ON A METHOD OF SOLVING TORSION AND BENDING PROBLEMS OF CONTINUOUS PANEL STRUCTURES: 2ND REPORT

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ON A METHOD OF SOLVING TORSION AND BENDING PROBLEMS OF CONTINUOUS PANEL STRUCTURES. (2ND REPORT)

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In the 1st report we have treated the general solution of the torsion and bending problems of continuous panel structures under transverse forces, twisting and bending moments, the members of which cross each other perpendicularly and distributions of the external forces acting on the members are quite arbitrary and positions of the supported bars are also arbitrary.

In the present report, we extend the method described in the 1st report to the problems of the structures, the members of which cross each other with angles different from rectangle. That is to say, the rotating angle being regarded as a vector as well known when the value of its angle is small, we can decompose the unknowns, say the twisting and bending moments and the rotating angles occurring at the intersecting points of the members, to the two directions of rectangular axes respectively, the directions of which we take in the directions of the axis of the member and the perpendicular to the member at the each point. By doing so, as to the decomposed quantities stated above, we can treat in the same manner as the method described in the 1st report. Thereupon, similarly as in the previous report, we induce also the simultaneous equations of the first degree referring to the inclinations and deflections at the intersecting points of the members, the number of which is the same as that of the unknown quantities. The simultaneous equations of this type can always be solved by means of the iteration methods practically and, moreover, using these quantities obtained above, all the unknowns in the present problems can be easily found from the formulae of bending as well known in the 1st report.

1. Introduction. In this report as mentioned above, we treat the general solution of the torsion and bending problems of the continuous panel structures under transverse forces, twisting and bending moments, the members of which cross each other with angles different from right angle and the distributions of the external forces acting on the members are quite arbitrary and positions of the supported bars are also arbitrary. First, under the assumptions that the deflections and the twisting angles of the members are small, we decompose all the unknowns, say the twisting and bending moments and the rotating angles experienced at the intersecting points of the members, to the two directions of the rectangular axes, the directions of which we take in the directions of the axis of the member and the perpendicular to the member at the each point. Thereupon, similarly as in the 1st report, considering the conditions of continuity of the inclinations, equilibriums of the twisting and bending moments, and equilibriums of the shearing forces at the intersecting points of the members as to the decomposed quantities stated above, the present problems reduce to the simultaneous equations of the first degree referring to the twisting and bending moments and the deflections at the intersecting points of the members and the number of the simultaneous equations is the same as that of unknown quantities and these equations suffice to define one set of the unknown quantities and, moreover, finding the solution of the equations, the problems are solved. But, for solving the simultaneous equations practically, we need the iterations methods as well known. Then, for the convenience of practical calculations, we induce the type of the simultaneous equations of the 1st degree referring to the inclinations and the deflections at the intersecting points.

2. Notations. First, let us show the notations used in the present report altogether for the convenience of explanation.

$F, M, T, i, \varphi, y.$: Shearing Force, Bending Moment, Twisting Moment, Inclination, Twisting angle, Deflection.

$E, C, I.$: Young's Modulus, Torsional Constant, Moment of Inertia.

mark ($\rightarrow$): Direction of the outward-drawn normal of a cross-section.

suffix (0): The value at the starting point.

suffix (ε): Distance apart infinitesimally small quantity from the intersecting point.

suffix (s): The suffix s attached on the left side relates to the two cross members (1, 2), and the suffix s on the right side stands for the numbers (0, 1, 2 and 3).

$P, M's, R.$: External force, External Bending Moment, Reaction.

$l, A, h.$: Length of a span, Cross-sectional area, Length of a supported bar.

$\theta_{n\cdot r}, L_{n\cdot r}, N_{n\cdot r}$: Angle different from right angle at an intersecting point (n, r) of the members, Cosine and Sine of the angle $\theta_{n\cdot r}$.

$v(x), M(x)$: Accumulation functions of exteral forces and bending moments.

$$V_s(x) = \alpha_s(x) - s\,\beta_{s-1}(x), \quad \alpha_s(x) = \int_0^x (x-t)^s\, dv(t),$$

$$\beta_s(x) = \int_0^x (x-t)^s\, dM(t),$$

$$U_s(x) = \gamma_s(x) + s\,\delta_{s-1}(x), \quad \gamma_s(x) = \int_0^x t^s\, dv(t),$$

$$\delta_s(x) = \int_0^x t^s\, dM(t),$$

$\overline{V}_s(l)$, $\overline{U}_s(l)$: $V_s(l)$, $U_s(l)$ without the effects of the external forces including the external bending moments, which act on a member at both the ends.

Hereupon, let us show the relations among the suffix used in all the following notations.

$(s=1:\ q=n+1\cdot r,\ q'=n-1\cdot r)$, $(s=2:\ q=n\cdot r+1,\ q'=n\cdot r-1)$

and in such cases when p joins the above suffix

$(s=1:\ p=2,\ q=n\cdot r+1,\ \ q'=n\cdot r-1)$,

$(s=2:\ p=1,\ q=n+1\cdot r,\ q'=n-1\cdot r)$.

$$\Delta y_1=(y_1-y_0)/l_1,\quad \Delta y_1'=(y_1-y_2)/l_2,\quad {}_s\Delta y_{n\cdot r}=(y_{n\cdot r}-y_{q'})/{}_sl_{n\cdot r},$$

$${}_s\Delta\,{}_si_{n\cdot r}={}_s\Delta y_{n\cdot r}-{}_s\Delta y_{q'},\quad {}_s\Delta\,{}_si'_{n\cdot r}={}_s\Delta y_{n\cdot r}-{}_s\Delta y_q,$$

$${}_s(\Delta',\Delta'')\,{}_s[i,B,Q]_{n\cdot r}=(T_g\,\theta,\ \sec\theta)_{n\cdot r}\,{}_s[i,B,Q]_{n\cdot r}-(T_q\,\theta,\ \sec\theta)_{q's}[i,B,Q]_{q'},$$

$$B_1=\mu_1(2M_{\underset{1-\varepsilon}{\longrightarrow}}-M_{\underset{\varepsilon}{\leftarrow}}),\quad B'_1=\mu_2\{(2M_{\underset{1+\varepsilon}{\longrightarrow}}-M_{\underset{2-\varepsilon}{\longleftarrow}})+(2M_{1+\varepsilon}-M_{2-\varepsilon})\},$$

$${}_1B_{n\cdot r}={}_1\mu_{n\cdot r}(2\,{}_1M_{\underset{n-\varepsilon\cdot r}{\longrightarrow}}-{}_1M_{\underset{n-1+\varepsilon\cdot r}{\longleftarrow}}),\quad {}_2B_{n\cdot r}={}_2\mu_{n\cdot r}(2\,{}_2M_{\underset{n\cdot r-\varepsilon}{\longrightarrow}}-{}_2M_{\underset{n\cdot r-1+\varepsilon}{\longleftarrow}}),$$

$${}_1B'_{n\cdot r}={}_1\mu_{n+1\cdot r}\{2\,{}_1M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}-{}_1M_{\underset{n+1-\varepsilon\cdot r}{\longleftarrow}}+(2\,{}_1M_{n+\varepsilon\cdot r}-{}_1M_{n+1-\varepsilon\cdot r})\},$$

$${}_2B'_{n\cdot r}={}_2\mu_{n\cdot r+1}\{2\,{}_2M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}-{}_2M_{\underset{n\cdot r+1-\varepsilon}{\longleftarrow}}+(2\,{}_2M_{n\cdot r+\varepsilon}-{}_2M_{n\cdot r+1-\varepsilon})\},$$

$$Q_1=\mu_1\left\{-2V_1(l_1)+\frac{3}{l_1}V_2(l_1)-\frac{1}{l^2}V_3(l)\right\},$$

$$Q_1'=\mu_2\left\{-2U_1(l_2)+\frac{1}{l_2}U_2(l_2)-\frac{1}{l_2^{\,2}}U_3(l_2)\right\},$$

$${}_sQ_{n\cdot r}={}_s\left[\mu\left\{-2V_1(l)+\frac{3}{l}V_2(l)-\frac{1}{l^2}V_3(l)\right\}\right]_{n\cdot r},\quad \mu=l/6EI,\quad \lambda=l/GC,$$

$${}_sQ'_{n\cdot r}={}_s\left[\mu_q\left\{-2U_1(l_q)+\frac{3}{l_q}U_2(l_q)-\frac{1}{l_q^{\,2}}U_3(l_q)\right\}\right],$$

$$\Theta_{n\cdot r}={}_1[\overline{U}_1(l_{n\cdot r})/l_{n\cdot r}+\overline{V}_1(l_q)/l_q]+{}_2[\overline{U}_1(l_{n\cdot r})/l_{n\cdot r}+\overline{V}_1(l_q)/l_q]+P_{n\cdot r},$$

