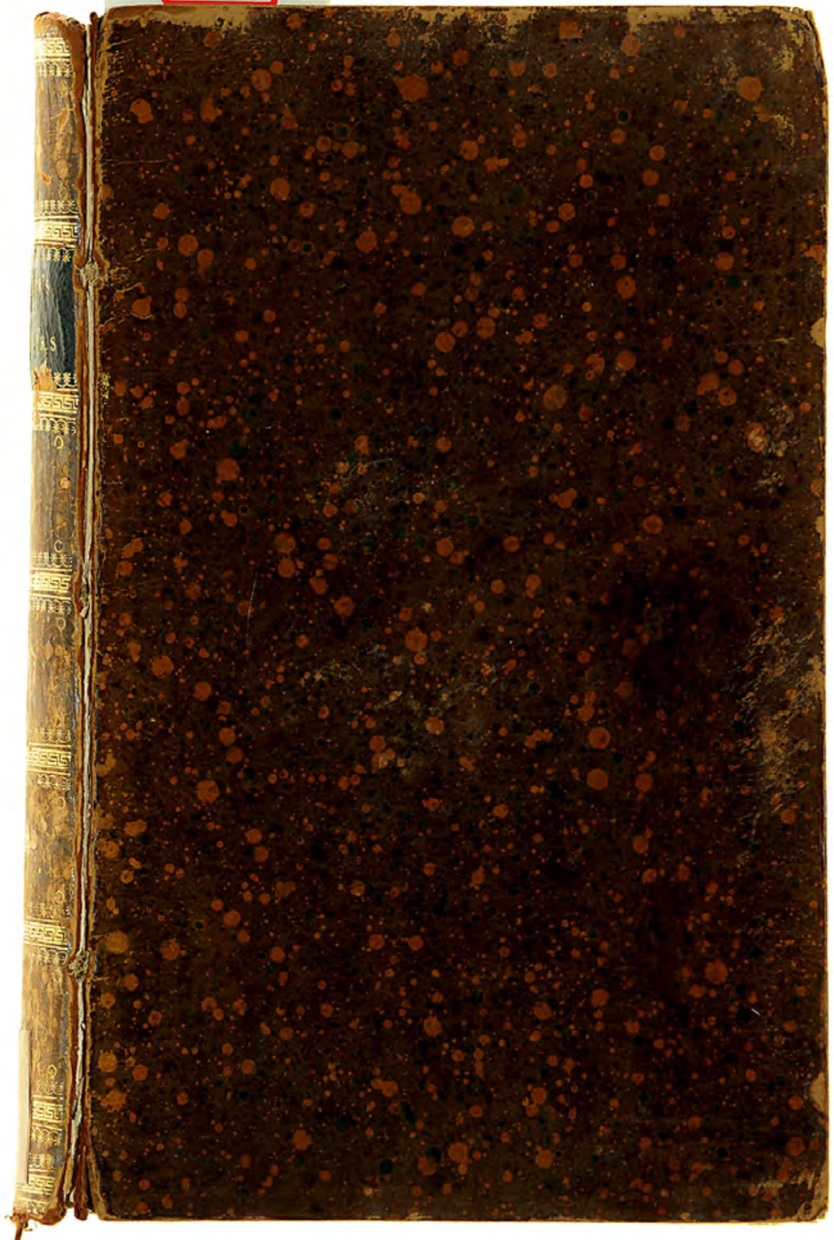

LETTERS

OF

EULER.

1816

LETTERS

OF

EULER

ON DIFFERENT SUBJECTS

IN

PHYSICS AND PHILOSOPHY.

ADDRESSED TO

A GERMAN PRINCESS.

TRANSLATED FROM THE FRENCH BY

HENRY HUNTER, D.D.

WITH

ORIGINAL NOTES,

And a Gloſſary of Foreign and Scientific Terms.

Second Edition.

IN TWO VOLUMES.

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PREFACE.

IT was long a matter of surprize to me, that a Work so well known, and so justly esteemed, over the whole European Continent, as EULER's *Letters to a German Princess*, should never have made it's way into our Island, in the language of the Country. While Petersburg, Berlin, Paris, nay the capital of every petty German principality, was profiting by the ingenious labours of this amiable man, and acute philosopher, the name of EULER was a sound unknown to the ear of youth in the British metropolis. I was mortified to reflect that the specious and seductive productions of a *Rousseau*, and the poisonous effusions of a *Voltaire*, should be in the hands of so many young men, not to

say

ſay young women, to the perverſion of the underſtanding, and the corruption of the moral principle, while the ſimple and uſeful inſtructions of the virtuous EULER were hardly mentioned.

I frequently ſuggeſted the idea of a tranſlation to more than one literary friend, in whoſe ability for the taſk I could place greater confidence than in my own: but not finding it undertaken, I determined, at length, to attempt it myſelf, with the ability which I had; and, in doing this, I conſidered myſelf as rendering a meritorious ſervice to my country.

As ſoon as Providence had beſtowed on me the bleſſing of children, I felt it to be my duty to charge myſelf with their inſtruction. How I have ſucceeded it becomes not me to ſay: but every day I live, the importance of early and proper

proper culture is more deeply impreſſed on my mind. There ſeems to be ſtill a *deſideratum* towards completing the plan of an uſeful education—ſomething that ſhall ſuggeſt to the opening mind, ſuitable ſubjects of thought, and aſſiſt it in purſuing a ſimple train of reflection—ſomething that ſhall convey knowledge in the guiſe of amuſement; that ſhall not be impoſed as a taſk, but conferred as a favour.

The ſubjects of theſe Letters, and the Author's method of treating them, ſeem to me much adapted to this purpoſe. With the aſſiſtance of a very moderate apparatus, they might conduct youth of both ſexes, with equal delight and emolument, to a very competent knowledge of natural philoſophy: very little previous elementary knowledge is neceſſary to a profitable peruſal of them, and that little may be very eaſily acquired.

A con-

A confiderable part of our common fchool education, it is well known, confifts of the ftudy of the elegant and amufing poetical fictions of Antiquity. Without meaning to decry this, may I not be permitted to hint, that it might be of importance frequently to recall young minds from an ideal world, and it's ideal inhabitants, to the real world, of which they are a part, and of which it is a fhame to be ignorant. Let your pupil, by all means, read the poets; let him read Ovid, and, after he has amufed himfelf with the golden age of old Saturn, lead him out into the open firmament of heaven, and fhew him the venerable planet of that name, coeval with time, yet fhining with unimpaired luftre, after fo many revolutions of ages. Having adminiftered the antidote that may repel the poifon, which a difplay of the lewd intrigues of a fabulous Jupiter or Venus naturally inftill; let him view, through the

the telefcope, the two beautiful ftars fo called, emitting their chafte and modeft light to the unpolluted eye of fober reafon. When he has diverted himfelf with the transformation of a lady into a bear, and that bear into a conftellation, point out to him the heavenly northern light, which never changes it's place, and, with undeviating fidelity, conducts the mariner through the feas of a hemifphere. Let him accompany Phaeton to the palace of the fun, and fmile at beholding the adventurous boy mount the flaming chariot; and then check his mirth by pointing to the glorious orb of day, travelling in the greatnefs of his ftrength; not dragged round the earth by fiery-footed fteeds, but wheeling worlds on worlds, each in his feveral orbit, around him, with irrefiftible force.

Why fhould not the boy be taught the principle on which his kite flies? What

What more pleaſant amuſement can he have than to communicate to the needle the magnetic virtue, and to ſteer his courſe through the hazel grove, by a compaſs of his own conſtructing? Why not teach him to elicit the electric ſpark; and to aſtoniſh and delight his ſiſters with the wonders of the magic lantern?

Euler wrote theſe Letters for the inſtruction of a young and ſenſible female, and in the ſame view that they were written, they are tranſlated, namely, the improvement of the female mind; an object of what importance to the world! I rejoice to think I have lived to ſee female education conducted on a more liberal and enlarged plan. I am old enough to remember the time when well-born young women, even of the north, could ſpell their own language but very indifferently, and ſome hardly

read it with common decency; when the young lady's hand-writing preſented a medley of outlandiſh characters; and when a column of pounds, ſhillings and pence preſented a labyrinth as inextricable as the extraction of the cube root. While the boys of the family were converſing with Virgil, perhaps with old Homer himſelf, the poor girls were condemned to croſs-ſtitch, on a piece of gauze-canvaſs, and to record their own age at the bottom of a ſampler.

They are now treated as rational beings, and ſociety is already the better for it. And wherefore ſhould the terms *female* and *philoſophy* ſeem a ridiculous combination? Wherefore preclude to a woman any ſource of knowledge to which her capacity, and condition in life, entitle her to apply? It is cruel and ungenerous to expoſe the frivolity of the ſex, after reducing it to the neceſſity of being ſilly and frivolous. Cultivate a young woman's underſtanding, and her perſon will become, even to herſelf, only a ſecondary concern; let her time be filled up in the acquiſition of attainable and uſeful knowledge, and then ſhe will ceaſe to be a burden to herſelf and to every body about her; make her acquainted with the world of nature, and the world of art will delude her no longer.

The time, I truſt, is at hand, when the Letters of EULER, or ſome ſuch book, will be daily on the breakfaſting table, in the parlour of every female academy in the kingdom; and when a young woman, while learning the uſeful arts of paſtry and plain-work, may likewiſe be acquainting herſelf with the phaſes of the moon, and the flux and reflux of the tides. And I am perſuaded ſhe may thrum on the guitar, or touch the keys of the harpſichord, much more agreeably both to herſelf and others, by ſtudying

ſtudying a little the theory of ſound. I have put the means of this in her power; it will be at once her fault and her folly if ſhe neglect it.

In tranſlating the Work, I have followed the laſt Paris Edition, given by Meſſrs. *de Condorcet & de la Croix*, in 1787, for the purpoſe of introducing the uſeful notes of theſe gentlemen; but I have taken the liberty to reſtore, from the original edition, that of Mietau and Leipſic, in 1770, ſeveral paſſages which the French Editor had thought proper to ſuppreſs. To ſome notes of my own I have added ſeveral others, furniſhed by two ingenious friends, whoſe names I am not at liberty to publiſh. The courſe of thirty-four years of a ſcientific age, muſt have ſupplied abundance of new facts and experiments, by which the philoſophy of even a EULER may be corrected and improved. The tranſlated notes of the Paris Edition, I have, for the ſake of diſtinction, marked with the characters *F. E.* and the original notes of this Edition, with the initials *E. E.* And I think it my duty, in this place, to vindicate to our ingenious countryman, Mr. *Dollond*, the optician, the diſcovery of achromatic glaſſes for teleſcopes, mentioned in the letters on dioptrics; for that gentleman is, in truth, the Author of this valuable improvement.

I have had the illuſtrative plates engraved in a better ſtyle and manner than French artiſts generally employ on mathematical figures: and to do credit to myſelf, not to ſay EULER, he appears in his Engliſh dreſs with every advantage which the ſtationer and printer could beſtow. At the ſame time, in order to keep down the price as much as poſſible, inſtead of dividing the Work into

Three Volumes, as in all the foreign editions, I have reduced mine to *Two*; as the diviſion is altogether indifferent to the ſubjects.

It being generally acceptable to the Reader to know ſomething about the *man* with whom he is converſing as an *author*; to gratify this curioſity, I have likewiſe given a tranſlation of the *Elogium of* EULER, read before the Academy of Sciences, and prefixed to M. de *Condorcet's* edition, becauſe it contains ſome intereſting traits of the character and events of the life of this diſtinguiſhed perſonage. But what is the life of a literary or ſcientific man, and where are we to find the hiſtory of it? In his works. NEWTON and EULER are their own beſt biographers; and the library of every ſcholar in Europe exhibits a never-dying repreſentation of what they were, and what they atchieved. We have hardly a trace of *Wren's* perſonal and domeſtic habits; but every ſtone of St. Stephen's Walbrook, and of St. Paul's, is inſcribed with his name, and tranſmits a memoir of the Architect.

The frequent, tireſome, courtly addreſs of YOUR HIGHNESS, except at the firſt ſetting out, I have entirely omitted; out of no diſreſpect to Princes, but becauſe it ſeemed to me a mere unneceſſary waſte of words, which only encumber and disfigure a work of ſcience. The Princeſs and her inſtructor are both gone to that awful world, in which the diſtinctions of the preſent, thoſe of virtue excepted, are for ever obliterated.

As every book ſhould be as complete in itſelf as poſſible, and this being deſtined to the uſe of the unlearned, I have ſubjoined a gloſſary of the foreign and ſcientific words which occur in the courſe

of

of theſe Letters. Some will, perhaps, think I may have ſwelled this beyond the neceſſary ſize, and given an explanation of many terms already ſufficiently underſtood. If this be an error, it is on the ſafe ſide. I would rather inſert twenty words of this deſcription, than omit one with which an ordinary reader might be unacquainted, and his progreſs thereby retarded. And I well know, that there is often a vague and obſcure idea of words floating in the brain, which a ſhort deſcription or an example would inſtantly render preciſe and diſtinct: and many young perſons would, without heſitation, conſult a gloſſary, who might be afraid, or aſhamed, or, perhaps, too proud, to ask a queſtion.

H. H.

Hoxton, 1802.

ADVERTISEMENT.

BY THE FRENCH EDITOR.

THE Letters of EULER to a German Princefs have acquired, over all Europe, a celebrity, to which the reputation of the Author, the choice and importance of the feveral fubjects, and the clearnefs of elucidation, juftly entitle them. They have defervedly been confidered as a treafury of fcience, adapted to the purpofes of every common feminary of learning. They may be ftudied to advantage without much previous elementary knowledge; they convey accurate ideas refpecting a variety of objects, highly interefting in themfelves, or calculated to excite a laudable curiofity; they infpire a proper tafte for the fciences, and for that found philofophy which, fupported by fcience, and never lofing fight of her cautious, fteady, methodical advances, runs no rifk of perplexing, or mifleading the attentive ftudent.

