then the agent exhibits $1.2s$, otherwise $2.0s$.

In the next time period of the auction, the agent goes backward to ”div” node, and restarts the action (go to right branch). Since the node is ”wait”, then the agent takes
Figure 3: Individual structure in English auction

no action for bidding at this time period. Further, in the next time period, the agent goes back to root node again, and select appropriate action. The agent repeat these behavior until the end of underlying auction.

4.3 GP learning

The GP learning of agents in English auctions are similar to cases in sealed-bid auctions, then we concisely summarize the procedure.
A. Selection of private evaluation \( v_i \) and evaluate \( AV_{MAX}, MIN \).
B. Select one \((kth)\) individual from the pool proportional to fitness.
C-1. Throughout one auction from time 1 through \( T_E \), an agent use a tree of an individual repeatedly. If the agent reach the terminal node, then decide "wait" or "join". In case of "join", the agents exhibit bid price by considering current bid price.
C-2. At the end of one auction, Seller determines the contract price \( CP(t) \).
D. Re-evaluate fitness of individuals using current \( CP(t) \) and equations (5),(6),(7).
E. Iterate procedures from A through D for sufficient times, and then apply following GP.
F. Since an agents use only one individual for underlying auction, then we assume that the agent iterate the same evaluation procedure for other individuals in the pool. Then, apply the GP (crossover and mutation operations).

5 Applications

5.1 Changes of profit in sealed-bid auctions

In the following simulation studies, we examine the ability of the GP method to realize artificial auction market by changing parameters of the system. At first, we show the result for artificial auction market of the sealed-bid auctions.

The parameters for simulation studies are selected as follows.
\[ t_1 = t_2 = t_3 = 3, \]
Number of bidder agents: 10
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\[ N_1 = 500000 \text{ (apply GP for 1000 iterations)} \]
\[ N_2 = 50000, \text{ upper limit of bid price}=150 \]
Number of individuals for each pool=100
Maximum number of nodes in trees=50
Probability of crossover:0.05
Probability of each mutation:0.03

Originally, as probabilities for selecting terminal nodes as arithmetic operations, if-then-else condition, constants and variables we assign 0.4, 0.1, 0.25, 0.25, respectively. However, if it is necessary to generate intermediate nodes, then the probabilities for arithmetic operators and if-then-else conditions are changed to \(0.4/(0.4+0.1), 0.1/(0.4+0.1)\), respectively. The constants used for if-then-else conditions and terminal nodes range from 0 \(\sim\) 150 and \(-150 \sim 150\), respectively.

We assume three cases for the definition of private evaluations \(v_i\) as follows.

(Case 1) identical: each agent has the same \(v_i = 100\)
(Case 2) uniformly distributed: select one \(v_i\) from set \((90, 91, \ldots, 110)\) at each iteration of auctions.
(Case 3) piecewise constant: assign two values depending on agent, such as \(V = (100, 100, \ldots, 100, 90, 90, \ldots, 90)\). Namely, five agents take 100 as \(v_i\), and other five agents take 90 as \(v_i\).

Table 1 shows the result for average profit of bidders depending on the private evaluations. In Table 1, \(Prf_1, Prf_2, Prf_3\) mean average profit of agents in Case 1, 2 and 3 defined as the value obtained by final bid price minus the private evaluation. In Table 1, \(Prc_2\) and \(Prc_3\) mean the bid price in Case 2 and 3.

As is seen from the result, in Case A if \(\omega = 1\), \(Prf_1=0\), then every bid prices greater than \(v_i\) are smoothly removed from the system, and no bid price greater than 100 is realized. But if \(\omega\) becomes less than 1, agents pay more attentions to the rate of successful bid, and the profit decreases. The fact implies that if there exist many bidder agents who pay more attention to the rate of the successful bid, then the seller can enjoys higher price.

In Case 3, we see also almost the same decrease of profit as Case 1 (then, the average price of bidding almost increases) along the decrease of \(\omega\) form 1. Even though agents use different private evaluation from 90 and 100, final bid price is almost always realized at 100. Then, the average profit of agents using their private evaluation at 90 becomes around -10.00.

In case 2, we find several different features of result. In case of \(\omega = 1\), the average profit of bidder is positive, and the final bid price is slightly greater than lowest limit of \(v_i\) (90). Even more, along the decrease of \(\omega\) from 1, the average profit of bidder becomes negative, but the range of the decrease is relatively small compared to Case 1 and 3. The fact implies that the random behavior of bidder agents help them to get more profit, and affect seller to decrease price. To examine the agents' behavior, in Fig.4 we show the average length of individuals in the pools along the time of learning, and in Fig.5 the average value of fitness of individuals is depicted. As is seen from these figures, two average values bear very little fluctuations, and the learning process...
of agents is seemed to be relatively monotonic. After learning, agents find ultimately their appropriate formulas (predictions) for future bid price.