$${}_s\Gamma_{n\cdot r}={}_s[i_{n\cdot r}+2i_{q'}],\quad {}_s\Gamma_{n\cdot r}'={}_s[2i_{n\cdot r}+i_{q'}],\quad {}_sK_{n\cdot r}={}_s\left[\frac{U_2(l)}{l}-\frac{U_3(l)}{l^2}\right]_{n\cdot r},$$

$${}_s\overline{K}_{n\cdot r}={}_sK_{n\cdot r}+{}_sM_{q''}\quad (s=1:\ q''=n-\varepsilon\cdot r,\ s=2:\ q''=n\cdot r-\varepsilon),$$

$${}_sS_{n\cdot r}={}_s\left[-\frac{V_2(l)}{l}+\frac{V_3(l)}{l^2}\right]_{n\cdot r},$$

$${}_s\overline{S}_{n\cdot r}={}_sS_{n\cdot r}+{}_sM_{q''}\quad (s=1:\ q''=n+\varepsilon\cdot r,\ s=2:\ q''=n\cdot r+\varepsilon),$$

$${}_sK'_{n\cdot r}={}_s\left[3\frac{U_2(l)}{l^2}-2\frac{U_3(l)}{l^3}\right]_{n\cdot r},\quad {}_sS_q'={}_s\left[-3\frac{V_2(l)}{l^2}+2\frac{V_3(l)}{l^3}\right]_q,$$

$${}_sa_1=2\,{}_s\left[\frac{EI}{l}\right]_{n\cdot r},\quad {}_sa_2=2\,{}_s\left[\frac{EI}{l}\right]_q,\quad a'=a/l,\quad a''=a/l^2,$$

$${}_sb_1=\frac{1}{L_{n\cdot r}}\,{}_p\left[\frac{GC}{l}\right]_{n\cdot r},\quad {}_sb_1'=\frac{1}{L_{q'}}\,{}_p\left[\frac{GC}{l}\right]_{n\cdot r},$$

$$ {}_sb_2 = \frac{1}{L_q}\,{}_p\left[\frac{GC}{l}\right]_q, \quad {}_2b_2' = \frac{1}{L_{n\cdot r}}\,{}_p\left[\frac{GC}{l}\right]_q, $$

$$ {}_sc_1 = \left(\frac{N}{L}\right)_{n\cdot r}\,{}_s\left[\frac{GC}{l}\right]_{n\cdot r}, \quad {}_sc_2 = \left(\frac{N}{L}\right)_q\,{}_s\left[\frac{GC}{l}\right]_q, $$

$$ {}_sc_1' = \left(\frac{N}{L}\right)_{q'}\,{}_s\left[\frac{GC}{l}\right]_{n\cdot r}, \quad {}_sc_2' = \left(\frac{N}{L}\right)_{n\cdot r}\,{}_s\left[\frac{GC}{l}\right]_q $$

and so on.

3. Formulae of Bending. First, for the conveniene of explanation, we show the formulae of bending obtained in the Ist report as follow.

The formulae of bending:

$$ \left.\begin{aligned} &-\underset{x}{F_{\rightarrow}} = \underset{0}{F_{\leftarrow}} + V_0(x), \quad \underset{x}{M_{\rightarrow}} = -\underset{0}{M_{\leftarrow}} + \underset{0}{F_{\leftarrow}}x + V_1(x), \\ &EIi = EIi_0 - \underset{0}{M_{\leftarrow}}x + \frac{1}{2}\underset{0}{F_{\leftarrow}}x^2 + \frac{1}{2}V_2(x), \\ &EIy = EI(y_0 + i_0x) - \frac{1}{2}\underset{0}{M_{\leftarrow}}x^2 + \frac{1}{6}\underset{0}{F_{\leftarrow}}x^3 + \frac{1}{6}V_3(x) \end{aligned}\right\} \quad (3.1) $$

and from the Eqs. (3.1), we have

$$ i_1 = \varDelta y_1 + B_1 + Q_1, \quad i' = \varDelta y_1' + B_1' + Q_1' \quad (3.2) $$

and

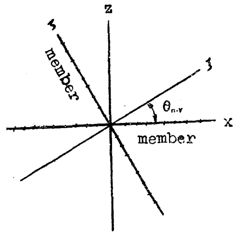
$$ \left.\begin{aligned} \underset{0}{F_{\leftarrow}} &= \frac{6EI}{l_1^3}\{(i_0+i_1)l_1 + 2(y_0-y_1)\} - \frac{3}{l_1^2}V_2(l_1) + \frac{2}{l_1^3}V_3(l_1) \\ \underset{0}{M_{\leftarrow}} &= \frac{2EI}{l_1^2}\{(2i_0+i_1)l_1 + 3(y_0-y_1)\} - \frac{1}{l_1}V_2(l_1) + \frac{1}{l_1^2}V_3(l_1) \\ \underset{1}{F_{\rightarrow}} &= -\frac{6EI}{l_1^3}\{(i_0+i_1)l_1 + 2(y_0-y_1)\} - \frac{3}{l_1^2}U_2(l_1) + \frac{2}{l^3}U_3(l_1) \\ \underset{1}{M_{\rightarrow}} &= \frac{2EI}{l_1^2}\{(i_0+2i_1)l_1 + 3(y_0-y_1)\} + \frac{1}{l_1}U_2(l_1) - \frac{1}{l_1^2}U_3(l_1) \end{aligned}\right\} \quad (3.3_1) $$

or

$$ \left.\begin{aligned} \underset{0}{F_{\leftarrow}} &= \{\underset{1}{M_{\rightarrow}} + \underset{0}{M_{\leftarrow}} - V_1(l_1)\}/l_1, \quad \underset{0}{F_{\rightarrow}} = -\{\underset{1}{M_{\rightarrow}} + \underset{0}{M_{\leftarrow}} - \overline{V}_1(l_1)\}/l_1 \\ \underset{1}{F_{\rightarrow}} &= -\{\underset{1}{M_{\rightarrow}} + \underset{0}{M_{\leftarrow}} + U_1(l_1)\}/l_1, \quad \underset{1}{F_{\leftarrow}} = \{\underset{1}{M_{\rightarrow}} + \underset{0}{M_{\leftarrow}} + \overline{U}_1(l_1)\}/l_1 \end{aligned}\right\} \quad (3.3_2) $$

4. Fundamental principle of the theory. As stated above, the rotating angles, say deflections of the members of the panel structures, being small in the present problems, vector analysis can be applied for its angles as well-known. Then we decompose the unknowns, namely the twisting and bending moments and the rotating angles of the members taking place at the intersecting points of the members, to the two directions of the rectangular axes respectively. If so, as to these decomposed quantities, we are able to treat in the same manner as the method stated in the 1st report.

Thereupon, let us take x- and ζ-axes in the two directions of the members at the intersecting point $(n \cdot r)$ and z- and ξ-axes in the two directions of the perpendiculars to the formers and y axis in the perpendicular to the z-x plane and, moreover, express the angle between x- and ξ-axes by the notations $\theta_{n \cdot r}$.



Fig

If so, we have the following relations between the axes (ξ, ζ) and the axes (x, z) from the formulae of the analytical geometry.

$$\left.\begin{aligned} \xi &= L_1\, x + N_1\, z \\ \zeta &= L_2\, x + N_2\, z = -N_1\, x + L_1\, z \end{aligned}\right\} \quad (4.1),$$

Now, let us first consider the equilibriums of the moments at the intersecting points $(n \cdot r)$.

Putting the twisting and bending moments referring to the ζ axis

$$\left.\begin{aligned} {}_2M_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2M_{\overleftarrow{n\cdot r-\varepsilon}} + {}_2M_{n\cdot r} &\equiv M_\xi \\ {}_2T_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2T_{\overleftarrow{n\cdot r-\varepsilon}} &\equiv T_\zeta , \end{aligned}\right\}$$

we have the following quantities referring to the x- and z-axes.

$$\left.\begin{aligned} M_\xi \cdot L_{n\cdot r} &= M_x , \quad & M_\xi \cdot N_{n\cdot r} &= -M_z \\ T_\zeta \cdot L_{n\cdot r} &= T_z , \quad & T_\zeta \cdot N_{n\cdot r} &= T_x \end{aligned}\right\}$$

in which $L_{n\cdot r}$, $N_{n\cdot r}$ are direct cosines of the ξ-axis for the x-and z-axes. Therefore, from the conditions of the equilibriums of the moments, we have

$$\left.\begin{aligned} &{}_1M_{\overrightarrow{n+\varepsilon\cdot r}} + {}_1M_{\overleftarrow{n-\varepsilon\cdot r}} + L_{n\cdot r}\big({}_2T_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2T_{\overleftarrow{n\cdot r-\varepsilon}}\big) \\ &\qquad - N_{n\cdot r}\big({}_2M_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2M_{\overleftarrow{n\cdot r-\varepsilon}} + {}_2M_{n\cdot r}\big) + {}_1M_{n\cdot r} = 0 \\ &{}_1T_{\overrightarrow{n+\varepsilon\cdot r}} + {}_1T_{\overleftarrow{n-\varepsilon\cdot r}} + N_{n\cdot r}\big({}_2T_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2T_{\overleftarrow{n\cdot r-\varepsilon}}\big) \\ &\qquad + L_{n\cdot r}\big({}_2M_{\overrightarrow{n\cdot r+\varepsilon}} + {}_2M_{\overleftarrow{n\cdot r-\varepsilon}} + {}_2M_{n\cdot r}\big) = 0 . \end{aligned}\right\} \quad (4.2)$$