The only cenfure that can be paffed on thefe Letters is, now and then, a digreffive detail, fomewhat ſomewhat too tedious, on queſtions rather foreign to the ſciences, and conſiderable inaccuracy in point of ſtyle. Without failing in the reſpect due to EULER, I thought myſelf at liberty to omit ſome paſſages altogether, and to correct the ſtyle of others. Few Readers, ſurely, will be ſo faſtidious as to refuſe the admiration attached to the name of this illuſtrious man, for the ſake of ſome ſlight blemiſhes, in a work of ſuch conſiderable length. A genius, like his, which has ſignalized itſelf by ſo many important diſcoveries, can ſuffer no diminution of greatneſs, from his not having written a foreign language with claſſical purity. A man whoſe tranſcendant powers have aſtoniſhed and confounded even thoſe whom habits of profound reflection muſt have rendered hard to pleaſe, reſpecting prodigies of this ſort, is not leſs worthy of veneration, that he did not apply the whole force of his mind to every object which preſented itſelf. It is of the laſt indifference to his glory, whether theſe ſmall ſpecks are effaced, or ſuffered to remain.

But the caſe is widely different as to the perſons for whoſe uſe the peruſal of this work is particularly deſigned. It is of importance for young people, whether of France, or of any other country, to defer reading till they thoroughly underſtand the language of books, in which the rules

rules of that language may be frequently violated. And the youth of the French nation muſt be cautioned againſt turning into ridicule a few uncouth expreſſions which, in the hurry of compoſition, may have dropped from the pen of a man of genius. Reſpect for every thing which merits this appellation is one of the ſentiments which education ought moſt powerfully to inculcate, as it is one of the moſt infallible preſervatives againſt prejudice of every kind, againſt the illuſions of vanity and ſelf-love, nay, againſt the paſſions which deprive us of the force neceſſary to our approximation toward theſe objects of univerſal admiration.

As to other retrenchments, they affect, almoſt all of them, reflections which relate leſs to the ſciences and philoſophy, than to theology, and frequently even to the peculiar doctrines of that eccleſiaſtical communion in which EULER lived. It is unneceſſary to aſſign a reaſon for omiſſions of this deſcription.

I have prefixed to this edition the Elogium of EULER, read before the Academy of Sciences, omitting only ſome ſcientific details, which might have appeared tedious to certain Readers.

As the Letters of EULER contain nothing, on ſeveral queſtions, capable of intereſting the generality of mankind, I have made ſome additions, but

but without throwing them into the form of letters. Thoſe publiſhed at the ſame time with this edition, have for their only object the calculation of probabilities. I took for granted that perſons diſpoſed to give them a peruſal, muſt have already made a conſiderable progreſs in mathematical knowledge. This branch of knowledge occupies at preſent, and ever muſt, a diſtinguiſhed place in a courſe of liberal education. If it is not abſolutely impoſſible to do without it, in order to the attainment of accurate ideas in phyſics, and reſpecting the laws of the univerſe, and the calculations of probability, we ſhall, at leaſt, by the ſtudy of mathematics, ſave much time and trouble, and acquire a habit of thinking and reaſoning on other ſubjects with greater exactneſs.

The idea which ſome have formed of the difficulty of this ſcience, obſtructs the progreſs of knowledge, and is not founded in truth. There are few minds, unleſs a previous education has already impreſſed falſe ideas, and a factitious delicacy, but what are capable of receiving the ideas neceſſary to mathematical combination, of acquiring the habit of purſuing them, and a reliſh for the ſimple truths which they preſent. As to the extraordinary powers which are deemed requiſite, I venture to affirm, that there are few perſons,

perſons, even of moderate capacity, who may not, by employing a little more time, and purſuing a courſe ſomewhat more deliberate, by entering more attentively into detail, and from frequently repeated applications, attain a degree of mathematical knowledge, far beyond what is really uſeful, nay, I add, neceſſary, to all men of liberal education.

ELOGIUM

ELOGIUM OF EULER.

LEONARD EULER, Prefident of the Mathematical School, in the Academy of Peterfburg, and previoufly in that of Berlin; Fellow of the Royal Society in London; and of the Academies of Turin, Lifbon and Bâle; Foreign Affociate of that of the Sciences, was born at Bâle, April the 15th, 1707, being the fon of *Paul Euler* and *Margaret Brucker*.

His father who, in 1708, undertook the paftoral charge of the village of Riechen, in the vicinity of Bâle, was his firft inftructor; and he enjoyed betimes the pleafure of contemplating the progrefs of his fon's expanding faculties, and dawning glory, a cordial fo reviving to the heart of a parent, advance under his own eye, and gather ftrength from his own affiduities.

He had ftudied mathematics under *James Bernouilli*. It is well known, that this celebrated fcholar united to a great genius for the fciences, a profound philofophy, which is not always the companion of this genius, but which ferves to give it a wider range, and to render it's exertions more ufeful. In teaching, he endeavoured to imprefs on his pupils, that geometry is not a detached fcience, but exhibited it to them as, at once, the bafis and the key-ftone of all human knowledge; as the ſcience in which the progreſs of the mind may be the moſt diſtinctly obſerved; the ſcience, the cultivation of which exerciſes our faculties to the greateſt advantage, as giving to the underſtanding, at one and the ſame time, ſtrength and accuracy; finally, as a ſtudy equally valuable, from the number and the variety of it's applications, and from it's tendency to inure the ſtudent to a method of reaſoning, which may, afterwards, be ſucceſsfully employed, in the inveſtigation of every ſpecies of truth, and as a guide in the conduct of life.

Paul Euler, who had fully imbibed the principles of his maſter, inſtructed his ſon in the elements of mathematics, though he had deſtined him, ultimately, to the ſtudy of theology; and ſuch was young EULER's early proficiency, that on being ſent to the univerſity of Bâle, he was deemed not unworthy of the attention and particular inſtructions of *John Bernouilli*. Such was his application, and ſuch his happy diſpoſitions, as quickly to ſecure to him the friendſhip of *Daniel* and *Nicolas Bernouilli*, the pupils, and, by this time, the rivals of their father. Nay, he had the felicity of getting into the good graces of the ſevere *John Bernouilli* himſelf, who carried his condeſcenſion ſo far as to give him a private leſſon, once a week, in the view of removing any difficulties which might occur in the courſe of reading and ſtudy. EULER employed the other days of the week in ſuch a manner as would enable him to make the moſt of this diſtinguiſhed mark of favour.

This

This excellent method of proſecuting his ſtudies, preſerved opening genius from exhauſting it's ſtrength, in combating inſuperable difficulties, and from wandering in unknown mazes, which it might attempt to unravel: it directed and ſeconded his own exertions; but, at the ſame time, laid him under the neceſſity of calling forth all his powers, which, accordingly, received conſtant increaſe from an exerciſe proportioned to his age, and to the progreſs in knowledge which he had already made.

But of this ſingular advantage he was ſoon deprived; for ſcarcely had he attained the degree of Maſter of Arts, when his father, who intended him for his own ſucceſſor, enjoined him to exchange the ſtudy of mathematics for that of theology. Happily, the effect of this act of authority was of ſhort duration. It proved no difficult matter to perſuade the father, that his ſon was deſtined to ſupply, to the learned world, the place of *John Bernouilli*, and not to ſink into the obſcure parſon of Riechen.

An eſſay, compoſed by EULER in his nineteenth year, on the maſting of ſhips, a ſubject propoſed by the Academy of Sciences, procured him, in 1727, an addition to his academical honours, ſo much the more reſpectable, that the youthful native of the Alps could have derived no aſſiſtance from practical knowledge, and that he yielded the palm to Mr. *Bouguer* alone, an able geometrician, then at the zenith of his reputation, and, for ten years before, profeſſor of hydrography in a maritime city.

About the ſame period, EULER ſtood candidate

for

for a vacant chair in the univerſity of Bâle: but it is fate, or chance, that ſettles the diſpute between competitors for offices of this ſort, and, on this occaſion, it was unfavourable, I do not ſay to Euler, but to his country, which, a few days afterward, loſt him for ever.

Two years before this, *Daniel* and *Nicolas Bernouilli* had been invited to Ruſſia. Euler felt the ſincereſt regret at parting with the friends of his youth, and engaged them to promiſe their utmoſt exertions to procure him a ſimilar invitation, which he was eager to participate. This needs to excite no ſurprize. The ſplendor of the capital of a vaſt empire, the glare diffuſing itſelf over the purſuits of which it is the theatre, and over the very perſons of it's inhabitants, ſeems to confer a glory on them, capable of eaſily ſeducing a youthful imagination, and of dazzling the free, but poor and obſcure, citizen of a petty republic.

The brothers, *Bernouilli*, were conſcientiouſly faithful to their promiſe, and exerted themſelves as ſtrenuouſly, to bring forward a competitor ſo formidable, as ordinary men would have done to keep a rival out of ſight.

Euler's journey to Ruſſia commenced under auſpices the moſt melancholy and diſcouraging. It was not long before he received intelligence, that *Nicolas Bernouilli* had fallen a victim to the ſeverity of the climate; and the very day he ſet foot on Ruſſian ground, Catharine I. paid the debt of nature. This event, at firſt, ſeemed to threaten the approaching

ing diſſolution of the Academy, whoſe eſtabliſhment that Princeſs had juſt completed, in compliance with the will of the deceaſed Czar, her huſband.

Euler, at a prodigious diſtance from his native country, deſtitute of the advantage which *Daniel Bernouilli* poſſeſſed, that of an illuſtrious and reſpected name, to prepare his way, formed the reſolution of entering into the Ruſſian marine ſervice. One of the admirals of Peter I. had already promiſed to procure him a ſituation; when, happily for geometry, the ſtorm, which lowered over the ſciences, ſpent itſelf. *Daniel Bernouilli* retired to his own country: Euler was declared Profeſſor of Geometry, and ſucceſſor to his illuſtrious friend, in 1733. The ſame year he married a young lady of the name of *Gſell*, a compatriot of his own, the daughter of a painter, whom Peter I. had brought with him to Ruſſia, on returning from his firſt voyage.

From this time forward, to uſe *Bacon*'s expreſſion, Euler felt that he had given hoſtages to fortune: and that the country, in which he could hope to form an eſtabliſhment for his family, was neceſſarily tranſformed into his native country. Born and educated in the boſom of a nation, all whoſe governments preſerve, at leaſt, the appearance and the language of a republican conſtitution; in which, notwithſtanding diſtinctions more real, than thoſe which ſeparate between the higheſt ſlave of a deſpot and the loweſt of his ſubjects, the forms of equality have always been ſcrupulouſly obſerved; in which the reſpect due to the laws extends to uſages the moſt indifferent, provided

vided they have the fanction of antiquity, and of vulgar opinion: EULER found himfelf in a country, where the Prince exercifes unlimited authority; where the moft facred law of abfolute governments, that which regulates the fucceffion to the throne, was at that time uncertain, or treated with contempt; where grandees, enflaved to the fovereign, rule with a defpotic fway over an enflaved people; where, at the very moment, a vaft empire, under the government of an ambitious, jealous and cruel foreign defpot, was enduring all the tyranny of the unrelenting *Biren*, and prefenting a fpectacle as terrifying as inftructive to men of letters, who had been enticed to feek in it's bofom, glory, fortune, and the power of enjoying, in perfect fecurity, the calm delights of literary refearch.

It is eafier to conceive, than to defcribe, what muft have been the feelings of EULER, bound to remain in fuch a fituation, by chains which it was impoffible for him to burft afunder. To this circumftance, however, we are, perhaps, indebted for that unremitted application to literary purfuits, of which he then acquired the habit, and which became his only refource, in a capital, filled with the parafites, or the enemies, of a violent Minifter; the former intent on feeding and flattering his fufpicious temper; the latter employed in fecuring themfelves from it's fanguinary effects. This had made fo deep an impreffion on the mind of EULER, that we find the traces of it fo late as 1741, the year after *Biren*'s fall, when tyranny had given place to a government more mild and humane. At that period he went from Peterfburg to Berlin, on the earneft folicitation of the King of Pruffia. He was prefented to the Queen-mother. This Princefs took great pleafure in the converfation of enlightened men: fhe received them with that noble familiarity which announces, in Princes, the fentiment of a perfonal greatnefs, independent of rank and title, and which has become one of the characteriftic marks of that auguft family. The Queen of Pruffia, however, could extract from EULER monofyllables only: fhe taxed him with a timidity and referve, which the cordiality of his reception could not poffibly have infpired: *Why, then, will you not talk to me*, faid the Queen? *Becaufe Madam*, replied he, *I have juft come from a country, where people are hanged, if they talk.*

Feeling myfelf now called upon to give fome account of EULER's immenfe fcientific labours, I fhrink from the impoffibility of following him in detail, of conveying any thing like an accurate idea of that multiplicity of difcoveries, of new methods of inveftigation, of ingenious views, diffufed over more than thirty feparate publications, and over near feven hundred memoirs, of which about two hundred, depofited in the Academy of Peterfburg previous to his death, are deftined to enrich, in their order, the future collections publifhed by that learned body.

But a particular character feems, to me, to diftinguifh EULER from the other illuftrious men who, in purfuing the fame career, have attained a glory which his has not eclipfed; that character is, his having embraced the mathematical ſciences in their univerſality; his having brought to perfection, one after another, the different parts; and, enriching the whole by important diſcoveries, his having produced a very beneficial revolution in the manner of treating them. I imagined, therefore, that in ſketching a methodical repreſentation of the different branches of theſe ſciences, in pointing out the progreſs of each, and the happy improvements to be aſcribed to the genius of EULER, I ſhould give, at leaſt as far as my ability permits, a juſter idea of this wonderful man, who, by uniting ſo many extraordinary talents, has preſented a phenomenon, if the expreſſion may be allowed, of which the hiſtory of ſcience has hitherto furniſhed no example.

Algebra had long been a ſcience of very limited uſe and application. The mode of conſidering the idea of magnitude, only in the higheſt degree of abſtraction of which the human mind is ſuſceptible; it's rigorouſly ſeparating from that idea every thing which, by employing imagination, might give ſupport, or repoſe, to the underſtanding; finally, the extreme generality of the ſigns which this ſcience makes uſe of, render it in ſome meaſure too foreign to our nature, too remote from ordinary conception, to admit of the mind's taking extraordinary pleaſure in it, and of eaſily acquiring a habit of tracing it's operations. The algebraic method is apt to diſcourage even perſons the moſt diſpoſed to abſtract ſpeculation. If the object of purſuit be ever ſo little complicated, we are forced to loſe ſight of it entirely,

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and to confine our whole attention to dry algebraic characters; the road is ſafe and ſure, but the point which is aimed at, and that from whence we took our departure, equally vaniſh from the eye of the geometrician; and it required no ſlight degree of courage, to venture out of ſight of land, without any other pilot than a recently diſcovered ſcience. Accordingly, on examining the works of the great geometricians of the laſt age, even of thoſe to whom algebra is indebted for the moſt important diſcoveries, we ſhall ſee how little they were accuſtomed to handle this very weapon, which has been brought to ſuch a ſtate of perfection; and it is impoſſible to refuſe to EULER the praiſe of having effected a revolution, which renders algebraic analyſis a mode of calculation luminous, univerſal, of general application and of eaſy acquiſition.