### 5.2 English auction

The parameters for simulation studies are the same as in sealed-bid auction, and the definitions of Case 1 and 2 are assumed, and Case 3 is not used in English auction. And duration time of auction is $T_E = 200$.

But, following points are changed from previous definition. Probabilities for if-then-else nodes and branching nodes are 0.2, 0.1, respectively. Probabilities for terminal nodes for ”wait” and for ”bid” are 0.6, 0.1, respectively.

Table 2 shows the result for average profit of bidders depending on the private evaluations. As is seen from the result, if we choose $\omega = 1$ the profit of bidder is large compared to sealed-bid auction, while agents know the current bid price and decide to finish bidding earlier. However, if $\omega$ becomes 0.8, 0.7 or 0.6, the profit becomes negative, an the result is similar to the cases of sealed-bid auction. The situation is more favorable to sellers. The fact reflects that from the sellers’ viewpoint, the sufficient where bidder pay no attention to successful bid, and the supply of commodity is large enough.

We also obtain the average length of individuals and the average value of fitness of
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![Graph showing average fitness of individuals](image)

**Figure 5: Average fitness of individuals**

**Table 2: Simulation result (English auction)**

<table>
<thead>
<tr>
<th></th>
<th>$\omega = 1$</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prf1</td>
<td>98.99</td>
<td>17.37</td>
<td>-16.03</td>
<td>-6.15</td>
<td>-47.42</td>
</tr>
<tr>
<td>Prf2</td>
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<td>0.055</td>
<td>-4.38</td>
<td>-6.55</td>
<td>-21.55</td>
</tr>
<tr>
<td>Prc2</td>
<td>1.00</td>
<td>106.08</td>
<td>105.99</td>
<td>106.10</td>
<td>121.18</td>
</tr>
</tbody>
</table>

individuals similar to the sealed-bid auctions, but they are monotonic and each other, and then we omitte the details.

6 Extension to multi-sellers auctions

6.1 Price changes under multi-sellers auctions

In previous discussions, we assumed a single good (item) and a single seller in the auction market. Then, we extend the model to the auction markets with multiple sellers (multi-sellers) which is resemble to ordinary trade markets of merchandises. We can find such kind of multi-sellers auction market in the trade of electricity [20, 21]. For example, under the deregulation of electricity market, many firms join the market to sell the electricity. Then, the conventional customers join the market as buyer (bidders) who search optimal price (lowest price) by showing bid price.

It is observed that most notable characteristics of deregulated electricity market is the remarkable changes (fluctuation) of prices[20, 21]. In the USA electricity market, sometime a sudden and very large rise (impulsive) in bid prices is realized, and the price reaches about 20 times larger than steady prices [20].

In the auction market with multiple sellers, the bidders think about the trading amount (called as the volume in the following) of item as well as the price. If a bidder predicts the shortage of item, then the bidder increases the bid price to successfully get...
the item. On the other hand, if a bidder predicts relatively sufficient volume of item, then the bidder show relatively low bid price, and does not increase the price if possible.

Different from single-seller cases, sellers also think about the volume to obtain better result of auctions [10, 16]. If a seller predicts relatively high bid price, then the seller offer larger volume of item into the market. On the other hand, if a seller predicts relatively lower price, then the seller abandons (stop) to offer item to the market to avoid the loss. As a result, the time series of trading volume and the price of item in the auction market fluctuate [10, 16].

### 6.2 Prediction of sellers

In simulation studies, under multi-seller assumption, sellers are allowed to offer items into the market depending on the prediction of the average price of items in the market.

We assume that seller agents utilize privious average contract price realized in the market. Finally, seller agents predict the average price of items at time $t + 1$ $\pi(t + 1)$ by using exponential smoothing as follows, to avoid sudden change of prediction of bid price.

$$\pi(t + 1) = \theta \pi(t) + (1 - \theta)p(t),$$  \hspace{1cm} (9)

where the weight $\theta$ is $0 \leq \theta \leq 1$.

We assume that each seller can offer one item for one auction. So, items of the same number as sellers are offered at the maximum. We assume that a minimum amount of item (one item) is always offered in the auction market.