Similarly, as to the twisting angles and the inclinations of the members at the intersecting points $(n \cdot r)$, we have

$$\left.\begin{aligned} ({}_2\varphi \cdot L)_{n\cdot r} &= (\varphi_{z\cdot 1})_{n\cdot r} , \quad & ({}_2\varphi \cdot N)_{n\cdot r} &= (\varphi_{x\cdot 1})_{n\cdot r} \\ ({}_2i \cdot L)_{n\cdot r} &= (\varphi_{x\cdot 2})_{n\cdot r} , \quad & ({}_2i \cdot N)_{n\cdot r} &= (-\varphi_{z\cdot 2})_{n\cdot r} . \end{aligned}\right\}$$

Therefore, the twisting angle and the inclination referring to the x-member are expressed by

$${}_1\varphi_{n\cdot r} = ({}_2\varphi \cdot N)_{n\cdot r} + ({}_2i \cdot L)_{n\cdot r} , \qquad {}_1i_{n\cdot r} = ({}_2\varphi \cdot L)_{n\cdot r} - ({}_2i \cdot N)_{n\cdot r}$$

and the twisting angles about the x-and ζ-members at the points $(n \cdot r)$ are

$$\left.\begin{aligned} {}_1\varphi_{n\cdot r} &= \frac{1}{L_{n\cdot r}}\big(N \cdot {}_1i + {}_2i\big)_{n\cdot r} \\ {}_2\varphi_{n\cdot r} &= \frac{1}{L_{n\cdot r}}\big({}_1i + N \cdot {}_2i\big)_{n\cdot r} . \end{aligned}\right\} \quad (4.3_1)$$

On the other hand, as to the relations between the inclinations and the moments, we can also take the following equations described in the 1st report without any modifications

$$ {}_s i_{n\cdot r} = {}_s[\varDelta y + B + Q]_{n\cdot r} \,. \tag{4.3$_2$} $$

Then from the Eqs. (4.3$_1$) and (4.3$_2$), the relations between the twisting moments and the twisting angles are expressed by

$$ \left. \begin{aligned} {}_1[\varphi_{n\cdot r} - \varphi_{n-1\cdot r}] &= {}_1[\lambda_{n\cdot r} T_{\underset{n-1+\varepsilon\cdot r}{\longrightarrow}}] = {}_1[\lambda_{n\cdot r} T_{\underset{n-\varepsilon\cdot r}{\longrightarrow}}] \\ &= {}_1[\varDelta'\,{}_1 i + \varDelta'\,{}_1 B + \varDelta'\,{}_1 Q]_{n\cdot r} + {}_1[\varDelta''\,{}_2 i + \varDelta''\,{}_2 B + \varDelta''\,{}_2 Q]_{n\cdot r} \\ {}_2[\varphi_{n\cdot r} - \varphi_{n\cdot r-1}] &= {}_2[\lambda_{n\cdot r} T_{\underset{n\cdot r-1+\varepsilon}{\longrightarrow}}] = {}_2[\lambda_{n\cdot r} T_{\underset{n\cdot r-\varepsilon}{\longrightarrow}}] \\ &= {}_2[\varDelta''\,{}_1 i + \varDelta''\,{}_1 B + \varDelta''\,{}_1 Q]_{n\cdot r} + {}_2[\varDelta'\,{}_2 i + \varDelta'\,{}_2 B + \varDelta'\,{}_2 Q]_{n\cdot r} \,. \end{aligned} \right\} \tag{4.4} $$

Next, similarly, as to the conditions of the continuity of inclinations and the equilibrium of the shearing forces, the relations stated in the 1st report are also applicable to the present problem without any modifications.

That is to say, from the conditions of the continuities of the inclinations

$$ {}_s[\varDelta\,{}_s i' + B + B' + Q + Q']_{n\cdot r} = 0 \tag{4.5} $$

and from the conditions of the equilibriums of the shearing forces

$$ \left. \begin{aligned} -R_{n\cdot r} = {}&{}_1[(M_{\underset{n-\varepsilon\cdot r}{\longrightarrow}} + M_{\underset{n-1+\varepsilon\cdot r}{\longleftarrow}})/l_{n\cdot r} - (M_{\underset{n+1-\varepsilon\cdot r}{\longrightarrow}} + M_{\underset{n+\varepsilon\cdot r}{\longleftarrow}})/l_q] \\ &+ {}_2[(M_{\underset{n\cdot r-\varepsilon}{\longrightarrow}} + M_{\underset{n\cdot r-1+\varepsilon}{\longleftarrow}})/l_{n\cdot r} - (M_{\underset{n\cdot r+1-\varepsilon}{\longrightarrow}} + M_{\underset{n\cdot r+\varepsilon}{\longleftarrow}})/l_q] + \Theta_{n\cdot r} \,. \end{aligned} \right\} \tag{4.6} $$

Further, the supported bars being elastic and the forms of the cross-sections being uniform, we have

$$ y_{n\cdot r} = -[h\,R/E\,A]_{n\cdot r} \tag{4.7$_1$} $$

for the relations between the reactions and the deflections and, moreover, the bars being approximately regarded as a rigid bodies

$$ E_{n\cdot r} \to \infty \,, \quad y_{n\cdot r} \fallingdotseq 0 \,. \tag{4.7$_2$} $$

Here, having the supported bar at the intersecting point $(n \cdot r)$, we have one independent equations as to the moments and the deflection by considering the Eqs. (4.6) and (4.7) together and having not a supported bar at the point, the Eqs. (4.6) is an independent relation as to the moments.

After all, in the present problems, the Eqs. (4.2), (4.4), (4.5), and (4.6) are the fundamental equations, corresponding to these in the 1st report and these equations are also the simultaneous equations of the 1st degree with the 9 unknown quantities and same number of the equations.

Next, let us consider the above relations at the intersecting points on the boundary frame.

(a) In such cases when the intersecting points $(n \cdot r)$ are on the boundary members.

In these cases, first, we have the following boundary conditions:—

$$\left.\begin{array}{rllll}
(r = \text{max.}): & {}_2[F_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & T_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}] = 0, \\
(r = \text{min.}): & {}_2[F_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & M_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & T_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}] = 0, \\
(n = \text{max.}): & {}_1[F_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & T_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}] = 0, \\
(n = \text{min.}): & {}_1[F_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & M_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & T_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}] = 0, \\
((n\cdot r) = \text{max.}): & {}_1[F_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & T_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}] = 0, \\
 & {}_2[F_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & T_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}] = 0, \\
((n\cdot r) = \text{min.}): & {}_1[F_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & M_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & T_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}] = 0, \\
 & {}_2[F_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & M_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & T_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}] = 0, \\
(n = \text{max.},\ r = \text{min.}): & {}_1[F_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}, & T_{\underset{n+\varepsilon\cdot r}{\longrightarrow}}] = 0, \\
 & {}_2[F_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & M_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}, & T_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}] = 0, \\
(n = \text{min.},\ r = \text{max.}): & {}_1[F_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & M_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}, & T_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}] = 0, \\
 & {}_2[F_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}, & T_{\underset{n\cdot r+\varepsilon}{\longrightarrow}}] = 0.
\end{array}\right\} \quad (4.8)$$

Thereupon, similarly as the 1st report, applying the above relations to the previous fundamental equations and considering that the equations referring to the fictitious members do not hold, we have also the simultaneous equations consisting with the same number of the linear equations as that of the unknown quantities at these intersecting points, if we consider all the cases together.

(b) In such cases when the ends $(n\cdot r)$ of the members are built in the frame.

In these cases, as well known, having always the boundary conditions

$$_s i_{n\cdot r} = 0, \quad _s\varphi_{n\cdot r} = 0, \quad y_{n\cdot r} = 0, \qquad (4.9)$$

the fundamental equations are as follows.

(i) case r = max. or min.:—

In these cases, first, we have the Eqs. (4.9) and the 1st or the 2nd of the Eqs. (4.8) for the boundary conditions.