Thus, at certain epochs, when after ſtrenuous exertions the mathematical ſciences ſeemed to have exhauſted all the reſources of genius, and to have reached the *ne plus ultra* of their career; all at once a new method of calculation is introduced, and the face of the ſcience is totally changed. We find it immediately, and with inconceivable rapidity, enriching the ſphere of knowledge, by a ſolution of an incredible number of important problems, which geometricians had not dared to attempt, intimidated by the difficulty, not to ſay the phyſical impoſſibility, of purſuing calculation to a real iſſue. Juſtice would, perhaps, demand, in favour of the man who invented

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and introduced these methods, and who first taught their use and application, a share in the glory of all those who have practised them with success; he has, at least, claims upon their gratitude, which cannot be contested without a crime.

EULER had neglected no part of analysis: he has demonstrated some of the theorems of *Fermat*, on indeterminate analysis, and has discovered many others, no less curious, and of no less difficult investigation. The knight's movement, in the game of chess, and different other problems of situation, have likewise excited his curiosity, and exercised his genius. He blended with researches the most important, amusements of this sort, frequently more difficult, but of little use either to the progress of the science itself, or to the applications hitherto attempted. EULER was too discreet, not to be sensible of the impropriety of devoting much of his attention to researches of mere curiosity; but at the same time of a mind too enlarged not to discern, that their inutility could only be momentaneous, and that the only means of exposing their inutility, was to attempt to unfold and generalize them.

The particular questions which do not strictly belong to the regular body of mathematical science, which do not enter into the applications of which it is susceptible, ought not to be considered merely as the means of exercising the powers, and displaying the genius of the geometrician: in cultivating the sciences, we almost always set out with attaching

ourselves to some detached parts, in preference; in proportion as successive discoveries multiply, the relations which unite the parts gradually appear; and to the illumination resulting from this union, we are most frequently indebted for the great discoveries, which form an era in the history of the human mind.

I shall conclude this brief representation of EULER's labours, on pure analysis, with observing, that it would be injust to limit it's influence on the progress of mathematics, to the innumerable discoveries with which his works abound. The communications which he has opened between all the parts of a science so extensive; those general views which sometimes he does not so much as indicate, but which cannot escape an attentive observer; the paths, whose entrance he has satisfied himself with clearing by removing the first obstacles which opposed; these are so many more benefits conferred on the sphere of science, and of which posterity will undoubtedly avail itself, while perhaps the hand which bestowed them may be forgotten.

The treatise on mechanics, which EULER gave to the world in 1736, is the first great work in which analysis has been applied to the science of motion. The number of things, entirely new, or exhibited in a new light, which this book contains, would have astonished geometricians, had not EULER already published, separately, the greatest part of it.

In his endless labours on the same science, he was ever faithful to analysis, and the happy use he made of it, has at laſt procured for the analytical method a preference to every other.

The problem of vibrating chords, and all thoſe which belong to the theory of ſound, or the laws of the oſcillation of the air, have been ſubjected to analyſis, by the new methods with which he enriched the calculation of partial differences. A theory of the motions of fluids, founded on the ſame calculation, aſtoniſhed every one by the clearneſs which he has diffuſed over queſtions ſo intricate, and the facility which he has communicated to modes of operation, founded on an analyſis ſo profound.

All the problems of phyſical-aſtronomy, treated in the preſent age, have been reſolved, by the analytical method peculiar to EULER. His calculation of the perturbations of the earth's orbit, and eſpecially his theory of the moon, may be held up as models of the ſimplicity, of the preciſion, to which this method may be carried; and in reading this laſt work, we ſee, with no leſs aſtoniſhment, how far a man of great genius, animated with a deſire of omitting nothing eſſential on an important queſtion, has been able to carry the patience and perſeverance of application.

Aſtronomy employed only the geometric method: EULER was ſenſible of every thing which that ſcience had to expect from the aid of analyſis, and he demonſtrated it by examples which, imitated ſince by men of ability and reputation, may in time beſtow a new form on aſtronomy.

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He treated, in all it's extent, the naval ſcience; in an elaborate work, to which an intelligent analyſis ſerves as baſis, and in which queſtions of the greateſt difficulty are ſubjected to this general and fertile method, which he underſtood ſo well to create and to employ. He publiſhed, many years afterward, on the ſame ſubject, an elementary abridgment of this treatiſe, containing, under the ſimpleſt form, every thing uſeful in practice, and neceſſary to be known by perſons who devote themſelves to the marine ſervice. This work, though deſigned by the Author merely for the ſchools of the Ruſſian empire, procured for him a liberal gratification from the King of France, who judged, that labours beneficial to mankind demanded the grateful acknowledgments of all Sovereigns, and who wiſhed to demonſtrate to Europe, from one extremity to another, that talents ſo rare could neither be overlooked, nor remain unrewarded. EULER was abundantly ſenſible of the value of this mark of reſpect from a great Prince; and it derived an additional charm, in his eyes, from the hand through which it was tranſmitted, that of Mr. *Turgot*, a miniſter univerſally reſpected for his talents and for his virtues; a man formed for commanding opinion, rather than following it, and whoſe ſuffrage, ever dictated by truth, and never by the deſire of attracting to himſelf the applauſe of the public, might be an acceptable piece of flattery, even to a wiſe man, too much accuſtomed to glory to be ſtill awake to the voice of fame.

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In men of a ſuperior genius, extreme ſimplicity of character may eaſily conſiſt with thoſe qualities of mind, which moſt forcibly announce ability and delicacy of feeling. EULER, accordingly, notwithſtanding that ſimplicity which never forſook him, knew, however, to diſtinguiſh with a ſagacity, always indulgent it is true, the homage of enlightened admiration from that which vanity laviſhes on great men, to ſecure to itſelf at leaſt the merit of enthuſiaſm.

His dioptrical reſearches are founded on an analyſis leſs profound, and we are tempted to give him credit for it, as being a kind of ſacrifice. The different rays of which a ſolar ray is formed, ſubſiſt in the ſame medium of different refractions; ſeparated thus from adjacent rays, they appear ſingle, or leſs blended, and give the ſenſation of the colour proper to them. This refrangibility varies in different mediums for every ray, and in conformity to a law which is not the ſame with that of the mean refraction in theſe mediums. This obſervation ſuggeſted a belief, that two unequal priſms, and of different ſubſtances, combined, might divert a ray from it's direction, without decompounding it, or rather by replacing the elementary rays, by refraction, in a parallel direction. On the truth of this conjecture might depend, in teleſcopes, the deſtruction of the iris, which colours objects viewed through lenticular glaſſes. EULER was convinced of the poſſibility of ſucceſs, conformably to this metaphyſical idea, that, *if the eye is compoſed of different humours, it is only in the view of deſtroying the effect of the aberration of refrangibility*. The only thing requiſite, therefore, was an attempt to imitate the operation of nature, and he propoſed the means of execution according to a theory which he had formed. His firſt eſſays induced naturaliſts to attend to an object which they ſeemed to have neglected. Their experiments did not correſpond to EULER's theory, but they confirmed the views he entertained reſpecting the perfection of teleſcopes. And, inſtructed by theſe, in the laws of diſperſion, in different mediums, he abandoned his firſt ideas, ſubjected to calculation the reſult of their experiments, and enriched dioptrics with analytical formules, ſimple, commodious, general, and applicable to inſtruments of every poſſible conſtruction.

We have, beſides, ſome eſſays of EULER, on the general theory of light, the phenomena of which he endeavoured to reconcile with the laws of the oſcillations of a fluid; becauſe the hypotheſis of the emiſſion of rays in a ſtraight line, appeared to him to preſent inſurmountable difficulties. The theory of the loadſtone, that of the propagation of fire, the laws of the coheſion of bodies, and thoſe of friction, furniſhed him, likewiſe, with ſubjects of ingenious calculations, but, unfortunately, ſupported by hypotheſis, rather than by experiment.

The calculation of probabilities and political arithmetic were farther objects of his indefatigable application. I ſhall here only mention his reſearches on on bills of mortality, and the means of deducing them from phenomena with greater exactnefs; his method of ftriking a medium from the obfervations made; his calculations refpecting the eftablifhment of a reverfionary fund, in the view of fecuring to widows, or orphans, either a fixed fum, or an annual revenue, payable after the death of a hufband or father; an ingenious and humane method, devifed by philofophic geometricians to counterbalance the moral evil refulting from the fettlement of life-annuities, and to convert, to the relief of families, the fmalleft favings from the principal's daily earnings, or from the revenue of a commiffion, a place or a penfion.

We have feen in the elogium of *Daniel Bernouilli*, that he had divided with EULER alone the glory of having carried off thirteen prizes, propofed by the Academy of Sciences: They often contended for the fame object, and occupied the fame ground: and the honour of triumph over a competitor was likewife divided between them; but this rivalfhip never encroached on the expreffions of reciprocal efteem, nor cooled the ardor of mutual friendfhip. On examining the fubjects for which the one or the other obtained the victory, we find that fuccefs depended principally on the character of talent peculiar to each. When the queftion required addrefs in the manner of taking it up, a dexterous application of experiment, or new and ingenious phyfical views, *Daniel Bernouilli* had the advantage: but did it prefent

fent difficulties, which profound and accurate calculation could refolve; was it neceffary to create a new method of analyfis, victory declared for EULER. Were any one fo prefumptuous as pretend to judge between them, he would find that he had to pronounce, not between two men, but between minds of a different genius, between two methods of employing genius.

I fhould have conveyed but a very imperfect idea of EULER's fertility of invention, unlefs I added to this faint fketch of his labours, that there are very few fubjects of importance, once treated by him, that he did not retrace; nay, fo far as to recompofe his firft work feveral times over. Sometimes he fubftituted a direct and analytical method, in place of one more indirect: fometimes he extended his firft folution to cafes which had at firft efcaped him; adding almoft always new examples, which he knew how to felect with fingular fkill among thofe which prefented, or fome ufeful obfervation, or curious remark.

The intention merely of giving to one of his productions a form more methodical, of rendering it fomewhat more luminous, of beftowing on it a higher degree of fimplicity was to him motive fufficient for engaging in labours incredible. Never did geometrician write fo much, and no one ever carried his works to fuch a height of perfection. When he publifhed a memoir on a new fubject, he fimply explained the track which he purfued; he pointed out to to his pupils it's intricacies and aberrations, and having, with fcrupulous exactnefs, made them accompany the progrefs of his own mind, in his firft effays, he fhewed them afterwards how he had been enabled to trace a fimpler path. It is evident, that he preferred the inftruction of his difciples to the filly fatisfaction of dazzling them by his own fuperiority; and that he did not believe he had done enough for fcience, unlefs he added, to the new truths with which he was enriching it, a candid expofition of the ideas which led to difcovery.

On reading the life of a great man, whether it be a conviction of the imperfection attached to frail humanity; whether it be, that the juftice of which we are capable, does not rife fo high as to induce us to acknowledge a fuperiority for which nothing can be an adequate compenfation; or, finally, whether it be, that the idea of perfection in another mortifies or humbles us ftill more than that of his greatnefs, but fome how or another it feems neceffary for us to find out fome weak part; we hunt after the difcovery of a defect in him, which may reconcile us to ourfelves; and we are involuntarily difpofed to call in queftion the impartiality of the Biographer, unlefs he points out the weak part, unlefs he withdraws the impertinent veil which conceals the defect.

EULER fometimes appeared to be taken up with the mere pleafure of calculation, and to confider the point of mechanics, or phyfics, which he was examining, only as an occafion of exercifing his genius, and of

of following the bent of his reigning paffion. Some of the learned have accordingly accufed him of lavifhing his talent for calculation on phyfical hypothefes, or even on metaphyfical principles, of which he had not fufficiently examined either the probability or the folidity. He has likewife been accufed of depending too much on the refources of calculation, and of having neglected thofe with which he might have been fupplied, by the examination of the very queftions which he propofed to refolve.

We muft admit, that the firft of thefe charges is not altogether deftitute of foundation. In EULER, undoubtedly, the metaphyfician, or even the naturalift was not fo great as the geometrician; and we are conftrained to regret, that in many parts of his works, thofe, for inftance, which he compofed on the naval fcience, on artillery, have been of little ufe, except to the progrefs of the fcience of calculation.

But the fecond charge appears by no means fo well founded. We obferve uniformly, through all the works of EULER, an unremitting effort to add to the riches of analyfis, to extend, and to multiply the applications of it: at the fame time that it appears to be his only inftrument, we fee clearly that it is his wifh to make it univerfally fo. The natural progrefs of the mathematical fciences muft have, in time, brought about this revolution; but he faw it, if I may fay fo, completed under his own eye: to his genius we are indebted for it; and it has been the

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reward of all his exertions and diſcoveries. Accordingly, even when he appears to be miſapplying analyſis, and exhauſting all it's ſecret ſtores in reſolving a queſtion, of which a few reflections, foreign to calculation, would have given him an eaſy and ſimple ſolution, he was frequently only aiming at a demonſtration of the power and reſources of his art; and he merits forgiveneſs at leaſt, if ſometimes, while he ſeemed taken up with another ſcience, it was ſtill to the progreſs and propagation of analyſis that his attention was devoted; and the revolution which this has effected in the world of ſcience, is one of his firſt claims on the gratitude of mankind, and the faireſt title to glory.

I thought myſelf obliged not to interrupt the detail of *Euler*'s ſcientific purſuits, by a recital of the few and ſimple events of his life.

He ſettled at Berlin in 1741, and remained there till 1766.

The Princeſs *d'Anhalt Deſſau*, niece to Frederick II, King of Pruſſia, was deſirous of receiving from him ſome leſſons in natural philoſophy. Theſe leſſons have been publiſhed, under the title of LETTERS TO A GERMAN PRINCESS, a work ineſtimable for the ſingularly clear light in which he has diſplayed the moſt important truths of mechanics, of phyſical-aſtronomy, of optics, and of the theory of ſound; and for the ingenious views, leſs philoſophical but more ſage, than thoſe which have made *Fontenelle*'s Plurality of Worlds outlive the Syſtem of Vortices.