Then, we assume that seller agents determine the volume of item to offer in the next auction as follows.

$$I_t(\pi) = \begin{cases} 
\Lambda_l, & (0 \leq \pi_t \leq d) \\
\Lambda_h(\pi - d)/(a - d) + \Lambda_l, & (d < \pi_t < a) \\
\Lambda_h + \Lambda_l, & (a \leq \pi_t)
\end{cases}$$  \hspace{1cm} (10)

The relation means if the low price of the item is predicted ($0 \leq \pi \leq d$), then sellers offer minimum volume ($\Lambda_l$) of item. On the other hand, if the high price of item is predicted in the market ($\pi \geq a$), then sellers decide to supply the item at the maximum volume ($\Lambda_h + \Lambda_l$). If the prediction of item price is predicted to exist between the highest ($a$) and lowest values ($d$), then sellers offer the item in proportion to the strength of predicted price.

### 6.3 Simulation result

Different form ordinary auctions with single seller, we are mainly interested in the change of bid price caused by the agents’ behavior. The conditions for simulation studies are same as simulations in previous sections. We assume $\Lambda_l = 1$, $\Lambda_h = 4$, $a = 110$, $d = 90$, $\omega = 0.9$, $\theta = 0$, the definition of private evaluations is case2, and only sealed-bid auction is used.
Fig. 6 shows an example of change of average contract price along the time after learning process. Figs. 7 and Fig. 8 depict examples of average value of the length of individuals and average value of fitness of individuals, both in the learning process. In the multi-seller auctions, it is assumed that items are traded based on the bidding process among multiple sellers and multiple bidders. Then, the difference of bid prices realized in each trade is usually observed.

As is seen from Fig. 6, the average contract price bears changes and fluctuations. The facts implies us that the offers of item done by sellers induce the reaction of bidders, and as a result, a small change of bid price is enlarged (exaggerate) in the bidding process, and the fluctuations in contract prices are observed. If sellers think the high average price of items in the market, then they intend to offer more item in the next time period. But in the bidding process, the volume of item is large enough, then the price fall to low value. However, at the end of bidding, the sellers think about the average contract price of item in the current bid, then they offer little item in the next auction. As a result, due to the shortage of item, the bid price rises and sometime becomes large.

The unstability of contract price is also explained from Fig. 7. In the learning process, since the differentiations of individuals in the pools is exist, and agents can not find useful and stable method for prediction in the bidding process, the values of average length of individuals show fluctuations.

### 6.4 Dependency on parameters

Now, we examine the rise/fall changes of bid price depending on the parameter $\theta$ which represents the smoothing characteristics of contract price predicted by seller agents. We consider that the parameter $\theta$ corresponds to the degree of response of sellers to the conditions of market. For example, if the value of $\theta$ is relatively large, then seller agents more surely depend on own predictions.

We also examine the effect of parameters $a, d$ in the piece-wise linear function in equation (10) used to determine next offer of items done by seller agents. It is easily seen that two parameters $a, d$ affect to the highest and lowest price of bidding.
By considering these characteristics, we define the changes of contract price as the standard deviation of average contract prices.

Table 3 shows the change depending on the value $\theta$. It is seen that if the value of $\theta$ close to zero, then the standard deviation (denoted as S.D. in Tables) of average contract prices becomes relatively large. On the other hand if the value $\theta$ close to 1, then the standard deviation becomes to be relatively small. The fact implies us that if the seller agents react in the price sensitively, then the contract price of the market reveals a kind of unstability. If all

Table 4 summarizes the result for some cases of combination of parameters $a$ and $d$. If the range $(a - d)$ is the same as range of private evaluations $v_i$ of bidder agents, the volume of item offered into the market becomes unstable, and the fluctuation of the bid price is large. On the contrary, if the range of value $a$ and $d$ is smaller than range of $v_i$, then change of volume of item is small, because contract prices becomes a value besides the range $a$ and $d$ and sellers offer constant number of items. If the range of value $a$ and $d$ is larger than range of $v_i$, then the influence of price to the volume of items is decreases ,and the standard deviation of contract prices becomes small.
Table 3: standard deviation of contract prices about θ

<table>
<thead>
<tr>
<th>θ</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D.</td>
<td>7.775</td>
<td>4.596</td>
<td>4.698</td>
<td>3.860</td>
<td>2.895</td>
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</table>

Table 4: standard deviation of contract prices about a and d

<table>
<thead>
<tr>
<th>(a, d)</th>
<th>(105, 95)</th>
<th>(110, 90)</th>
<th>(115, 85)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.D.</td>
<td>5.66</td>
<td>7.10</td>
<td>2.66</td>
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</table>

7 Conclusion

In this paper, we showed the agent-based simulation of artificial auction markets using the GP where bidder agents learn from past experiences. Simulation studies for two types of auctions were given to show the ability of our method. Then, we extended the system to multi-seller auction, and the change of bid price was shown.

For further works, the extension of the method to various types of auctions, and further works will be done by the authors.

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