Then, from the Eqs. (3.2)

$$\left.\begin{array}{ll}
y_{n\cdot r-1}/{}_2l_{n\cdot r} = {}_2[B+Q]_{n\cdot r} & (r = \text{max.}) \\
y_{n\cdot r+1}/{}_2l_{n\cdot r+1} = {}_2[B'+Q']_{n\cdot r} & (r = \text{min.})
\end{array}\right\}$$

and from the Eqs. (4.2)

$$_2[M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}} + M_{\underset{n\cdot r-\varepsilon}{\longleftarrow}} + M_{n\cdot r}] = 0, \quad _2[T_{\underset{n\cdot r+\varepsilon}{\longrightarrow}} + T_{\underset{n\cdot r-\varepsilon}{\longleftarrow}}] = 0$$

$$(r = \text{max. or min.}).$$

Moreover, from the Eqs. (4.4)

$$-{}_2\varphi_{n\cdot r-1} = {}_2[\lambda_{n\cdot r}\, T_{\underset{n\cdot r-1+\varepsilon}{\longrightarrow}}] = {}_2[\lambda_{n\cdot r}\, T_{\underset{n\cdot r-\varepsilon}{\longrightarrow}}]$$
$$= -\{(\sec\theta)_{n\cdot r-1}\, {}_1[\varDelta y + B + Q]_{n\cdot r-1} + (T_g\,\theta)_{n\cdot r-1}\, {}_2[\varDelta y + B + Q]_{n\cdot r-1}\} \qquad (r = \text{max.})$$

and from the Eqs. (3.3_2)

$$\left.\begin{aligned} -R_{n\cdot r} &= {}_2F_{\underset{n\cdot r-\varepsilon}{\longleftarrow}} = {}_2[\{M_{\underset{n\cdot r-\varepsilon}{\longrightarrow}} + M_{\underset{n\cdot r-1+\varepsilon}{\longleftarrow}} + \overline{U}_1(l_{n\cdot r})\}/l_{n\cdot r}] && (r = \text{max.}) \\ -R_{n\cdot r} &= {}_2F_{\underset{n\cdot r+\varepsilon}{\longrightarrow}} = -{}_2[\{M_{\underset{n\cdot r+1-\varepsilon}{\longrightarrow}} + M_{\underset{n\cdot r+\varepsilon}{\longleftarrow}} - \overline{V}_1(l_{n\cdot r+1})\}/l_{n\cdot r+1}] && (r = \text{min.}) \end{aligned}\right\}$$

That is, these equations stated above are the fundamental equations for the present cases and these are also the simultaneous equations of the 1st degree with the 8 unknown quantities as to the moments and the same number of the equations, if we consider both the cases together.

(ii) case $n = \text{max.}$ or min. :—

In these cases, we have the Eqs. (4.9) and the 3rd or the 4th of the Eqs. (4.8) for the boundary conditions.

Then, from the Eqs. (3.2)

$$\left.\begin{aligned} y_{n-1\cdot r}/{}_1l_{n\cdot r} &= {}_1[B + Q]_{n\cdot r} && (n = \text{max.}) \\ y_{n+1\cdot r}/{}_1l_{n+1\cdot r} &= {}_1[B' + Q']_{n\cdot r} && (n = \text{min.})\,, \end{aligned}\right\}$$

and from the Eqs. (4.2)

$${}_1[M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}} + M_{\underset{n-\varepsilon\cdot r}{\longleftarrow}} + M_{n\cdot r}] = 0\,, \quad {}_1[T_{\underset{n+\varepsilon\cdot r}{\longrightarrow}} + T_{\underset{n-\varepsilon\cdot r}{\longleftarrow}}] = 0 \qquad (n = \text{max. or min.})\,.$$

Moreover, from the Eqs. (4.4)

$$-{}_1\varphi_{n-1\cdot r} = {}_1[\lambda_{n\cdot r}\, T_{\underset{n-1+\varepsilon\cdot r}{\longrightarrow}}] = {}_1[\lambda_{n\cdot r}\, T_{\underset{n-\varepsilon\cdot r}{\longrightarrow}}]$$
$$= -\{(T_g\,\theta)_{n-1\cdot r}\, {}_1[\varDelta y + B + Q]_{n-1\cdot r} + (\sec\theta)_{n-1\cdot r}\, {}_2[\varDelta y + B + Q]_{n-1\cdot r}\} \qquad (n = \text{max.})$$

and, from the Eqs. (3.3_2)

$$\left.\begin{aligned} -R_{n\cdot r} &= {}_1F_{\underset{n-\varepsilon\cdot r}{\longleftarrow}} = {}_1[\{M_{\underset{n-\varepsilon\cdot r}{\longrightarrow}} + M_{\underset{n-1+\varepsilon\cdot r}{\longleftarrow}} + \overline{U}_1(l_{n\cdot r})\}/l_{n\cdot r}] \\ &\qquad (n = \text{max.}) \\ -R_{n\cdot r} &= {}_1F_{\underset{n+\varepsilon\cdot r}{\longrightarrow}} = -{}_1[\{M_{\underset{n+1-\varepsilon\cdot r}{\longrightarrow}} + M_{\underset{n+\varepsilon\cdot r}{\longleftarrow}} - \overline{V}_1(l_{n+1\cdot r})\}/l_{n+1\cdot r}] \\ &\qquad (n = \text{min.}) \end{aligned}\right\}.$$

That is to say, similarly as the case (i), the equations stated above are the fundamental equations for these cases and these are also the simultaneous equations of the 1st degree with the 8 unknown quantites as to the moments and the same number of the equations, if we consider both of the cases together.

In conclusion, the above descriptions tell us that we have as many equations of the forms of the fundamental equations as the intersecting points $(n \cdot r)$ of the members of the structure and the simultaneous equations of these equations are consisted with the same number of linear equations as that of the unknown quantities ${}_s(M,\ T,\ y)_{n\cdot r}$ at all the intersecting points. Then, similarly as in the 1st report, the present problems are solved, provided that we find the solution of the simultaneous equations. It is, however, practically impossible to obtain the solutions of the equations by usual method of employing the determinant, as the number of the unknown quantities is too large in general, so that the well-known iteration methods are needed for solving the equations practically.

Hence, for the convenience of the actual calculation, we rewrite the simultaneous equations with the unknown quantities $({}_si,\ y)_{n\cdot r}$ as to the inclinations and the deflections at the points $(n\cdot r)$ instead of the previous fundamental equations in a form more convenient for our purpose.

First, from the Eqs. (3.3), we have

$$\left.\begin{aligned}
{}_1M_{\underset{n+\varepsilon\cdot r}{\longleftarrow}} &= -{}_1[M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}} + M_{n+\varepsilon\cdot r}] = {}_1[a_2(\Gamma_q + 3\,\Delta y'_{n\cdot r}) + S_q]\,,\\
{}_1M_{\underset{n+\varepsilon\cdot r}{\longrightarrow}} &= -{}_1[a_2(\Gamma_q + 3\,\Delta y'_{n\cdot r}) + \bar{S}_q]\\
{}_1M_{\underset{n+1-\varepsilon\cdot r}{\longrightarrow}} &= -{}_1[M_{\underset{n+1-\varepsilon\cdot r}{\longleftarrow}} + M_{n+1-\varepsilon\cdot r}] = {}_1[a_2(\Gamma_q' + 3\Delta y'_{n\cdot r}) + K_q]\,,\\
{}_1M_{\underset{n+1-\varepsilon\cdot r}{\longleftarrow}} &= -{}_1[a_2(\Gamma_q' + 3\,\Delta y'_{n\cdot r}) + \overline{K}_q]\\
{}_2M_{\underset{n\cdot r+\varepsilon}{\longleftarrow}} &= -{}_2[M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}} + M_{n\cdot r+\varepsilon}] = {}_2[a_2(\Gamma_q + 3\,\Delta y'_{n\cdot r}) + S_q]\,,\\
{}_2M_{\underset{n\cdot r+\varepsilon}{\longrightarrow}} &= -{}_2[a_2(\Gamma_q + 3\,\Delta y'_{n\cdot r}) + \bar{S}_q]\\
{}_2M_{\underset{n\cdot r+1-\varepsilon}{\longrightarrow}} &= -{}_2[M_{\underset{n\cdot r+1-\varepsilon}{\longleftarrow}} + M_{n\cdot r+1-\varepsilon}] = {}_2[a_2(\Gamma_q' + 3\,\Delta y'_{n\cdot r}) + K_q]\,,\\
{}_2M_{\underset{n\cdot r+1-\varepsilon}{\longleftarrow}} &= -{}_2[a_2(\Gamma_q' + 3\,\Delta y'_{n\cdot r})\,\overline{K}_q]
\end{aligned}\right\}\quad (4.9).$$

Then, we have only to rewrite the moments in the fundamental equations by the inclinations using the Eqs. (4.9).