The name of EULER, ſo great in the ſphere of ſcience; the reſpectful idea attached to his works, employed in unfolding all that is intricate and abſtract in analyſis, diffuſe a ſingular charm over theſe letters, ſo ſimple, and ſo eaſy. Thoſe who have not ſtudied mathematics, aſtoniſhed, perhaps flattered, at being able to underſtand a work of EULER, will feel grateful to him for having deſcended to their level; and theſe elementary details of the ſciences acquire a ſpecies of greatneſs, from their approximation to the glory, and the genius, of the illuſtrious man who traced them.

The King of Pruſſia employed EULER in calculations reſpecting the coinage; on conſtructing the aqueduct of Sans-Soucis; on the formation of ſeveral navigable canals. That great Prince had a mind too enlarged to believe that extraordinary talents, and profound knowledge, ever could be uſeleſs or dangerous qualities; and the felicity of being able to do good, an advantage reſerved by nature for ignorance and mediocrity.

In 1750, EULER made a journey to Frankfort, to receive his mother, then a widow, and to conduct her to Berlin. He had the happineſs to preſerve her till 1761. For eleven years, then, ſhe enjoyed the glory of her highly diſtinguiſhed ſon, in the way that the maternal heart knows how to enjoy, and was ſtill more happy, perhaps, in the tender and aſſiduous expreſſions of filial affection, the value of which that glory greatly enhanced.

During his refidence at Berlin, EULER, united to Mr. *de Maupertuis* by the ties of gratitude, thought himfelf obliged to defend the principle of *the leaft action*, on which the Prefident of the academy of Pruffia had founded the hope of a reputation fo exalted. The means which EULER thought proper to ufe could hardly have been employed by any other perfon but himfelf; it was to refolve, on this principle, feveral of the principal and moft difficult problems of mechanics. Thus, in the age of fable, the Gods vouchfafed to forge, for their favourite warriors, armour impenetrable by all the blows of their enemies. It were to have been wifhed, that Euler's gratitude had confined itfelf to a protection fo noble, and fo worthy of himfelf; but it cannot be denied, that there is an infufion of afperity, rather too ftrong, in his replies to *Kœnig*; and with forrow we are conftrained to recognize a great man, among the enemies of an unfortunate and perfecuted fcholar. Happily for EULER, the whole tenor of his life fhelters him from a more ferious fufpicion. But for that fimplicity, that indifference to the voice of fame, which he uniformly manifefted, it might have been fufpected, that the pleafantries of an illuftrious partifan of *Kœnig* (pleafantries which *Voltaire* himfelf has juftly configned to oblivion) had fomewhat foured the temper of the gentle and fage geometrician; but if on this occafion he is chargeable with a fault, it muft be imputed folely to an excefs of gratitude; and if once in his life he acted wrong, the motive at leaft is refpectable.

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The Ruffian forces having, in 1760, penetrated into the marches of Brandeburg, plundered a farm of EULER's, near Charlottenburg: but General *Tottleben* had not come to make war on the fciences. Being informed of the lofs which EULER had fuftained, he haftened to repair it, by ordering payment far beyond the real value of the property, and having communicated to the Emprefs *Elizabeth*, an account of this involuntary difrefpect, fhe was pleafed to add a gratuity of four thoufand florins to an indemnification already more than fufficient. This anecdote is not fo generally known as it deferves to be, while we quote, with enthufiaftic admiration, fimilar actions tranfmitted to us from antiquity. Is not this difference in the judgments we form, a proof of the happy progrefs of the human fpecies, which certain authors ftill obftinately perfevere in denying, apparently to fhun the imputation of having contributed to it?

The government of Ruffia had never treated EULER as a ftranger. Notwithftanding his abfence, part of his falary was always regularly paid; and in 1766, the Emprefs having given him an invitation to return to Peterfburg, he complied.

In 1735, the exertion occafioned by an aftronomical calculation, for which other academicians demanded feveral months, but completed by him in a few days, brought on an indifpofition, which iffued in the lofs of one of his eyes. He had reafon to apprehend a total lofs of fight, if he continued to ex-

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pofe himfelf in a climate, the influence of which was unfavourable to his conftitution. The intereft of his family got the better of this apprehenfion; and if we reflect that, to EULER, ftudy was an exclufive paffion, we fhall readily conclude, that few examples of paternal tendernefs have more completely demonftrated, that it is the moft powerful, and the fweeteft of all our affections.

A few years after, he was overtaken by the calamity which he forefaw and dreaded: but happily for himfelf, and for the fciences, he preferved ftill the faculty of diftinguifhing large characters traced on a flate with chalk. His fons, his pupils, copied his calculations; wrote, as he dictated, the reft of his memoirs; and if we may form a judgment of thefe from their number, and frequently from the genius transfufed through them, it will appear abundantly credible, that from the abfence ftill more abfolute of all diftraction, and from the new energy which this conftrained recollection gave to all his faculties, he gained more, both as to facility and means of labour, than he loft by a diminution of fight.

Befides, EULER, by the nature of his genius and his habits of life, had even involuntarily laid up for himfelf extraordinary fupplies. On examining thofe great analytical formules, fo rare before his time, but fo frequent in his works, the combination and difplay of which unite fo much fimplicity and elegance, whofe very form pleafes the eye as well as the mind, it will be evident, that they are not the refult of a calcalation traced on paper, but that, produced entirely in the head, they are the creation of an imagination equally vigorous and active.

There exift in analyfis, and EULER greatly multiplied their number, formules of a common and almoft daily application; he had them always prefent to his mind, knew them by heart, repeated them in converfation; and Mr. *d'Alembert*, when he faw him at Berlin, was aftonifhed at an effort of memory, which demonftrated, that EULER poffeffed at once a ftrength and a clearnefs of recollection almoft incredible. At length his facility of calculation by the head was carried to fuch a degree as would exceed all belief, had not the hiftory of his labours accuftomed us to prodigies. He has been known, in the view of exercifing his little grandfon in the extraction of the fquare and cube roots, to have formed to himfelf the table of the fix firft powers of all numbers from 1 to 100, and to have preferved it exactly in his memory. Two of his pupils had calculated as far as to the feventeenth term of a convergent feries, abundantly complicated; their refults, though formed after a written calculation, differed one unit at the fiftieth figure: they communicated this difference to their mafter: EULER went over the whole calculation in his head, and his decifion was found to be the true one.

From the time he loft his fight, his chief amufement was to make artificial magnets, and to give leffons in the mathematics to one of his grand-children,

dren, who feemed to have a promifing difpofition to that fcience.

He made a point of ftill going occafionally to the Academy, efpecially if delicate circumftances demanded his attendance, or when he deemed his prefence neceffary to the maintenance of liberty. It is eafy to conceive how much it is in the power of a perpetual prefident, appointed by the court, to difturb the peace of an Academy, and how much fuch a feminary has to apprehend from one who, not being elected from their own number, does not feel himfelf reftrained even by a fenfe of that fupport which his reputation needs from the fuffrages of his colleagues. How is it poffible for men, employed folely in calm literary purfuits, and underftanding no language but that of the fciences, to defend themfelves in fuch a cafe; efpecially if ftrangers, unconnected, far from their country, they derive their whole fupport from that government, to which they would appeal for juftice againft an imperious prefident, whom that very government had placed over them.

But there is a degree of glory, which places a man beyond the reach of fear: it is, when all Europe would roufe itfelf to refent a perfonal injury offered to a great man, that he can without rifk oppofe to injuftice the authority of his reputation, and elevate, in fupport of the fciences, a voice which will make itfelf heard. EULER, gentle, modeft as he was, was fenfible of his power, and oftener than once made a very happy ufe of it.

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In 1771, the city of Peterfburg fuffered feverely from a terrible conflagration: the flames had caught the houfe of EULER. One *Peter Grimm*, a native of Bâle, whofe name well deferves to be tranfmitted to pofterity, apprized of the danger of his illuftrious compatriot, now blind and enfeebled, burft through the midft of the fire, reaches his apartment, places him on his fhoulders, and faves EULER's life, at the hazard of his own. His library, his furniture was deftroyed, but the zeal and exertions of Count *Orloff* preferved his manufcripts. The attention paid to this, at the height of a calamity fo dreadful, is the moft honourable and flattering homage which public authority could have offered to fcience. The houfe of EULER was one of the Emprefs's gifts to him; a fimilar act of munificence fpeedily repaired the lofs.

He had by his firft wife thirteen children, eight of whom died young. His three fons furvived him, but he had the misfortune to lofe both his daughters, the laft year of his own life. Of thirty-eight grand-children, twenty-fix were living at the time of his death. In 1776 he entered a fecond time into the married ftate, by efpoufing a Mifs *Gfell*, fifter to his firft wife's father. He had always retained all that fimplicity of manners, of which his father's houfe had fet the example. As long as his fight remained, he every evening collected, to domeftic devotion, his grand-children, his domeftics, and fuch of his pupils as lodged in the houfe; he read to them a portion of of Scripture, and fometimes accompanied it with an expofition.

He was of a very religious turn of mind. He publifhed a new demonftration of the exiftence of God, and of the fpirituality of the foul: this laft treatife has been admitted as a ftandard book into feveral colleges of divinity. With fcrupulous exactnefs he adhered to the religion of his country, which is rigid Calvinifm: and it does not appear that, after the example of moft fcholars of the proteftant perfuafion, he ever took the liberty of adopting peculiar ideas, or of forming a fyftem of religion for himfelf.

His erudition was very extenfive, efpecially in the hiftory of mathematics. It is alleged that he had carried his curiofity fo far as to acquire the knowledge of the proceffes and rules of aftrology; and that he had even made fome applications of them. However, when in 1740 he was commanded to calculate the nativity of Prince *Ivan*, he excufed himfelf, by reprefenting that this was the proper bufinefs of Mr. *Kraaff*, in quality of royal aftronomer. Credulity of this fort, which we are aftonifhed to find at fo recent a period in the Court of Ruffia, prevailed, the age before, in all the Courts of Europe: thofe of Afia have not yet fhaken off this abfurd yoke, and it muft be acknowledged, that if we except the common maxims of morality, there is no one truth which can boaft of having been fo generally adopted, and through fuch a fucceffion

fucceffion of ages, as certain ridiculous or pernicious errors.

Euler had ftudied almoft every branch of phyfics, anatomy, chemiftry, botany; but his fuperiority in mathematics did not permit him to attach the flighteft importance to his proficiency in any other branch of fcience, though it was fuch as might have induced a perfon more fufceptible of the flattery of felf-love to afpire to the title of an univerfal fcholar.

The ftudy of ancient literature, and of the learned languages, had formed part of his education: he retained a tafte for thefe to the end of life, and never forgot any thing he had once acquired; but he had neither time nor inclination to profecute farther his attainments in claffic literature. He had not fo much as read the modern poets, but knew the Eneid by heart. Euler, however, did not lofe fight of the mathematics, even in reciting the verfes of *Virgil*. Every thing concurred to prefent him with this darling object of his thoughts, and we find among his works, an ingenious memoir on a queftion in mechanics, the firft idea of which, he tells us, was fuggefted by a line of *Virgil*.

It has been faid that, to men of great talents, the pleafure of exertion is a reward ftill more gratifying than glory itfelf: were it neceffary to prove this truth by examples, that of Euler would put it beyond a doubt.

In his moft profound difcuffions with celebrated geometricians, he never betrayed the flighteft fymptom

tom which could excite a fufpicion of his being actuated by motives of felf-love. He difcovered no eagernefs to affert his title to the merit of his difcoveries; and if any thing in his works was claimed as the difcovery of another, he was at pains to repair the involuntary offence, even without enquiring too fcrupuloufly, whether rigid juftice demanded an abfolute renunciation. Did any one pretend to have detected him in error, if the charge was unfounded, he forgot it; if juft, he corrected it, without ftopping to obferve that, in many cafes, the merit of thofe who boafted of having made the detection, confifted wholly in an eafy application of the methods which he himfelf had taught them, to theories, the greateft difficulties of which he had before-hand removed.

Men of middling ability almoft always endeavour to make themfelves of confequence, by an affected feverity, proportional to the lofty idea which they wifh to convey of their underftanding, or of their genius. Inexorable to all that rifes above them, they give no quarter even to inferiority; fo that we are tempted to fay, a fecret confcioufnefs fhews them the neceffity which they are under of lowering others. An inftinctive emotion engaged EULER, on the contrary, to celebrate genius the moment that it's firft exertions had challenged his attention, and without waiting till public opinion courted the fanction of his fuffrage.

He has been known to employ his time in refolving problems

problems already folved, which was to procure for him, at moft, the inferior praife of greater elegance, or exactnefs of method; and this with the fame ardor and perfeverance that he could have exerted in the profecution of a new truth, the difcovery of which might have brought him an increafe of reputation. Befides, had an ardent defire of glory actually exifted in his breaft, it would have been impoffible for him, fuch was the franknefs of his character, to conceal it's emotions. But the glory which he was fo little folicitous to purfue, fought and found him out. The fingular fertility of his genius was a ftriking phenomenon, even to perfons who were not in a condition to underftand his works.

Though wholly devoted to geometry, his reputation challenged the attention of men little verfed in that fcience; and he appeared in the eyes of all Europe not only the firft of geometricians, but a great man. It is the cuftom of Ruffia to beftow military titles on men wholly unconnected with the fervice. This is paying homage to a prejudice which would reprefent the profeffion of a foldier as the only title to nobility, but the practice is at the fame time a direct acknowledgment of it's complete falfity. Some of the *Literati* have even arrived at the rank of Major-General: EULER never had, and indeed never would have, any diftinction of this fort; but what title in the power of Princes to beftow, could do honour to the name EULER? And then, regard for the prefervation of the natural rights of humanity,

humanity, impoſes, in ſome meaſure, the duty of ſetting the example of a ſage indifference to theſe baubles of human vanity, ſo childiſh and yet ſo dangerous.

Moſt of the Princes of the North, to whom he was perſonally known, gave him marks of their eſteem, or rather of a veneration which they could not with-hold from the union of a virtue ſo ſimple with a genius ſo vaſt and elevated. When the Prince Royal of Pruſſia travelled to Peterſburg, he did not wait for a viſit from EULER, but went firſt to his houſe, and paſſed ſome hours by the bed-ſide of the venerable old man, holding his hands in his own, with one of EULER's grand-children in his lap, whom early ſymptoms of a genius for geometry had rendered the particular object of paternal affection.

All the noted mathematicians of the preſent day are his pupils: there is no one of them who has not formed himſelf by the ſtudy of his works, who has not received from him the formules, the method which he employs; who is not directed and ſupported by the genius of EULER in his diſcoveries. This honour he owes to the revolution effected in the mathematical ſciences, by ſubjecting all to analyſis; to his indefatigable application, which has enabled him to embrace the whole extent of theſe ſciences; to the order in which he has arranged his great works; to the ſimplicity, to the elegance, of his formules; to the clearneſs of his methods and demonſtrations;

demonſtrations; and all this greatly enhanced, by the multiplicity and the choice of his examples. Neither *Newton*, nor *Deſcartes*, whoſe influence was once ſo powerful, has arrived at this pitch of glory; and hitherto, EULER alone, of geometricians, has poſſeſſed it entirely, and without a rival.

But, as Profeſſor, he has formed pupils in a peculiar ſenſe his own. Among theſe, we mention his eldeſt ſon, whom the Academy of Sciences elected to ſupply his place, without any apprehenſion that this honourable ſucceſſion granted to the name of EULER, as to that of *Bernouilli*, could ever become a dangerous precedent: a ſecond ſon, now engaged in the ſtudy of medicine, but who, in his youth, obtained from that Academy the prize propoſed for determining the alterations of the mean motion of the planets; Mr. *Lexell*, whoſe premature death has juſt left a blank in the world of ſcience; and, to mention no more, Mr. *Fuſs*, the youngeſt of his ſcholars, and the companion of his laſt labours; who, ſent from Bâle to EULER by *Daniel Bernouilli*, has, by his works, done credit to *Bernouilli*'s recommendation, and EULER's inſtructions, and who, after having paid public homage in the Academy of Peterſburg, to his illuſtrious maſter, married his granddaughter.

Of ſixteen profeſſors belonging to the Academy of Peterſburg, eight had been formed by him; and all of them, well known from their productions, and decorated with academic honours, value themſelves

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on being able to add, to all the reſt, that of diſciple to EULER.

He had retained all his facility of thought, and, apparently, all his mental vigour: no decay ſeemed to threaten the ſciences with the ſudden loſs of their great ornament. On the 7th of September, 1783, after amuſing himſelf with calculating on a ſlate the laws of the aſcending motion of air-balloons, the recent diſcovery of which was then making a noiſe all over Europe, he dined with Mr. *Lexell* and his family, talked of *Herſchell*'s planet, and of the calculations which determine it's orbit. A little after he called his grand-child, and fell a playing with him as he drank tea, when ſuddenly, the pipe, which he held in his hand, dropped from it, and he ceaſed to calculate and to breathe.

Such was the end of one of the greateſt and moſt extraordinary men ever produced by the hand of nature: a man whoſe genius was equally capable of the greateſt efforts, and of the moſt unwearied application; who multiplied his productions far beyond what could have been expected from powers merely human, and was, nevertheleſs, original in every one; whoſe head was inceſſantly employed, and his ſpirit always tranquil; who, finally, by a deſtiny unfortunately too rare, united, and that deſervedly, a felicity hardly ever interrupted, to a glory which no one ever diſputed with him.

His death was conſidered as a public loſs, even in the country which he inhabited. The Academy of Peterſburg

Peterſburg went into deep mourning for him, and voted a marble buſt of him, at their own expence, to be placed in their Aſſembly-Hall. An honour ſtill more diſtinguiſhed had already been conferred on him, by that learned body, in his life-time. In an allegorical painting, a figure of Geometry is repreſented leaning on a tablet, exhibiting mathematical calculations, and the characters inſcribed, by order of the Academy, are the formules of his new theory of the moon. Thus, a country which, at the beginning of the preſent century, we conſidered as ſcarcely emerged out of barbariſm, is become the inſtructor of the moſt enlightened nations of Europe, in doing honour to the life of great men, and in embalming their memory: it is ſetting theſe nations an example, which ſome of them may bluſh to reflect, that they have had the virtue neither to propoſe, nor to imitate.

LETTERS

ON

DIFFERENT SUBJECTS

IN

PHYSICS AND PHILOSOPHY.

LETTER I.

Of Magnitude, or Extenſion.

MADAM,

THE hope of having the honour to communicate, in perſon, to your Highneſs, my leſſons in Geometry, becoming more and more diſtant, which is a very ſenſible mortification to me, I feel myſelf impelled to ſupply perſonal inſtruction by writing, as far as the nature of the objects can permit.

I begin my attempt, by aſſiſting you to form a juſt idea of *magnitude;* producing, as examples, the ſmalleſt as well as the greateſt extenſions of matter actually diſcoverable in the ſyſtem of the Univerſe. And firſt, it is neceſſary to fix on ſome one determinate diviſion of meaſure, obvious to the ſenſes, and of which we have an exact idea, that of a foot, for inſtance. The quantity of this, once eſtabliſhed, and rendered familiar to the eye, will enable us to form the idea of every other quantity, as to length, great or or ſmall; the former, by aſcertaining how many feet it contains; and the latter, by aſcertaining what part of a foot meaſures it. For, having the idea of a foot, we have that alſo of it's half, of it's quarter, of it's twelfth part, denominated an inch, of it's hundredth, and of it's thouſandth part; which is ſo ſmall as almoſt to eſcape the ſight. But it is to be remarked, that there are animals, not of greater extenſion than this laſt ſubdiviſion of a foot, which, however, are compoſed of members through which the blood circulates, and which again contain other animals, as diminutive compared to them, as they are compared to us. Hence it may be concluded that animals exiſt, whoſe ſmallneſs eludes the imagination; and that theſe again are diviſible into parts inconceivably ſmaller. Thus, for example, though the ten thouſandth part of a foot be too ſmall for ſight, and, compared to us, ceaſes to be an object of ſenſe, it nevertheleſs ſurpaſſes in magnitude certain complete animals; and muſt, to one of thoſe animals, were it endowed with the power of perception, appear extremely great.

Let us now make the tranſition from theſe minute quantities, in purſuing which the mind is loſt, to thoſe of the greateſt magnitude. You have the idea of a mile;* the diſtance from hence to Magdeburg is computed to be 18 miles;† a mile contains 24,000 feet, and we employ it in meaſuring the diſtance of the different regions of the globe, in order to avoid

* The German mile is equal to 4 3-5ths miles Engliſh, nearly.
† About 83 Engliſh miles.

numbers

numbers inconceivably great, in our calculations, which muſt be the caſe if we uſed foot inſtead of mile. A mile then, containg 24,000 feet, when it is ſaid that Magdeburg is 18 miles from Berlin, the idea is much clearer, than if the diſtance of theſe two cities were ſaid to be 432,000 feet: A number ſo great almoſt overwhelms the underſtanding. Again, we ſhall have a tolerably juſt idea of the magnitude of the earth, when we are told that it's circumference is about 5,400 miles. And the diameter being a ſtraight line paſſing through the centre, and terminating, in oppoſite directions, in the ſurface of the ſphere, which is the acknowledged figure of the earth, for which reaſon alſo we give it the name of *globe*, the diameter of this *globe* is calculated to be 1720 miles;* and this is the meaſurement which we employ for determining the greateſt diſtances diſcoverable in the heavens. Of all the heavenly bodies the moon is neareſt to us, being diſtant only about 30 diameters of the earth, which amount to 51,600 miles,† or 1,238,400,000 feet; but the firſt computation of 30 diameters of the earth, is the cleareſt idea. The ſun is about 300 times farther from us than the moon; and when we ſay his diſtance is 9,000 diameters of the earth, we have a much clearer idea, than if it were expreſſed in miles, or in feet.

* About 7,920 Engliſh miles. The diameter of our earth is really 7,964 Engliſh miles, it's circumference 25,020. The mean diſtance of the moon is 240,000 miles, which ſcarcely exceeds the 400th part of the ſun's mean diſtance, or 93,720,000 miles.
† About 237,360 miles Engliſh.

You

You know that the earth performs a revolution round the fun in the fpace of a year, but that the fun remains fixed. Befide the earth, there are five other fimilar bodies, named planets, which revolve round the fun; two of them at fmaller diftances, Mercury and Venus; and three at greater, namely Mars, Jupiter and Saturn. All the other ftars which we fee, comets excepted, are called fixed; and their diftance from us is incomparably greater than that of the fun. The diftances are undoubtedly very unequal, which is the reafon that fome of thefe bodies appear greater than others. But the neareft of them is, unqueftionably, above 5,000 times more diftant than the Sun: it's diftance from us, accordingly, exceeds 45,000,000 of times the earth's diameter, that is 77,400,000,000 of miles;* and this again multiplied by 24,000 will give that prodigious diftance expreffed in feet. And this, after all, is the diftance only of thofe fixed ftars which are the neareft to us; the moft remote which we fee, are perhaps a hundred times farther off. It is probable, at the fame time, that all thefe ftars taken together, conftitute only a very fmall part of the whole univerfe, relatively to which thefe prodigious diftances are not greater than a grain of fand compared to the earth. This immenfity is the work of the Almighty, who governs the greateft bodies and the fmalleft.†

Berlin, 19th April, 1760.

* 356,050,000,000 miles Englifh.

† This letter, in the original edition, that of Leipfig, 1770, is dated, *Berlin* 19th April, 1760, and concludes with thefe words, (which are with great propriety omitted by the philofophic French editor of the work, twenty-feven years afterwards) *and who is now crowning with fuccefs the arms in which we are fo deeply interefted.* This is, no doubt, a dreadful "falling off" from the majefty of the fubject. Who cares *now* about the fuccefs of the Pruffian arms in 1760? But philofophers, as well as other men, are under the dominion of local and temporary circumftances. Frederick II. was then in the zenith of his glory; Euler was living at Berlin, and giving leffons in philofophy to the niece of that illuftrious prince. Is it to be wondered, then, that he fhould fink for a moment into the courtier, and offer a drachm of incenfe to a great lady; or, that a foul fo uniformly devout, fhould acknowledge the providence of the Almighty in a particular inftance?

LETTER II.

Of Velocity.

FLATTERING myfelf that your Highnefs may be pleafed to accept the continuation of my inftructions, a fpecimen of which I took the liberty of prefenting to you in a former letter, I proceed to unfold the idea of velocity, which is a particular fpecies of extenfion, and fufceptible of increafe and of diminution. When any fubftance is tranfported, that is, when it paffes from one place to another, we afcribe to it a velocity. Let two perfons, the one on horfeback, the other on foot, proceed from Berlin to Magdeburg, we have, in both cafes, the idea of a certain velocity; but it will be immediately affirmed, that the velocity of the former exceeds that of the latter. The queftion then is, Wherein confifts the difference which we obſerve between theſe ſeveral degrees of velocity? The road is the ſame to him who rides and to him who walks: but the difference evidently lies in the time which each employs in performing the ſame courſe. The velocity of the horſeman is the greater of the two, as he employs leſs time on the road from Berlin to Magdeburg; and the velocity of the other is leſs, becauſe he employs more time in travelling the ſame diſtance. Hence it is clear, that in order to form an accurate idea of velocity, we muſt attend at once to two kinds of quantity, namely, to the length of the road, and to the time employed. A body, therefore, which in the ſame time paſſes through double the ſpace which another body does, has double its velocity; if, in the ſame time, it paſſes through thrice the diſtance, it is ſaid to have thrice the velocity, and ſo on. We ſhall comprehend, then, the velocity of a body, when we are informed of the ſpace through which it paſſes in a certain quantity of time. In order to know the velocity of my pace, when I walk to Lytzow,* I have obſerved that I make 120 ſteps in a minute, and one of my ſteps is equal to two feet and a half. My velocity, then, is ſuch, as to carry me 300 feet in a minute, and a ſpace ſixty times greater, or 18,000 feet in an hour, which however does not amount to a mile, for this, being 24,000 feet, would require an hour and 20 minutes. Were I, therefore, to walk from hence to Magdeburg, it would take exactly 24 hours. This conveys an accurate idea of the velo-

* A village about a league from Berlin.

city

city with which I am able to walk. Now it is eaſy to comprehend what is meant by a greater or leſs velocity. For if a courier were to go from hence to Magdeburg in 12 hours, his velocity would be the double of mine: if he went in eight hours, his velocity would be triple. We remark a very great difference in the degrees of velocity. The tortoiſe furniſhes an example of a velocity extremely ſmall. If ſhe advances only one foot in a minute, her velocity is 300 times leſs than mine, for I advance 300 feet in the ſame time. We are likewiſe acquainted with velocities much greater. That of the wind admits of great variation. A moderate wind goes at the rate of 10 feet in a ſecond, or 600 feet in a minute; its velocity therefore is the double of mine. A wind that runs 20 feet in a ſecond, or 1200 in a minute, is rather ſtrong; and a wind which flies at the rate of 50 feet in a ſecond is extremely violent, though its velocity is only 10 times greater than mine, and would take two hours and twenty-four minutes to blow from hence to Magdeburg.

The velocity of ſound comes next, which moves 1000 feet* in a ſecond, and 60,000 in a minute. This velocity, therefore, is 200 times greater than that of my pace; and were a cannon to be fired at Magdeburg, if the report could be heard at Berlin, it

* The velocity of ſound is generally computed at 1,142 feet each ſecond, but varies with the elaſticity and denſity of the air. The earth travels in her orbit 1,612,000 miles in the ſpace of 24 hours, and therefore with a velocity more than 50 times greater than that of a cannon ball. Light moves about 13 millions of miles every minute.

 would

would arrive there in ſeven minutes. A cannon ball moves with nearly the ſame velocity; but when the piece is loaded to the utmoſt, the ball is ſuppoſed capable of flying 2,000 feet in a ſecond, or 120,000 in a minute. This velocity appears prodigious, though it is only 400 times greater than that of my pace in walking to Lytzow; it is at the ſame time the greateſt velocity known upon earth. But there are in the heavens velocities far greater, though their motion appears to be extremely deliberate. You know that the earth turns round on it's axis in 24 hours: every point of it's ſurface, then, under the equator, moves 5,400 miles* in 24 hours, while I am able to get through only 18 miles.† It's velocity is accordingly 300 times greater than mine, and leſs notwithſtanding than the greateſt poſſible velocity of a cannon ball. The earth performs it's revolution round the ſun in the ſpace of a year, proceeding at the rate of 128,250 miles‡ in 24 hours. It's velocity, therefore, is 18 times more rapid than that of a cannon ball. The greateſt velocity of which we have any knowledge is, undoubtedly, that of light, which moves 2,000,000 of miles § every minute, and exceeds the velocity of a cannon ball 400,000 times.

22d April, 1760.