Now, let us consider the Eqs. (4.5). First, calculating ${}_sB_{n\cdot r}$ and ${}_sB'_{n\cdot r}$ in the Eqs. (4.3) and (4.5), we have

$$\left.\begin{aligned}
{}_sB_{n\cdot r} &= {}_s[i_{n\cdot r} + \Delta y'_{q'} + \mu_{n\cdot r}(2\,K_{n\cdot r} - S_{n\cdot r})]\,,\\
{}_sB'_{n\cdot r} &= -{}_s[i_{n\cdot r} + \Delta y'_{n\cdot r} + \mu_q(2\,S_q - K_q)]
\end{aligned}\right\}\quad \text{(a)}$$

and the following relations are satisfied

$${}_s[Q + \mu(2\,K - S)]_{n\cdot r} = 0\,,\qquad {}_s[Q'_{n\cdot r} + \mu_q(K_q - 2\,S_q)] = 0 \qquad \text{(b)}.$$

Thereupon, adopting the above relations to the Eqs. (4.5), we see that the Eqs. (4.5) are satisfied identically. The above results tell us that utilizing the Eqs. (4.9), we do not need to consider the Eqs. (4.5).

Next, we consider the relations between the twisting moments and twisting angles.

That is, substituting the Eqs. (a) and (b) for M's and B's in the Eqs. (4.4), we have

$$\left.\begin{aligned} {}_1T_{\underset{n-1+\varepsilon\cdot r}{\longrightarrow}} &= {}_1T_{\underset{n-\varepsilon\cdot r}{\longrightarrow}} = {}_1[c_1\, i_{n\cdot r} - c_1'\, i_{n-1\cdot r}] + {}_2[b_1\, i_{n\cdot r} - b_1'\, i_{n-1\cdot r}] \\ {}_2T_{\underset{n\cdot r-1+\varepsilon}{\longrightarrow}} &= {}_2T_{\underset{n\cdot r-\varepsilon}{\longrightarrow}} = {}_1[b_1\, i_{n\cdot r} - b_1'\, i_{n\cdot r-1}] + {}_2[c_1\, i_{n\cdot r} - c_1'\, i_{n\cdot r-1}] \end{aligned}\right\} \quad (c).$$

Then, applying the Eqs. (4.9) and (c) to the Eqs. (4.2) and (4.6), we have

$$\left.\begin{aligned} &{}_1[-(a_1\, i_{n-1\cdot r} + a_2\, i_{n+1\cdot r}) + L_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) \\ &\qquad - \{2\,(a_1 + a_2) + L_{n\cdot r}\,(b_1 + b_2')\}\, i_{n\cdot r}] \\ &+ {}_2[(N_{n\cdot r}\, a_1 + L_{n\cdot r}\, c_1')\, i_{n\cdot r-1} + (N_{n\cdot r}\, a_2 + L_{n\cdot r}\, c_2)\, i_{n\cdot r+1} \\ &\qquad + \{2\, N_{n\cdot r}\,(a_1 + a_2) - L_{n\cdot r}\,(c_1 + c_2')\}\, i_{n\cdot r}] \\ &- 3\, {}_1[a_1'\, y_{n-1\cdot r} - a_2'\, y_{n+1\cdot r} + (a_2' - a_1')\, y_{n\cdot r}] \\ &\qquad + 3\, N_{n\cdot r}\, {}_2[a_1'\, y_{n\cdot r-1} - a_2'\, y_{n\cdot r+1} + (a_2' - a_1')\, y_{n\cdot r}] = [{}_1H_{n\cdot r}]_0 \end{aligned}\right\} \quad [\mathrm{I}]_0$$

$$\left.\begin{aligned} &{}_1[c_1'\, i_{n-1\cdot r} + c_2\, i_{n+1\cdot r} + N_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) \\ &\qquad - \{c_1 + c_2' + N_{n\cdot r}\,(b_1 + b_2')\}\, i_{n\cdot r}] \\ &+ {}_2[b_1'\, i_{n-1\cdot r} + b_2\, i_{n+1\cdot r} + (N_{n\cdot r}\, c_1' - L_{n\cdot r}\, a_1)\, i_{n\cdot r-1} \\ &\qquad + (N_{n\cdot r}\, c_2 - L_{n\cdot r}\, a_2)\, i_{n\cdot r+1} \\ &\qquad - \{b_1 + b_2' + N_{n\cdot r}\,(c_1 + c_2') + 2\, L_{n\cdot r}\,(a_1 + a_2)\}\, i_{n\cdot r}] \\ &- 3\, L_{n\cdot r}\, {}_2[a_1'\, y_{n\cdot r-1} + (a_2' - a_1')\, y_{n\cdot r} - a_2'\, y_{n\cdot r+1}] = [{}_2H_{n\cdot r}]_0 \end{aligned}\right\} \quad [\mathrm{II}]_0$$

$$\left.\begin{aligned} &3\, [{}_1\{a_1'\, i_{n-1\cdot r} - a_2'\, i_{n+1\cdot r} + (a_1' - a_2')\, i_{n\cdot r}\} \\ &\qquad + {}_2\{a_1'\, i_{n\cdot r-1} - a_2'\, i_{n\cdot r+1} + (a_1' - a_2')\, i_{n\cdot r}\}] \\ &+ 6\, [{}_1\{a_1''\, y_{n-1\cdot r} + a_2''\, y_{n+1\cdot r}\} + {}_2\{a_1''\, y_{n\cdot r-1} + a_2''\, y_{n\cdot r+1}\} \\ &\qquad - \{{}_1(a_1'' + a_2'') + {}_2(a_1'' + a_2'')\}\, y_{n\cdot r}] = [{}_3H_{n\cdot r}]_0 \end{aligned}\right\} \quad [\mathrm{III}]_0$$

in which

$$[{}_1H_{n\cdot r}]_0 = -[{}_1M_{n\cdot r} - N_{n\cdot r}\, {}_2M_{n\cdot r} - ({}_1\bar{S}_{n+1\cdot r} + {}_1\bar{K}_{n\cdot r}) + N_{n\cdot r}({}_2\bar{S}_{n\cdot r+1} + {}_2\bar{K}_{n\cdot r})]$$

$$[{}_2H_{n\cdot r}]_0 = -L_{n\cdot r}\, {}_2[M_{n\cdot r} - (\bar{S}_{n\cdot r+1} + \bar{K}_{n\cdot r})]$$

$$[{}_3H_{n\cdot r}]_0 = -[{}_1(K'_{n\cdot r} - S'_q) + {}_2(K'_{n\cdot r} - S_q') - 3\, P_{n\cdot r} + R_{n\cdot r}]$$

namely, the above Eqs. $[\mathrm{I}]_0$, $[\mathrm{II}]_0$ and $[\mathrm{III}]_0$ are the required fundamental equations expressed in terms of the inclinations and the deflection at the intersecting point $(n \cdot r)$ of the members.

Further, in such cases when the intersecting points $(n \cdot r)$ are on the boundary members, we find the fundamental equations directly from the above Eqs. $[\mathrm{I}]_0$, $[\mathrm{II}]_0$ and $[\mathrm{III}]_0$, provided that we consider the suffix of the unknown quantities and their coefficients in the fundamental equations and put the quantities referring to the fictitious members equal to zero. Then, for the convenience of the actual calculations, we show the results as follow.

(a) In such cases when the intersecting points $(n \cdot r)$ are on the boundary members:—

(i) case $r = \text{mmx.}$:—

$$\left.\begin{aligned} &{}_1[-(a_1\, i_{n-1\cdot r} + a_2\, i_{n+1\cdot r}) + L_{n\cdot r}\, b_1'\, i_{n\cdot r-1} \\ &\qquad - \{2(a_1 + a_2) + L_{n\cdot r}\, b_1\} i_{n\cdot r}] \\ &+ {}_2[(N_{n\cdot r}\, a_1 + L_{n\cdot r}\, c_1')\, i_{n\cdot r-1} + (2\, N_{n\cdot r}\, a_1 - L_{n\cdot r}\, c_1)\, i_{n\cdot r}] \\ &\qquad - 3\, {}_1[a_1'\, y_{n-1\cdot r} - a_2'\, y_{n+1\cdot r} + (a_2' - a_1')\, y_{n\cdot r}] \\ &+ 3\, N_{n\cdot r}\, {}_2a_1'\, (y_{n\cdot r-1} - y_{n\cdot r}) = [{}_1H_{n\cdot r}]_1 \end{aligned}\right\} [\mathrm{I}]_1$$

$$\left.\begin{aligned} &{}_1[c_1'\, i_{n-1\cdot r} + c_2\, i_{n+1\cdot r} + N_{n\cdot r}\, b_1'\, i_{n\cdot r-1} - (c_1 + c_2' + N_{n\cdot r}\, b_1)\, i_{n\cdot r}] \\ &+ {}_2[b_1'\, i_{n-1\cdot r} + b_2\, i_{n+1\cdot r} + (N_{n\cdot r}\, c_1' - L_{n\cdot r}\, a_1)\, i_{n\cdot r-1} \\ &\qquad - (b_1 + b_2' + 2\, L_{n\cdot r}\, a_1 + N_{n\cdot r}\, c_1)\, i_{n\cdot r}] \\ &- 3\, L_{n\cdot r}\, {}_2a_1'\, (y_{n\cdot r-1} - y_{n\cdot r}) = [{}_2H_{n\cdot r}]_1 \end{aligned}\right\} [\mathrm{II}]_2$$