* 24,840 Engliſh miles
† About 83 Engliſh.
‡ 589,950 Engliſh.
§ 9,200,000 miles Engliſh.

LETTER

LETTER III.

Of Sound, and it's Velocity.

THE elucidations of the different degrees of velocity, which I have had the honour to lay before your Highneſs, carry me forward to the examination of ſound, or noiſe in general. It muſt be remarked, that a certain portion of time always intervenes before ſound can reach our ears, and that this time is longer in proportion to our diſtance from the place where the ſound is produced; a ſecond of time being requiſite to convey ſound 1000 feet.

When a cannon is fired, thoſe who are at a diſtance do not hear the report for ſome time after they have ſeen the flaſh. Thoſe who are a mile, or 24,000 feet diſtant, hear not the report till 24 ſeconds after they ſaw the flame. You muſt no doubt have frequently remarked, that the noiſe of thunder reaches not the ear for ſome time after the lightning: and it is by this we are enabled to calculate our diſtance from the place where the thunder is generated. If, for example, we obſerve that 20 ſeconds intervene between the flaſh and the thunder-clap, we may conclude that the ſeat of the thunder is 20,000 feet diſtant, allowing 1000 feet of diſtance for every ſecond of time. This primary property leads us to inquire, In what ſound conſiſts? Whether it's nature is ſimilar to that of ſmell, that is, whether ſound iſſues from the body which produces it, as ſmell is emitted

emitted from the flower, by filling the air with ſubtile exhalations, proper to affect our ſenſe of ſmelling. This opinion was formerly entertained, but it is now demonſtrated, that from a bell ſtruck nothing proceeds that is conveyed to our ear, and that the body which produces ſound loſes no part of it's ſubſtance. When we look upon a bell that is ſtruck, or the ſtring of an inſtrument when touched, we perceive that theſe bodies are then in a ſtate of trembling, or agitation, by which all their parts are affected; and that all bodies, ſuſceptible of ſuch an agitation of their parts, likewiſe produce ſound. Theſe ſhakings or vibrations are viſible in the ſtring of an inſtrument when it is not too ſmall; the tenſe ſtring A C B paſſes alternately into the ſituation A M B and A N B. *(See plate I. fig.* 1. *in which I have repreſented theſe vibrations much more obvious to ſenſe than they are in fact.)* It muſt be further obſerved, that theſe vibrations put the adjacent air into a ſimilar vibration, which is ſucceſſively communicated to the more remote parts of the air, till it come at length to ſtrike our organ of hearing. It is the air, then, which receives theſe vibrations, and which tranſmits the ſound to our ear. Hence it is evident, that the perception of ſound is nothing elſe but the impreſſion made on our ear by the concuſſion of the air, communicated to us through the organ of hearing; and when we hear the ſound of a ſtring touched, our ear receives from the air as many ſtrokes as the ſtring performs vibrations in the ſame time. Thus, if the ſtring performs 100 vibrations in a ſecond,

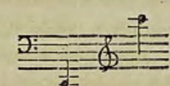
cond, the ear likewiſe receives 100 ſtrokes in the ſame time; and the perception of theſe ſtrokes is what we call ſound. When theſe ſtrokes ſucceed each other uniformly, or when their intervals are all equal, the ſound is regular, and ſuch as is requiſite to muſic. But when the ſtrokes ſucceed unequally, or when their intervals are unequal among themſelves, an irregular noiſe, incompatible with muſic, is the reſult. On conſidering ſomewhat more attentively the muſical ſounds, whoſe vibrations take place equally, I remark firſt, that when the vibrations, as well as the ſtrokes impreſſed on the ear, are more or leſs ſtrong, no other difference of ſound reſults from it, but that of ſtronger or weaker, which produces the diſtinction, termed by muſicians, *forte & piano.* But there is a difference much more eſſential, when the vibrations are more or leſs rapid, that is, when more or fewer of them are performed in a ſecond. When one ſtring makes 100 vibrations in a ſecond, and another ſtring makes 200 vibrations in the ſame time, their ſounds are eſſentially different; the former is lower or more flat, and the other higher or more ſharp. Such is the real difference between the flat and ſharp ſounds, on which all muſic hinges, and which teaches how to combine ſounds different in reſpect of flatneſs and ſharpneſs, but in ſuch a manner as to produce an agreeable harmony. In the flat ſounds there are fewer vibrations in the ſame time than in the ſharp ſounds; and every key of the harpſichord contains a certain and determinate number of vibrations, which are completed in a ſecond.

Thus

Thus the note marked by the letter C,* makes nearly 100 vibrations in a fecond; and the note marked $\bar{\bar{\bar{c}}}$ makes 1600 vibrations in the fame fpace of time. A ftring which vibrates 100 times in a fecond, will give precifely the note C; and if it vibrated only 50 times, the note would be lower or more flat. But with regard to our ear, there are certain limits beyond which found is no longer perceptible. It would appear that we are incapable of determining either the found of a ftring which makes lefs than 30 vibrations in a fecond, becaufe it is too low; or that of a ftring which would make more than 7552 in a fecond, becaufe fuch a note would be too high.

26th April, 1760.

* The note C is that which is produced by touching the thickeft ftring of a violoncello; the note $\bar{\bar{\bar{c}}}$ is the fourth octave of the firft; accordingly, thefe two notes, reprefented by the ufual method of pricking mufic, are



Mr. Euler marks the progreffion of octaves thus:

	1ft octave,	2d octave,	3d octave,	4th octave.
C, or *ut.*	*c.*	$\bar{c}$	$\bar{\bar{c}}$	$\bar{\bar{\bar{c}}}$

and in like manner for the other notes of the gamut; D. E. F. G. A. B. or *re, mi, fa, fol, la, fi.*

In writing the chromatic fcale, he employs the following figns:

C.	Cs,	D,	Ds,	E,	F,	Fs,	G,	Gs,	A,	B,	H,	c
ut,	*ut*♯,	*re,*	*re*♯,	*mi,*	*fa,*	*fa*♯,	*fol,*	*fol*♯,	*la,*	*fi*♭,	*fi*♮,	*ut.*

LETTER

LETTER IV.

Of Confonance and Diffonance.

I RESUME my remark, that on hearing a fimple mufical found, our ear is ftruck with a feries of ftrokes equally diftant from each other, the frequency and number of which, in a given fpace of time, conftitute the difference which fubfifts between low notes and high: fo that, the fmaller the number of vibrations or ftrokes produced in a given time, fay a fecond, the lower we eftimate that note; and the greater the number of fuch vibrations, the higher is the note. The perception of a fimple mufical found may, therefore, be compared to a feries of dots equidiftant from each other, as . . . . If the intervals between thefe dots be greater or fmaller, the found produced will be lower or higher. It cannot be doubted, that the perception of a fimple found is fomewhat fimilar or analogous to the fight of fuch a feries of dots equidiftant from each other: we are enabled thus to reprefent to the eye what the ear perceives on hearing found. If the diftances between the dots were not equal, or were thefe dots fcattered about confufedly, they would be a reprefentation of a confufed noife, inconfiftent with harmony. This being laid down, let us confider what effect two founds emitted at once muft produce on the ear. Firft, it is evident, that if two founds are equal, or if each performs the fame number of vibrations in

the

the fame time, the ear will be affected in the very fame manner as by a fingle note; and, in mufic, thefe two notes are faid to be in unifon, which is the fimpleft *accord*: we mean by the term *accord* the blending of two or more founds heard at once. But if two founds differ in refpect of low and high, we fhall perceive a mixture of two feries of ftrokes, in each of which the intervals are equal among themfelves, but greater in the one than in the other; the greater intervals correfponding to the lower note, and the fmaller to the higher. This mixture, or this accord of two notes, may be reprefented to the eyes by two feries of dots arranged on two lines A B and C D;

1 2 3 4 5 6 7 8 9 10 11
A . . . . B
C . . . . D
1 2 3 4 5 6 7 8 9 10 11 12

and in order to form a juft idea of thefe two feries, we muft have a clear perception of the order which fubfifts among them, or, in other words, of the relation between the intervals of the one line and of the other. Having numbered and marked the dots of each line, and placed No. 1. under No. 1; thofe marked with the figure 2, will not exactly correfpond, and ftill lefs thofe marked 3: but we find No. 11 exactly over No 12: from which we difcover that the higher note makes 12 vibrations, and the other only 11. If we had not affixed the figures, the eye would hardly have perceived this order; it is the fame with the ear, which would with much difficulty

difficulty have traced it in the two notes which I have reprefented by two rows of dots. But in the following figure,

. . . .
.   .   .   .   .   .

you difcover at the firft glance that the upper line contains twice as many dots as the under, or that the intervals in the under line are twice as great as thofe of the upper. This is undoubtedly, next to unifon, the fimpleft of all cafes, in which you can at once difcover the order which fubfifts between thefe two feries of dots; and the fame thing holds with refpect to the two notes reprefented by thefe two lines of dots: the number of vibrations contained in the one will be precifely the double of the vibrations contained in the other, and the ear will eafily perceive the pleafing relation of thefe two founds; whereas, in the preceding cafe, it was extremely difficult, if not impoffible, to difcriminate. When the ear readily difcovers the relation fubfifting between two notes, their accord is denominated *confonance*: and if it be very difficult, or even impoffible to catch this relation, the accord is termed *diffonance*. The fimpleft confonance, then, is that in which the high note produces precifely twice as many vibrations as the low note. This confonance, in the language of mufic, is called *octave*: every one knows what it means; and two notes which differ precifely an octave, harmonize fo perfectly, and poffefs fuch a complete refemblance, that muficians mark them by the fame letters. Hence it is that in church-mufic the

the women ſing an octave higher than the men, and yet imagine they are uttering the ſame ſounds. You may eaſily aſcertain the truth of this by touching the keys of a harpſichord, when you will perceive with pleaſure the delightful accord of all the notes which are juſt an octave diſtant, whereas any other two notes whatever will ſtrike the ear leſs agreeably.

29th April, 1760.

LETTER V.

Of Uniſon and Octaves.

YOUR Highneſs has by this time remarked, that the accord which muſicians call an octave, ſtrikes the ear in a manner ſo decided, that the ſlighteſt deviation is eaſily perceptible. Thus, having touched the Key marked F, that marked f, which is an octave higher, is eaſily attuned to it, by the judgment of the ear only. If the ſtring which is to produce this note be ever ſo little too high or too low, the ear is inſtantly offended, and nothing is eaſier than to put the two keys perfectly in tune. Thus we obſerve, that in ſinging the voice ſlides eaſily from one note to another, which is juſt an octave higher or lower. But were it required to paſs immediately from the note F to the note d, for example, an ordinary ſinger might eaſily fall into a miſtake, unleſs aſſiſted by an inſtrument. Having fixed the note F, it is almoſt impoſſible all at once to make the tranſition to the note d. What then is the reaſon of this

this difference, that it is ſo eaſy to make note f harmonize with note F, and ſo difficult to make note d accord with it? The reaſon is evident from the remarks already made: it is this, that note F and note f make an octave, and that the number of vibrations of note f is preciſely double that of note F. In order to have the perception of this accord, you have only to conſider the proportion of one to two, which, as it inſtantly ſtrikes the eye by the repreſentation of the dots I formerly employed, affects the ear in a ſimilar manner. You will eaſily comprehend, then, that the more ſimple any proportion is, or expreſſed by ſmall numbers, the more diſtinctly it preſents itſelf to the underſtanding, and conveys to it a ſentiment of ſatisfaction.* Architects likewiſe carefully attend to this maxim, as they uniformly employ in their works proportions as ſimple as circumſtances permit. They uſually make the height of doors and windows double the breadth, and endeavour to employ throughout proportions capable of being expreſſed by ſmall numbers, becauſe this is obvious and grateful to the underſtanding. The ſame thing holds good in muſic: accords are pleaſing only in ſo far as the mind perceives the relation ſubſiſting between the ſounds, and this relation is ſo much more

* In order to have a clear conception of what follows, it muſt be recollected, that the terms *relation* and *ratio* are ſynonimous, and that the author is here conſidering geometrical proportion, which conſiſts in the number of times that the firſt term is contained in the ſecond.—*F. E.*

eaſily

eaſily perceptible, as it is expreſſed by ſmall numbers. Now, next to the relation of equality, which denotes two ſounds in uniſon, the ratio of two to one is undoubtedly the moſt ſimple, and it is this which furniſhes the accord of an octave: hence it is evident, that this accord poſſeſſes many advantages above every other conſonance. Having thus explained the accord, or interval of two notes denominated by muſicians an octave, let us conſider ſeveral notes, as F, f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$ each of which is an octave higher than the one immediately preceding: ſince then the interval of F from f, of f from $\bar{f}$, of $\bar{f}$ from $\bar{\bar{f}}$, of $\bar{\bar{f}}$ from $\bar{\bar{\bar{f}}}$ is an octave, the interval of F to $\bar{f}$ will be a double octave, that of F to $\bar{\bar{f}}$ a triple octave, and that of F to $\bar{\bar{\bar{f}}}$ a quadruple octave. Now, while note F makes one vibration, note f makes two, note $\bar{f}$ makes four, note $\bar{\bar{f}}$ makes eight, and note $\bar{\bar{\bar{f}}}$ makes ſixteen: hence we ſee, that as an octave correſponds in the relation of 1 to 2, a double octave muſt be in the ratio of 1 to 4, a triple in that of 1 to 8, and a quadruple in that of 1 to 16. And the ratio of 1 to 4, not being ſo ſimple as that of 1 to 2, for it does not ſo readily ſtrike the eye, a double octave is not ſo eaſily perceptible to the ear as a ſingle; a triple is ſtill leſs perceptible, and a quadruple ſtill much leſs ſo. When, therefore, in tuning a harpſichord, you have fixed the note F, it is not ſo eaſy to attune the double octave $\bar{f}$ as the ſingle f; it is ſtill more difficult to attune the triple octave $\bar{\bar{f}}$ and the quadruple $\bar{\bar{\bar{f}}}$ without riſing through the intermediate octaves. Theſe

Theſe accords are likewiſe comprehended in the term conſonance; and as that of uniſon is moſt ſimple, they may be arranged according to the following gradations:

I. Degree, uniſon, indicated by the relation of 1 to 1.
II. Degree, the immediate octave, in the ratio of 1 to 2.
III. Degree, the double octave, in that of 1 to 4.
IV. Degree, the triple octave, in that of 1 to 8.
V. Degree, the quadruple octave, in that of 1 to 16.
VI. Degree, the quintuple octave, in that of 1 to 32.