$$\left.\begin{aligned} &3\, [{}_1\{a_1'\, i_{n-1\cdot r} - a_2'\, i_{n+1\cdot r} + (a_1' - a_2')\, i_{n\cdot r}\} + {}_2\{a_1'\, (i_{n\cdot r-1} + i_{n\cdot r})\}] \\ &+ 6\, [{}_1\{a_1''\, y_{n-1\cdot r} + a_2''\, y_{n+1\cdot r}\} + {}_2a_1''\, y_{n\cdot r-1} \\ &\qquad - ({}_1a_1'' + {}_1a_2'' + {}_2a_1'')\, y_{n\cdot r}] = [{}_3H_{n\cdot r}]_1 \end{aligned}\right\} [\mathrm{III}]_1$$

in which

$$[{}_1H_{n\cdot r}]_1 = -[{}_1M_{n\cdot r} - N_{n\cdot r}\, {}_2M_{n\cdot r} - ({}_1\overline{S}_{n+1\cdot r} + {}_1\overline{K}_{n\cdot r}) + N_{n\cdot r}\, {}_2\overline{K}_{n\cdot r}]$$

$$[{}_2H_{n\cdot r}]_1 = -L_{n\cdot r}\, {}_2[M_{n\cdot r} - \overline{K}_{n\cdot r}]$$

$$[{}_3H_{n\cdot r}]_1 = -[{}_1K'_{n\cdot r} - {}_1S'_{n+1\cdot r} + {}_2K'_{n\cdot r} - 2\, P_{n\cdot r} + R_{n\cdot r}]\,.$$

(ii) case $r = \text{min.}$:—

$$\left.\begin{aligned} &{}_1[-(a_1\, i_{n-1\cdot r} + a_2\, i_{n+1\cdot r}) + L_{n\cdot r}\, b_2\, i_{n\cdot r+1} \\ &\qquad - \{2(a_1 + a_2) + L_{n\cdot r}\, b_2'\} i_{n\cdot r}] \\ &+ {}_2[(N_{n\cdot r}\, a_2 + L_{n\cdot r}\, c_2)\, i_{n\cdot r+1} + (2\, N_{n\cdot r}\, a_2 - L_{n\cdot r}\, c_2')\, i_{n\cdot r}] \\ &+ 3\, [{}_1a_2'\, y_{n+1\cdot r} - {}_1a_1'\, y_{n-1\cdot r} - N_{n\cdot r}\, {}_2a_2'\, y_{n\cdot r+1} \\ &\qquad + ({}_1a_1' - {}_1a_2' + N_{n\cdot r}\, {}_2a_2')\, y_{n\cdot r} = [{}_1H_{n\cdot r}]_{1'} \end{aligned}\right\} [\mathrm{I}]_{1'}$$

$$\left.\begin{aligned} &{}_1[c_1'\, i_{n-1\cdot r} + c_2\, i_{n+1\cdot r} + N_{n\cdot r}\, b_2\, i_{n\cdot r+1} - (c_1 + c_2' + N_{n\cdot r}\, b_2')\, i_{n\cdot r}] \\ &+ {}_2[b_1'\, i_{n-1\cdot r} + b_2\, i_{n+1\cdot r} + (N_{n\cdot r}\, c_2 - L_{n\cdot r}\, a_2)\, i_{n\cdot r+1} \\ &\qquad - (b_1 + b_2' + N_{n\cdot r}\, c_2' + 2\, L_{n\cdot r}\, a_2)\, i_{n\cdot r}] \\ &- 3\, L_{n\cdot r}\, {}_2a_2'\, (y_{n\cdot r} - y_{n\cdot r+1}) = [{}_2H_{n\cdot r}]_{1'} \end{aligned}\right\} [\mathrm{II}]_{1'}$$

$$\left.\begin{aligned} &3\, [{}_1\{a_1'\, i_{n-1\cdot r} - a_2'\, i_{n+1\cdot r} + (a_1' - a_2')\, i_{n\cdot r}) - {}_2a_2'\, {}_2\{i_{n\cdot r+1} + i_{n\cdot r}\}] \\ &+ 6[{}_1\{a_1''\, y_{n-1\cdot r} + a_2''\, y_{n+1\cdot r}\} + {}_2a_2''\, y_{n\cdot r+1} \\ &\qquad - \{{}_1(a_1'' + a_2'') + {}_2a_2''\}\, y_{n\cdot r}] = [{}_3H_{n\cdot r}]_{1'} \end{aligned}\right\} [\mathrm{III}]_{1'}$$

in which

$$[{}_1H_{n\cdot r}]_{1'} = -[{}_1M_{n\cdot r} - N_{n\cdot r}\, {}_2M_{n\cdot r} + N_{n\cdot r}\, {}_2\overline{S}_{n\cdot r+1} - ({}_1\overline{S}_{n+1\cdot r} + {}_1\overline{K}_{n\cdot r})]$$

$${}_2[H_{n\cdot r}]_{1'} = -L_{n\cdot r}\, {}_2[M_{n\cdot r} - \overline{S}_{n\cdot r+1}]$$

$$[{}_3H_{n\cdot r}]_{1'} = -[{}_1K'_{n\cdot r} - {}_1S'_{n+1\cdot r} - {}_2S'_{n\cdot r+1} - 2\, P_{n\cdot r} + R_{n\cdot r}]$$

(iii) case $n = \max.$:—

$$\left.\begin{aligned}
&{}_1[-a_1\, i_{n-1\cdot r} + L_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) \\
&\qquad - \{2\,a_1 + L_{n\cdot r}\,(b_1 + b_2')\}\, i_{n\cdot r}] \\
&+ {}_2[(N_{n\cdot r}\,a_1 + L_{n\cdot r}\,c_1')\, i_{n\cdot r-1} + (N_{n\cdot r}\,a_2 + L_{n\cdot r}\,c_2)\, i_{n\cdot r+1} \\
&\qquad + \{2\,N_{n\cdot r}\,(a_1 + a_2) - L_{n\cdot r}(c_1 + c_2')\, i_{n\cdot r}\}] \\
&- 3\,{}_1a_1'\,(y_{n-1\cdot r} - y_{n\cdot r}) + 3\,N_{n\cdot r}\,{}_2[a_1'\, y_{n\cdot r-1} + (a_2' - a_1')\, y_{n\cdot r} \\
&\qquad - a_2'\, y_{n\cdot r+1}] = [{}_1H_{n\cdot r}]_2
\end{aligned}\right\}\,[\mathrm{I}]_2$$

$$\left.\begin{aligned}
&{}_1[c_1'\, i_{n-1\cdot r} + N_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) - \{c_1 + N_{n\cdot r}\,(b_1 + b_2')\} i_{n\cdot r}] \\
&+ {}_2[b_1'\, i_{n-1\cdot r} + (N_{n\cdot r}\,c_1' - L_{n\cdot r}\,a_1)\, i_{n\cdot r-1} + (N_{n\cdot r}\,c_2 - L_{n\cdot r}\,a_2)\, i_{n\cdot r+1} \\
&\qquad - \{b_1 + N_{n\cdot r}\,(c_1 + c_2') + 2\,L_{n\cdot r}\,(a_1 + a_2)\}\, i_{n\cdot r}] \\
&- 3\,L_{n\cdot r}\,{}_2[a_1'\, y_{n\cdot r-1} + (a_2' - a_1')\, y_{n\cdot r} - a_2'\, y_{n\cdot r+1}] = [{}_2H_{n\cdot r}]_2
\end{aligned}\right\}\,[\mathrm{II}]_2$$

$$\left.\begin{aligned}
&3\,[{}_1a_1'\,(i_{n-1\cdot r} - i_{n\cdot r}) + {}_2\{a_1'\, i_{n\cdot r-1} - a_2'\, i_{n\cdot r+1} + (a_1' - a_2')\, i_{n\cdot r}\}] \\
&+ 6\,[{}_1a_2''\, y_{n+1\cdot r} + {}_2a_1''\, y_{n\cdot r-1} + {}_2a_2''\, y_{n\cdot r+1} \\
&\qquad - \{{}_1a_2'' + {}_2(a_1'' + a_2'')\}\, y_{n\cdot r}] = [{}_3H_{n\cdot r}]
\end{aligned}\right\}\,[\mathrm{III}]_2$$

in which

$$[{}_1H_{n\cdot r}]_2 = -[{}_1M_{n\cdot r} - N_{n\cdot r}\,{}_2M_{n\cdot r} - {}_1\overline{K}_{n\cdot r} + N_{n\cdot r}\,({}_2\overline{S}_{n\cdot r+1} + {}_2\overline{K}_{n\cdot r})]\,,$$

$$[{}_2H_{n\cdot r}]_2 = -L_{n\cdot r}\,{}_2[M_{n\cdot r} - (\overline{S}_{n\cdot r+1} + \overline{K}_{n\cdot r})]$$

$$[{}_3H_{n\cdot r}]_2 = -[{}_1K'_{n\cdot r} + {}_2K'_{n\cdot r} - {}_2S'_{n\cdot r+1} - 2\,P_{n\cdot r} + R_{n\cdot r}]$$