And ſo on, as long as ſound is perceptible. Such are the accords denominated conſonances, to the knowledge of which we have been thus far conducted; but hitherto we know nothing of the other ſpecies of conſonance, and ſtill leſs of the diſſonances employed in muſic. Before I proceed to the explication of theſe, I muſt add one remark reſpecting the name octave, given to the interval of two notes, the one of which contains twice the vibrations contained in the other. You ſee the reaſon of it in the principal ſtops of the harpſichord, which riſe by ſeven degrees before you arrive at the octave C, D, E, F, G, A, B, c, ſo that ſtop c is the eighth, reckoning C the firſt. And this diviſion depends on a certain ſeries of muſical intervals, the nature of which ſhall be unfolded in the following letters.

3d *May*, 1760.

LETTER VI.

Of other Consonances.

IT may be affirmed, that the relations of one to 2, of 1 to 4, of 1 to 8, of 1 to 16, which we have hitherto confidered, and which contain the progreffion of octaves, are all formed by the number 2 only; fince 4 is 2 times 2; 8, 2 times four; 16, two times 8. Were we to admit, therefore, the number 2 alone into mufic, we fhould arrive at the knowledge of only the accords or confonances which muficians call the fingle, double, or triple octave; and as the number 2, by its reduplication, furnifhes only the numbers 4, 8, 16, 32, 64, the one being always double the preceding, all other numbers would remain unknown. Now, did an inftrument contain octaves only, as the notes marked C, c, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, and were all others excluded, it could not prqduce an agreeable mufic, on account of its too great fimplicity. Let us introduce, then, together with number 2, the number 3 likewife, and obferve what accords or confonances would be the refult. The ratio of 1 to 3 prefents at once two founds, the one of which makes 3 times more vibrations than the other in the fame time. This ratio is undoubtedly the moft eafily to be comprehended, next to that of 1 to 2; it will, accordingly, furnifh very pleafing confonances, but of a nature totally different from that of octaves. Let us fuppofe, then, that in the proportion of 1

to

to 3, number 1 correfponds to note C; fince note c is expreffed by number 2, number 3 gives a found higher than c, but at the fame time lower than note $\bar{c}$, which correfponds to number 4. Now, the note expreffed by 3 is that to which muficians affix the letter g, and they denominate the interval from c to g, *a fifth*, becaufe in the keys of a harpfichord that of g is the fifth from c, as c, d, e, f, g. If then number 1 produces the found C, number 2 will give c; number 3 gives g, number 4 the note $\bar{c}$; and note $\bar{g}$ being the octave of g, the number correfponding to it will be 2 times 3, or 6. Rifing ftill an octave, the found $\bar{\bar{g}}$ will correfpond to a number twice greater, that is 12. All the notes with which the two numbers 2 and 3 furnifh us, indicating note C by 1, therefore are,

C, c, g, $\bar{c}$, $\bar{g}$, $\bar{\bar{c}}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{c}}}$
1. 2. 3. 4. 6. 8. 12. 16.

Hence it is clear, that the ratio of 1 to 3 expreffes an interval, compounded of an octave and a fifth, and that this interval, on account of the fimplicity of the numbers which reprefent it, muft be, next to the octave, the moft grateful to the ear. Muficians accordingly affign the fecond rank among confonances to the fifth; and the ear catches it fo eafily, that there is no difficulty in tuning a fifth. For this reafon, in violins, the four ftrings rife by fifths, the loweft being g, the fecond $\bar{d}$, the third $\bar{a}$, and the fourth $\bar{\bar{e}}$;* and every mufician puts them in tune

* That is, in the language of *fol-faing*, *fol*, *re*, *la*, *mi*.

by

by the ear only. A fifth, however, is not fo eafily tuned as an octave; but the fifth above the octave, as from C to g, being expreffed by the proportion of 1 to 3, is more perceptible than a fimple fifth, as from C to G, or from c to g, which is expreffed by the proportion of 2 to 3: and it is likewife known by experience, that having fixed the note C, it is eafier to attune to it the higher fifth g, than the fimple G. If unity had marked the note F, number 3 would mark the note $\bar{c}$, fo that,

F, f, $\bar{c}$, $\bar{f}$, $\bar{\bar{c}}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{c}}}$, would be marked by

1. 2. 3. 4. 6. 8. 12. where, from f to c the interval is a fifth in the relation of 2 to 3; from $\bar{f}$ to $\bar{\bar{c}}$, from $\bar{\bar{f}}$ to $\bar{\bar{\bar{c}}}$ are alfo fifths, as the ratio of 4 to 6, and of 8 to 12, is the fame as that of 2 to 3. For if two ftrings perform, in the fame time, the one 4 vibrations, the other 6, the former ftring will make, in a time equal to half the firft fpace of time, two vibrations, and the fecond, in the fame time, will make three. Now the founds emitted from thefe ftrings are the fame in both cafes; of confequence the relation of 4 to 6 expreffes the fame interval as that of 2 to 3, that is, a fifth. Hence we have arrived at the knowledge of another interval contained in the ratio of 3 to 4, which is that of $\bar{c}$ to $\bar{f}$, and confequently alfo of c to f, or of C to F. Muficians call it *a fourth;* and being expreffed by greater numbers, it is not fo agreeable, by a great deal, as the fifth, and ftill lefs fo than the octave. Number 3 having furnifhed us new accords or confonances, namely the fifth and the fourth,

fourth, before we call in any others, let us take it again three times, in order to have the number 9, which will give a higher note than note 3*, or $\bar{c}$ one octave and one fifth. Now, $\bar{\bar{c}}$ is the octave of $\bar{c}$, and $\bar{\bar{g}}$ the fifth of $\bar{\bar{c}}$; number 9 then gives the note $\bar{\bar{g}}$, fo that $\bar{\bar{c}}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{c}}}$, will be marked by

6. 8. 9. 12; and if thefe notes be taken in the lower octaves, the relations remaining the fame, we fhall have:

C, F, G; c, f, g; $\bar{c}$, $\bar{f}$, g; $\bar{c}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$; $\bar{\bar{\bar{c}}}$.
6. 8. 9; 12. 16. 18; 24. 32. 36; 48. 64. 72. 96.
which leads us to the knowledge of new intervals.

The firft is that of F to G, contained in the ratio of 8 to 9, which muficians call a *fecond* or *tone.* The fecond is that of G to f, contained in the ratio of 9 to 16; called a *feventh*, and which is one fecond, or one tone lefs than an octave. Thefe proportions, being already expreffed by very great numbers, are not reckoned among the confonances, and muficians call them *diffonances* or *difcords.*

Again, if we take three times the number 9, or 27, it will mark a tone higher than $\bar{c}$, and precifely a fifth higher than g; it will be accordingly the tone $\bar{d}$, and it's octave $\bar{\bar{d}}$ will correfpond to twice the number 27, or 54, and it's double octave $\bar{\bar{\bar{d}}}$ to twice the

* Great care muft be taken to guard ourfelves from affixing to numbers the idea of a perfect identity with the founds which they reprefent. The firft express only the relation of the number of vibrations performed in the fame time, by the bodies which emit the founds in queftion. There is no real analogy between number and found.—*F. E.*

C 4 number,

number, 54, or 108. Let us reprefent thefe tones fome octaves lower, in the manner following:

C, D, F, G; c, d, f, g, $\bar{c}$, $\bar{d}$, $\bar{f}$,
24, 27, 32, 36; 48, 54, 64, 72; 96, 108, 128,
$\bar{g}$; $\bar{\bar{c}}$, $\bar{\bar{d}}$, $\bar{\bar{f}}$, $\bar{\bar{g}}$; $\bar{\bar{\bar{c}}}$.
144; 192, 216, 256, 288; 384.

Hence we fee, that the interval from D to F is contained in the ratio of 27 to 32, and that of F to d in the ratio of 32 to 54, the two terms of which are divifible by 2; and then in place of this relation we have that of 16 to 27. The firft interval is called a *tierce minor*, or *leffer third*, and the other a *greater fixth*. The number 27 might be ftill farther multiplied by 3, but mufic extends not fo far, and we limit ourfelves to number 27, refulting from 3, multiplied three times by itfelf: other mufical tones ftill wanting are introduced by means of number 5, and fhall be unfolded in my next Letter.

3d *May*, 1760.

LETTER VII.

Of the twelve Tones of the Harpfichord.

THE prefent fubject of my correfpondence with your Highnefs is fo dry, that I begin to apprehend it may be growing tirefome. That I may not wafte too much time on it, and be relieved from the neceffity of recurring frequently to a topic fo difgufting,

gufting, I fend you by this conveyance three letters at once. My intention, in undertaking it, was to render vifible the real origin of mufical notes, with which muficians themfelves are almoft totally unacquainted. It is not to theory they are indebted for the knowledge of all thefe founds; but rather to the fecret power of genuine harmony, operating fo efficacioufly on their ears, that they have been conftrained, if I may be allowed to fay it, to receive tones actually in ufe, though they are not hitherto perfectly agreed refpecting their juft determination. The principles of harmony are ultimately reducible to numbers,* as I have demonftrated; and it has been remarked, that the number 2 furnifhes octaves only, fo that having fixed, for example, the note F, we are conducted to the notes f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$. The number 3 afterwards furnifhes C, c, $\bar{c}$, $\bar{\bar{c}}$, $\bar{\bar{\bar{c}}}$, which differ one fifth from the preceding feries; and the repetition of this fame number 3, furnifhes again the fifths of the firft, namely G, g, $\bar{g}$, $\bar{\bar{g}}$, $\bar{\bar{\bar{g}}}$: and finally, the third repetition of this number 3 adds farther the notes D, d, $\bar{d}$, $\bar{\bar{d}}$, $\bar{\bar{\bar{d}}}$. The principles of harmony then being attached to fimplicity, feem to forbid our pufhing farther the

* This is true only to a certain degree; for, if we except the knowledge of the relation of notes, or the numerical expreffion of intervals, numbers cannot be introduced into mufic, as Mr. d'Alembert has juftly remarked, but as a piece of ufelefs parade; and the fcanty knowledge they furnifh is far fhort of the theory of compofition, which is founded on the pleafure of the ear, and hitherto no one has attempted to make this a fubject of calculation. —*F. E.*

repetition

repetition of number 3; hitherto, accordingly, we have only the following notes for each octave:

F. G. c. d. f.
16. 18. 24. 27. 32. which certainly would not furnish a very copious music. But let us introduce, in addition to these, number 5, and observe the tone which shall emit five vibrations while F emits only one. Now, f makes two vibrations in the same time; $\bar{f}$ makes four; and $\bar{\bar{c}}$ six. The note in question then, is between $\bar{f}$ and $\bar{\bar{c}}$. It is that which musicians indicate by letter $\bar{a}$, the accord of which, with note $\bar{f}$, is denominated a *greater third*, and is found to produce a very agreeable concord, being expressed by the very simple ratio of 4 to 5. Farther, note $\bar{a}$ with note $\bar{\bar{c}}$ produces an accord contained in the ratio of 5 to 6, which is almost as agreeable as the former, and which is denominated a *lesser third*, represented by the ratio of 27 to 32, and it's difference from the first is almost imperceptible to the ear. This same number 5 being applied to the other notes G, c, d, will give us, in like manner, their greater thirds, taken in the second octave below, that is to say, the notes $\bar{b}$, $\bar{\bar{e}}$, and $\bar{\bar{\bar{f}}}$, which, being transposed, will give the following notes, with their corresponding numbers.

F. Fs. G. A. B. c. d. e. f.
128. 135. 144. 160. 180. 192. 216. 240. 256.

Take away the notes Fs, and you will have the principal touches of the harpsichord, which, according to the ancients, constitute the genus denominated *diatonic*, resulting from number 2, from number 3, thrice repeated, and from number 5. Admitting these

these sounds only, we are in a condition to compose harmonies very agreeable and various, the beauty of which is founded on the simplicity alone of the numbers corresponding to the notes. Finally, upon applying, a second time, the number 5, we shall be furnished with the thirds of the four new tones, A, E, B, Fs, which we have just found, we shall have the notes Cs Gs Ds and B, so that now the octave is completed of the 12 tones received in music. All these tones derive their origin from the three numbers 2, 3, and 5, multiplying 2 by itself, as often as the octaves require; but we carry the multiplication of 3 only to the third stage, and of five to the second. All the tones of the first octave are contained in the following table, in which you will see how the fundamental numbers 2, 3, and 5, enter into the composition of those which express the relation of these notes.

				Difference.
ut or	C	2, 2, 2, 2, 2, 2, 2, 3 . . .	384	
ut ♯	Cs	2, 2, 2, 2, 5, 5	400	16
re	D	2, 2, 2, 2, 3, 3, 3.	432	32
re ♯	Ds	2, 3, 3, 5, 5.	450	18
mi	E	2, 2, 2, 2, 2, 3, 5.	480	30
fa	F	2, 2, 2, 2, 2, 2, 2, 2, 2. .	512	32
fa ♯	Fs	2, 2, 3, 3, 3, 5	540	28
sol	G	2, 2, 2, 2, 2, 2, 3, 3 . . .	576	36
sol ♯	Gs	2, 2, 2, 3, 5, 5	600	24
la	A	2, 2, 2, 2, 2, 2, 2, 5 . . .	640	40
si b.	B♭	3, 3, 3, 5, 5.	675	35
si n	B♮	2, 2, 2, 2, 3, 3, 5.	720	45
ut	c	2, 2, 2, 2, 2, 2, 2, 3, 3. .	768	48

While

While note C makes 384 vibrations, the tone Cs gives 400, and the others as many as are marked by their correfponding numbers: note c will give then, in the fame time, double the number of vibrations marked by 384, that is 768. And for the following octaves, you have only to multiply thefe numbers by 2, by 4, or by 8. Accordingly note $\bar{c}$ will give twice 768, or 1536 vibrations, note $\bar{\bar{c}}$ twice 1536, or 3072 vibrations, and note $\bar{\bar{\bar{c}}}$ twice 3072, or 6144 vibrations. In order to comprehend the formation of founds, by means of thefe numbers 2, 3, and 5, it muft be remarked, that the points placed between the numbers in the preceding table fignify that they are multiplied into each other; thus, taking the tone Fs, for example, the expreffion 2, 2, 3, 3, 3, 5, fignifies 2 multiplied by 2, that product by 3, that again by 3, that again by 3, and that by 5. Now 2 by 2 make 4, that by 3 make 12, that by 3 make 36, that by 3 make 108, and that by 5 make 540. Hence it is feen that the differences between thefe tones are not equal among themfelves; but that fome are greater, and others lefs. This is what real harmony requires. The inequality, however, not being confiderable, we commonly look on all thefe differences as equal, denominating the interval from one note to another, *femitone;* and thus the octave is divided into 12 *femitones*. Many modern muficians make them equal, though this be contrary to the principles of harmony, becaufe no one fifth or third is perfectly exact, and the effect is the fame as if thefe tones were not

not perfectly in tune.* They likewife admit, that we muft give up exactnefs of accord in order to obtain the advantage of equality of femitones, fo that the tranfpofition from any one tone whatever to another may in no refpect injure the melody. They acknowledge, however, that the fame piece played in the tone C, or a half tone higher, that is Cs, muft confiderably affect it's nature. It is evident, therefore, that in fact all femitones are not equal, whatever efforts may be made by muficians to render them fuch; becaufe true harmony refifts the execution of a defign contradictory to it's nature. Such, then, is the real origin of the mufical notes already in ufe; they are derived from the numbers 2, 3, and 5. Were we farther to introduce number 7, that of the tones of an octave would be increafed, and the art of mufic carried to a higher degree of perfection. But here the mathematician gives up the mufician to the direction of his ear.