(iv) case $n = \min.$:—

$$\left.\begin{aligned}
&{}_1[-a_2\, i_{n+1\cdot r} + L_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) \\
&\qquad - \{2\,a_2 + L_{n\cdot r}\,(b_1 + b_2')\}\, i_{n\cdot r}] \\
&+ {}_2[(N_{n\cdot r}\,a_1 + L_{n\cdot r}\,c_1')\, i_{n\cdot r-1} + (N_{n\cdot r}\,a_2 + L_{n\cdot r}\,c_2)\, i_{n\cdot r+1} \\
&\qquad + \{2\,N_{n\cdot r}\,(a_1 + a_2) - L_{n\cdot r}\,(c_1 + c_2')\}\, i_{n\cdot r}] \\
&- 3\,{}_1a_2'\,(y_{n\cdot r} - y_{n+1\cdot r}) \\
&\qquad + 3\,N_{n\cdot r}\,{}_2[a_1'\, y_{n\cdot r-1} + (a_2' - a_1')\, y_{n\cdot r} - a_2'\, y_{n\cdot r+1}] \\
&\qquad\qquad = [{}_1H_{n\cdot r}]_{2'}
\end{aligned}\right\}\,[\mathrm{I}]_{2'}$$

$$\left.\begin{aligned}
&{}_1[c_2\, i_{n+1\cdot r} + N_{n\cdot r}\,(b_1'\, i_{n\cdot r-1} + b_2\, i_{n\cdot r+1}) - \{c_2' + N_{n\cdot r}\,(b_1 + b_2')\}\, i_{n\cdot r}] \\
&+ {}_2[b_2\, i_{n+1\cdot r} + (N_{n\cdot r}\,c_1' - L_{n\cdot r}\,a_1)\, i_{n\cdot r-1} + (N_{n\cdot r}\,c_2 - L_{n\cdot r}\,a_2)\, i_{n\cdot r+1} \\
&\qquad - \{b_2' + N_{n\cdot r}\,(c_1 + c_2') + 2\,L_{n\cdot r}\,(a_1 + a_2)\}\, i_{n\cdot r}] \\
&- 3\,L_{n\cdot r}\,{}_2[a_1'\, y_{n\cdot r-1} + (a_2' - a_1')\, y_{n\cdot r} - a_2'\, y_{n\cdot r+1}] = [{}_2H_{n\cdot r}]_{2'}
\end{aligned}\right\}\,[\mathrm{II}]_{2'}$$

$$\left.\begin{aligned}
&3\,[{}_1\{-a_2'\,(i_{n+1\cdot r} + i_{n\cdot r})\} + {}_2\{a_1'\, i_{n\cdot r-1} - a_2'\, i_{n\cdot r+1} + (a_1' - a_2')\, i_{n\cdot r}\}] \\
&+ 6\,[{}_1a_2''\, y_{n+1\cdot r} + {}_2\{a_1''\, y_{n\cdot r-1} + a_2''\, y_{n\cdot r+1}\} \\
&\qquad - \{{}_1a_2'' + {}_2(a_1'' + a_2'')\}\, y_{n\cdot r}] = [{}_3H_{n\cdot r}]_{2'}
\end{aligned}\right\}\,[\mathrm{III}]_{2'}$$

in which

$$[{}_1H_{n\cdot r}]_{2'} = -[{}_1M_{n\cdot r} - N_{n\cdot r}\,{}_2M_{n\cdot r} - {}_1\bar{S}_{n+1\cdot r} + N_{n\cdot r}({}_2\bar{S}_{n\cdot r+1} + {}_2\bar{K}_{n\cdot r+1})],$$

$$[{}_2H_{n\cdot r}]_{2'} = -L_{n\cdot r}\,{}_2[M_{n\cdot r} - (\bar{S}_{n\cdot r+1} + \bar{K}_{n\cdot r})],$$

$$[{}_3H_{n\cdot r}]_{2'} = -[-{}_1S'_{n+1\cdot r} + {}_2K'_{n\cdot r} - {}_2S'_{n\cdot r+1} - 2P_{n\cdot r} + R_{n\cdot r}]$$

(v) case $(n\cdot r) = \max.$:—

$$\left.\begin{aligned} &{}_1[-a_1 i_{n-1\cdot r} + L_{n\cdot r} b_1' i_{n\cdot r-1} - (2a_1 + L_{n\cdot r} b_1) i_{n\cdot r}] \\ &+ {}_2[(N_{n\cdot r} a_1 + L_{n\cdot r} c_1') i_{n\cdot r-1} - (L_{n\cdot r} c_1 - 2N_{n\cdot r} a_1) i_{n\cdot r}] \\ &- 3\{{}_1a_1' y_{n-1\cdot r} - N_{n\cdot r}\,{}_2a_1' y_{n\cdot r-1} - ({}_1a_1' - N_{n\cdot r}\,{}_2a_1') y_{n\cdot r}\} = [{}_1H_{n\cdot r}]_3 \end{aligned}\right\} [\mathrm{I}]_3$$

$$\left.\begin{aligned} &{}_1[c_1' i_{n-1\cdot r} + N_{n\cdot r} b_1' i_{n\cdot r-1} - (c_1 + N_{n\cdot r} b_1) i_{n\cdot r}] \\ &+ {}_2[b_1' i_{n-1\cdot r} + (N_{n\cdot r} c_1' - L_{n\cdot r} a_1) i_{n\cdot r-1} - (b_1 + 2L_{n\cdot r} a_1 + N_{n\cdot r} c_1) i_{n\cdot r}] \\ &- 3L_{n\cdot r}\,{}_2a_1'(y_{n\cdot r-1} - y_{n\cdot r}) = [{}_2H_{n\cdot r}]_3 \end{aligned}\right\} [\mathrm{II}]_3$$

$$\left.\begin{aligned} &3[{}_1a_1'({}_1i_{n-1\cdot r} + {}_1i_{n\cdot r}) + {}_2a_1'({}_2i_{n\cdot r-1} + {}_2i_{n\cdot r})] \\ &+ 6[{}_1a_1'' y_{n-1\cdot r} + {}_2a_1'' y_{n\cdot r-1} - ({}_1a_1'' + {}_2a_1'') y_{n\cdot r}] = [{}_3H_{n\cdot r}]_3 \end{aligned}\right\} [\mathrm{III}]_3$$

in which

$$[{}_1H_{n\cdot r}]_3 = -[{}_1M_{n\cdot r} - N_{n\cdot r}\,{}_2M_{n\cdot r} - {}_1\bar{K}_{n\cdot r} + N_{n\cdot r}\,{}_2\bar{K}_{n\cdot r}],$$

$$[{}_2H_{n\cdot r}]_3 = -L_{n\cdot r}\,{}_2[M_{n\cdot r} - \bar{K}_{n\cdot r}],$$

$$[{}_3H_{n\cdot r}]_3 = -[{}_1K_{n\cdot r} + {}_2K_{n\cdot r} - P_{n\cdot r} + R_{n\cdot r}]$$

(vi) case $(n\cdot r) = \min.$:—

$$\left.\begin{aligned} &{}_1[-a_2 i_{n+1\cdot r} + L_{n\cdot r} b_2 i_{n\cdot r+1} - (2a_2 + L_{n\cdot r} b_2') i_{n\cdot r}] \\ &+ {}_2[(L_{n\cdot r} c_2 + N_{n\cdot r} a_2) i_{n\cdot r+1} - (L_{n\cdot r} c_2' - 2N_{n\cdot r} b_2') i_{n\cdot r}] \\ &+ 3\{{}_1a_2' y_{n+1\cdot r} - N_{n\cdot r}\,{}_2a_2' y_{n\cdot r+1} - ({}_1a_2' - N_{n\cdot r}\,{}_2a_2') y_{n\cdot r}\} = [{}_1H_{n\cdot r}]_{3'} \end{aligned}\right\} [\mathrm{I}]_{3'}$$

$$\left.\begin{aligned} &{}_1[c_2 i_{n+1\cdot r} + N_{n\cdot r} b_2 i_{n\cdot r+1} - (c_2' + N_{n\cdot r} b_2') i_{n\cdot r}] \\ &+ {}_2[b_2 i_{n+1\cdot r} - (L_{n\cdot r} a_2 - N_{n\cdot r} c_2) i_{n\cdot r+1} - (b_2' + 2L_{n\cdot r} a_2 + N_{n\cdot r} c_2') i_{n\cdot r}] \\ &- 3L_{n\cdot r}\,{}_2a_2'(y_{n\cdot r} - y_{n\cdot r+1}) = [{}_2H_{n\cdot r}]_{3'} \end{aligned}\right\} [\mathrm{II}]_{3'}$$

$$\left.\begin{aligned} &3[-{}_1a_2'({}_1i_{n+1\cdot r} + {}_1i_{n\cdot r}) - {}_2a_2'({}_2i_{n\cdot r+1} + {}_2i_{n\cdot r})] \\ &+ 6[{}_1a_2'' y_{n+1\cdot r} + {}_2a_2 y_{n\cdot r+1} - ({}_1a_2'' + {}_2a_2'') y_{n\cdot r}] = [{}_3H_{n\cdot r}]_{3'} \end{aligned}\right\} [\mathrm{III}]_{3'}$$

in which

$$[{}_1H_{n\cdot r}]_{3'} = -[{}_1M_{n\cdot r} - N_{n\cdot r}\,{}_2M_{n\cdot r} - {}_1\bar{S}_{n+1\cdot r} + N_{n\cdot r}\,{}_2\bar{S}_{n\cdot r+1}],$$

$$[{}_2H_{n\cdot r}]_{3'} = -L_{n\cdot r}\,{}_2[M_{n\cdot r} - S_{n\cdot r+1}],$$

$$[{}_3H_{n\cdot r}]_{3'} = -[-\{{}_1S_{n+1\cdot r} + {}_2S_{n\cdot r+1}\} - P_{n\cdot r} + R_{n\cdot r}]$$

(vii) case $n = \max.$ and $r = \min.$:—

$$\left.\begin{aligned} &{}_1[-a_1 i_{n-1\cdot r} + L_{n\cdot r} b_2 i_{n\cdot r+1} - (2a_1 + L_{n\cdot r} b_2') i_{n\cdot r}] \\ &+ {}_2[(L_{n\cdot r} c_2 + N_{n\cdot r} a_2) i_{n\cdot r+1} + (2N_{n\cdot r} a_2 - L_{n\cdot r} c_2') i_{n\cdot r}] \\ &- 3\{{}_1a_1' y_{n-1\cdot r} + N_{n\cdot r}\,{}_2a_2' y_{n\cdot r+1} - ({}_1a_1' + N_{n\cdot r}\,{}_2a_2') y_{n\cdot r}\} = [{}_1H_{n\cdot r}]_4 \end{aligned}\right\} [\mathrm{I}]_4$$

$$\left.\begin{array}{l} {}_1[c_1' i_{n-1\cdot r} + N_{n\cdot r} b_2 i_{n\cdot r+1} - (c_1 + N_{n\cdot r} b_2') i_{n\cdot r}] \\ + {}_2[b_1' i_{n-1\cdot r} - (L_{n\cdot r} a_2 - N_{n\cdot r} c_2) i_{n\cdot r+1} - (b_1 + 2 L_{n\cdot r} a_2 + N_{n\cdot r} c_2') i_{n\cdot r}] \\ - 3 L_{n\cdot r}\, {}_2a_2 (y_{n\cdot r} - y_{n\cdot r+1}) = [{}_2H_{n\cdot r}]_4 \end{array}\right\} [\mathrm{II}]_4$$

$$\left.\begin{array}{l} 3 [{}_1a_1'({}_1i_{n-1\cdot r} + {}_1i_{n\cdot r}) - {}_2a_2'({}_2i_{n\cdot r+1} + {}_2i_{n\cdot r})] \\ + 6[{}_1a_1'' y_{n-1\cdot r} + {}_2a_2'' y_{n\cdot r+1} - ({}_1a_1'' + {}_2a_2'') y_{n\cdot r}] = [{}_3H_{n\cdot r}]_4 \end{array}\right\} [\mathrm{III}]_4$$

in which

$$[{}_1H_{n\cdot r}]_4 = -[{}_1M_{n\cdot r} - N_{n\cdot r}\, {}_2M_{n\cdot r} - {}_1\overline{K}_{n\cdot r} + N_{n\cdot r}\, {}_2\overline{S}_{n\cdot r+1}],$$

$$[{}_2H_{n\cdot r}]_4 = -L_{n\cdot r}\, {}_2[M_{n\cdot r} - \overline{S}_{n\cdot r+1}],$$

$$[{}_3H_{n\cdot r}]_4 = -[{}_1K'_{n\cdot r} - {}_2S'_{n\cdot r+1} - P_{n\cdot r} + R_{n\cdot r}]$$

(viii) case $n = \min.$ and $r = \max.$: —

$$\left.\begin{array}{l} {}_1[-a_2 i_{n+1\cdot r} + L_{n\cdot r} b_1' i_{n\cdot r-1} - (2 a_2 + L_{n\cdot r} b_1) i_{n\cdot r}] \\ + {}_2[(L_{n\cdot r} c_1' + N_{n\cdot r} a_1) i_{n\cdot r-1} - (L_{n\cdot r} c_1 - 2 N_{n\cdot r} a_1) i_{n\cdot r}] \\ + 3 \{{}_1a_2' y_{n+1\cdot r} + N_{n\cdot r}\, {}_2a_1' y_{n\cdot r-1} - ({}_1a_2' + N_{n\cdot r}\, {}_2a_1') y_{n\cdot r}\} = [{}_1H_{n\cdot r}]_{4'} \end{array}\right\} [\mathrm{I}]_{4'}$$

$$\left.\begin{array}{l} {}_1[c_2 i_{n+1\cdot r} + N_{n\cdot r} b_1' i_{n\cdot r-1} - (c_2' + N_{n\cdot r} b_1) i_{n\cdot r}] \\ + {}_2[b_2 i_{n+1\cdot r} + (N_{n\cdot r} c_1' - L_{n\cdot r} a_1) i_{n\cdot r-1} - (b_2' + 2 L_{n\cdot r} a_1 + N_{n\cdot r} c_1) i_{n\cdot r}] \\ - 3 L_{n\cdot r}\, {}_2a_1' (y_{n\cdot r-1} - y_{n\cdot r}) = [{}_2H_{n\cdot r}]_{4'} \end{array}\right\} [\mathrm{II}]_{4'}$$

$$\left.\begin{array}{l} 3 [-{}_1a_2'({}_1i_{n+1\cdot r} + {}_1i_{n\cdot r}) + {}_2a_1'({}_2i_{n\cdot r-1} + {}_2i_{n\cdot r})] \\ + 6 [{}_1a_2'' y_{n+1\cdot r} + {}_2a_1'' y_{n\cdot r-1} - ({}_1a_2'' + {}_2a_1'') y_{n\cdot r}] = [{}_3H_{n\cdot r}]_{4'} \end{array}\right\} [\mathrm{III}]_{4'}$$

in which

$$[{}_1H_{n\cdot r}]_{4'} = -[{}_1M_{n\cdot r} - N_{n\cdot r}\, {}_2M_{n\cdot r} - {}_1\overline{S}_{n+1\cdot r} + N_{n\cdot r}\, {}_2\overline{K}_{n\cdot r}],$$

$$[{}_2H_{n\cdot r}]_{4'} = -L_{n\cdot r}\, {}_2[M_{n\cdot r} - K_{n\cdot r}],$$

$$[{}_3H_{n\cdot r}]_{4'} = -[-{}_1S'_{n+1\cdot r} + {}_2K'_{n\cdot r} - P_{n\cdot r} + R_{n\cdot r}].$$

(b) In such cases when the ends $(n \cdot r)$ of the members are built in the frames.

In theses cases, as the previous discriptions, we have

$$ {}_si_{n\cdot r} = 0, \quad {}_s\varphi_{n\cdot r} = 0, \quad y_{n\cdot r} = 0 \tag{4.9}$$

for the boundary conditions and these equations are used instead of the fundamental equations.

In conclusion, from the above results, we have as many equations of the form of the fundamental equations mentioned above as the intersecting points $(n \cdot r)$ of the members of the structure and the simultaneous equations of these equations are consisted with the same number of linear equations as that of the unknown quantities $({}_si,\, y)_{n\cdot r}$ at all the intersecting points in the structure and these equations can always be solved by the well-known iteration methods. Then, utilizing theses values obtained above for the Eqs. (3.3) and (3.1), all of the requirements are directly found and the present problems are solved.

5. Conclusion. By the present method, the torsion and bending problems of continuous panel structures can always be solved, the members of which cross each other with angles different from right angle and distributions of the external forces acting on the members are quite arbitrary and positions of the supported bars are also arbitrary, when the deflections and the inclinations of the members of the structures are all small.

Needless to say, putting the angles $\theta_{n \cdot r}$ different from right angle, namely the angles between the ξ- and x-axes at the intersecting points $(n \cdot r)$, equal to zero, the present results perfectly coincide with those described in the 1st report. But as the descriptions in the 1st report, the present method, as it is, can not be used for the large deflections, because the formulae of bending, say the Eqs. (3.1), are not applicable and both of the restriction conditions at the supported points and the boundary conditions become far more complex than stated in the report. Moreover, when the sizes of the cross-sections of the members are large, the restrictions received by the deformations occuring in the cross-sections of the members under the external forces are quantities of sensible magnitude, so that we need consider these points fully in the practical designs of the structures. Thus, we are still in the course of research of these questions mentioned above.

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