3d May, 1760.

* The alteration thus forced upon the fifths, in order that every key of the harpfichord may ferve equally for the higher note flattened, and for the lower fharpened, and that, at the end of the fubdivifion, the octaves may be exact, is called *temperament*. It has been remarked that fifths may be a little weakened without hurting the ear very much; whereas greater thirds become harfh and difagreeable when they are ftrengthened.—*F. E.*

LETTER

LETTER VIII.

Of the Pleaſure derived from fine Muſic.

IT is a queſtion as important as curious, Whence is it that a fine piece of muſic excites a ſentiment of pleaſure? The learned differ on this ſubject. Some pretend that it is mere caprice, and that the pleaſure produced by muſic is not founded on reaſon, becauſe what is grateful to one is diſguſting to another. This, far from deciding the queſtion, renders it only more complicated. The very point to be determined is, How comes it, that the ſame piece of muſic produces effects ſo different, ſeeing all admit that nothing happens without reaſon? Others maintain that the pleaſure derived from fine muſic conſiſts in the perception of the order which pervades it. This opinion appears at firſt ſight ſufficiently well founded, and merits a more attentive examination. Muſic preſents objects of two kinds, in which order is eſſential. The one relates to the difference of the ſharp or flat tones; and you will recollect, that it conſiſts in the number of vibrations performed by each note in the ſame time. This difference, which is perceptible between the quickneſs of the vibrations of all ſounds, is what is properly called harmony. The effect of a piece of muſic, of which we feel the relations of the vibrations of all the notes that compoſe it, is the production of harmony. Thus two notes which differ an octave, excite a perception of the re-lation

lation of 1 to 2; a fifth, of that of 2 to 3; and a greater third, of that of 4 to 5. We comprehend then the order which is found in harmony, when we know all the relations which pervade the notes of which it is compoſed, and it is the perception of the ear which leads to this knowledge. This perception more or leſs delicate, determines why the ſame harmony is felt by one, and not at all by another, eſpecially when the relations of the notes are expreſſed by ſomewhat greater numbers. Muſic contains, beſide harmony, another object equally ſuſceptible of order, namely the *meaſure*, by which we aſſign to every note a certain duration: and the perception of the meaſure conſiſts in the knowledge of this duration, and of the relations which reſult from it. The drum and tymbal furniſh the example of a muſic in which meaſure alone takes place, as all the notes are equal among themſelves, and then there is no harmony. There is likewiſe a muſic conſiſting wholly in harmony, to the excluſion of meaſure. This muſic is the *choral*, in which all the notes are of the ſame duration; but perfect muſic unites harmony and meaſure. Thus the connoiſſeur who hears a piece of muſic, and who comprehends, by the acute perception of his ear, all the proportions on which both the harmony and the meaſure are founded, has certainly the moſt perfect knowledge poſſible of that muſic; while another, who perceives theſe proportions only in part, or not at all, underſtands nothing of the matter, or poſſeſſes at moſt a very ſlender knowledge of it. But the ſentiment of pleaſure excited

cited by fine muſic muſt not be confounded with the knowledge of which I have been ſpeaking, though it may be confidently affirmed, that a piece of muſic cannot produce any, unleſs the relations of it are perceived. For this knowledge alone is not ſufficient to excite the ſentiment of pleaſure; ſomething more is wanting, which no one hitherto has unfolded. In order to be convinced that the perception alone of all the proportions of a piece of muſic is inſufficient to produce pleaſure, you have only to conſider muſic of a very ſimple conſtruction, ſuch as goes in octaves alone, in which the perception of proportions is undoubtedly the eaſieſt. Such muſic would be far from conveying pleaſure, though you might have the moſt perfect knowledge of it. It will be ſaid then that pleaſure requires a knowledge not quite ſo eaſily attained, a knowledge that occaſions ſome trouble; which muſt, if I may uſe the expreſſion, coſt us ſomething. But, in my opinion, neither is this a ſatisfactory ſolution. A diſſonance, the relations of which are expreſſed by the higheſt numbers, is caught with more difficulty; a ſeries of diſſonances, however, following without choice, and without deſign, cannot pleaſe. The compoſer muſt therefore have purſued in his work, a certain plan, executed in real and perceptible proportions. Then a connoiſſeur on hearing ſuch a piece, and comprehending, beſide the proportions, the very plan and deſign which the compoſer had in view, will feel that ſatisfaction which conſtitutes the pleaſure procured by exquiſite muſic to an ear accuſtomed to reliſh the beauties and delicacies

cacies of that enchanting art. It ariſes, then, from divining in ſome meaſure the views and feelings of the compoſer, whoſe execution, when fortunate, fills the ſoul with an agreeable ſenſation. It is a ſatisfaction ſomewhat ſimilar to that which is derived from the ſight of a well acted pantomime, in which you may conjecture, by the geſture and action, the ſentiments and dialogue intended to be expreſſed, and which preſents beſides a well digeſted plan. The enigma of the chimney-ſweeper,* which was ſo diverting to your Highneſs, furniſhes me with another excellent compariſon. When you can gueſs the ſenſe, and diſcover that it is perfectly expreſſed in the propoſition of the enigma, you feel a very ſenſible pleaſure on making the diſcovery; but inſipid and incongruous enigmas produce none. Such are, if I may be permitted to judge, the true principles on which deciſions reſpecting the excellency of muſical compoſitions are founded.

6th May, 1760.

LETTER IX.

Compreſſion of the Air.

THE explanation of ſound, which I have had the honour to preſent to your Highneſs, leads me forward to a more particular conſideration of air, which, being ſuſceptible of a movement of vibration, ſuch

* A celebrated enigma of La Mothe, publiſhed in his fugitive pieces.

fuch as that by which mufical ftrings, bells, and other fonorous bodies are agitated, tranfmits the concuffion to our ears. It will be immediately afked, What is air? For it does not appear, at firft fight, to be a material fubftance. As we perceive no fenfible body in it, furrounding fpace feems to contain no matter whatever. We feel nothing; we can walk, and move every limb in it, without meeting the flighteft obftacle. But you have only to move your hand brifkly, to be fenfible of fome refiftance, and even to perceive a ftream of wind excited by that rapid movement. Now the wind is nothing elfe but air put in motion; and feeing it is capable of producing effects fo furprifing, how is it poffible to doubt that air is a material fubftance, and confequently a body?* For the terms *body* and *matter* are fynonymous.

Body is divided into two great claffes, folid and fluid. The air, it is evident, muft be referred to the clafs of fluids. It has feveral properties in common with water; but it is much more fubtile and fine. Experiments have afcertained that air is about 800 times more fubtile and more rarefied than water;

* It is an erroneous principle that the air is diftinguifhed from other fluids by it's fufceptibility of compreffion. All fluids are perfectly elaftic, only the force required to produce a certain degree of compreffion differs very widely in each. Thus the fame force which caufes water to fuffer a contraction of only the thirty thoufandth part of it's bulk, condenfes air into one half. The real diftinction between the aeriform and liquid fluids feems to be, that the reaction of the former is proportional to their denfity, while that of the latter is proportional to the quantity of compreffion.—*E. E.*

and

and that if air were to be rendered 800 times denfer than it is, it would have the fame confiftency as the other fluid. A principal property of air, by which it is diftinguifhed from other fluids, is it's quality of being compreffed, or reduced into a fmaller fpace. This is demonftrated by the following experiment. Take a tube of metal or glafs A B C D (*plate* I. *fig.* 2.) clofe fhut at the end A B, and open at the other, into which is introduced a pifton, filling exactly the cavity of the tube. On pufhing the pifton inwards, when it has arrived at the middle E, the air which occupied at firft the cavity A B C D will be reduced one half, and confequently will have become twice as denfe. If the pifton is pufhed ftill farther in, as far as F, half way between B and E, the air will be reduced to a fpace four times fmaller than at firft; and if you continue to drive forward the pifton to G, fo that B G fhall be the half of B F, or the eighth part of the whole length B D, the fame air which in the beginning was expanded over the whole cavity of the tube, will be contracted to a fpace eight times fmaller. Going on in the fame manner to contract it into a fpace 800 times fmaller, you will obtain an air 800 times denfer than ordinary air. It would then be as denfe as water, which it would be eafy to prove by other experiments. Hence it appears, that air is a fluid fubftance, capable of compreffion, or, in other words, of being reduced to a fmaller fpace, and in this refpect it differs entirely from water. For, let the tube A B C D be filled with this laft fluid, and attempt to introduce the pifton, you will find it im-

poffible

poſſible to drive it forward. Employ what force you may, you will gain nothing; the tube will burſt ſooner than you can reduce the water to a ſpace ſenſibly ſmaller. This then is the eſſential difference between air and water: the latter is ſuſceptible of no compreſſion, but air may be compreſſed to any degree you pleaſe. The more the air is compreſſed, the denſer it becomes; thus the air which occupied a certain ſpace, when compreſſed or reduced to half that ſpace, becomes twice as denſe; if reduced to a ſpace 10 times ſmaller, it is rendered 10 times more denſe; and ſo on. I have already remarked, that could it be rendered 800 times more denſe, it would then be as denſe as water, and conſequently as heavy, for weight increaſes in the ſame proportion as denſity. Gold, the heavieſt ſubſtance with which we are acquainted, is likewiſe the moſt denſe. It is found by experiment to be 19 times heavier than water; and that a maſs of gold, in form of a cube of one foot, would weigh 19 times a maſs of water of the ſame dimenſions. Now ſuch a maſs of water weighs 70 pounds; the maſs of gold therefore would weigh 19 times 70, that is 1330 pounds. It follows that were it poſſible to compreſs air till it were reduced to a ſpace 19 times 800, that is, 15,200 times ſmaller, it would become as denſe and as weighty as gold.

But it is very far from being poſſible to carry the compreſſion of air to that degree. You may at firſt without difficulty drive forward the piſton, but the farther you advance, the reſiſtance becomes more powerful;

powerful; and, before you are able to reduce the air to a ſpace 10 times ſmaller, ſuch a force muſt be employed as would burſt the tube, unleſs it were of uncommon ſtrength. And not only would ſuch a force be neceſſary to drive the piſton farther, but an equal force would be requiſite to keep it in that ſtate, for on the ſlighteſt relaxation of the power, the compreſſed air would drive it backward. The more compreſſed the air is, the more violent are it's efforts to expand, and to recover it's natural ſtate. This is what we call the ſpring or elaſticity of the air, of which I propoſe to treat in my next letter.

10th May, 1760.

LETTER X.

Rarefaction and Elaſticity of the Air.

I HAVE remarked, that air is a fluid, about 800 times more ſubtile than water; ſo that could water, without being reduced to vapour, be expanded over a ſpace ſo many times greater, and could become of conſequence ſo many times more ſubtile, it would be of a ſimilar conſiſtence with the air which we breathe. But air has a property which water has not, that of ſuffering compreſſion into a ſmaller ſpace, and of being proportionably condenſed, as I demonſtrated in the preceding letter. And we diſcover in air another property no leſs remarkable: it is capable of being expanded over a greater ſpace, and thus rendered

rendered ftill more fubtile. This operation is called the rarefaction of air.

You have only to take, as before, a tube A B C D, (*plate* I. *fig.* 3.) at the bottom of which A C, let there be a fmall aperture O, fo that, on introducing the pifton as far as to F, the air may efcape by that aperture without being condenfed. The air which now occupies the cavity A C E F, will then be in it's natural ftate; let the aperture O be clofely ftopped. On drawing back the pifton, the air will gradually expand through the greater fpace, fo that when the pifton is brought back to the point G, the fpace C G being double the fpace C F, the fame air which was contained in the fpace A C E F, will fill a fpace twice as great; it will be of courfe only half as denfe, or, which is the fame thing, twice as rare. If you draw back the pifton to the point H, the fpace C H being four times as great as the fpace C F, the air will become four times as rare as it was at firft, as it is then expanded over a fpace four times as great. And could the pifton be drawn back till the fpace became 1000 times as great, the air would ftill equally expand through that fpace, and confequently become 1000 times as rare. Here then, likewife, air differs effentially from water: for if the cavity A C E F were filled with water, to no purpofe would you draw back the pifton; the water would continue to occupy the fame fpace as at firft, and the reft would remain empty. Hence we fee that the air poffeffes an intrinfic power of expanding itfelf more and more, which it exerts not only when it is condenfed,

but

but alfo when rarefied. In whatever ftate of condenfation or rarefaction the air may be, it makes unremitting efforts to extend itfelf over a larger fpace, and is continually expanding fo long as it meets no obftacle. This property is called the elafticity of air; and it has been demonftrated by experiments which I fhall prefently defcribe, that this elaftic power is in proportion to the denfity; in other words, the more the air is condenfed the greater are it's efforts to expand itfelf; and the more rarefied it is, the feebler are thofe efforts. It will be demanded, perhaps, why the air now in my chamber does not make it's efcape by the door, being endowed with an expanfibility continually impelling it to occupy a greater fpace? The anfwer is obvious. This would infallibly happen, did not the external air make equal efforts to extend itfelf; but the efforts of the air of the chamber to get out, and that of the external air to prefs in, being equal, they balance each other, and remain in a ftate of reft. Had the external air accidentally acquired a greater degree of denfity, that is, more elafticity, it would in part force it's way into the chamber, where the air being compreffed, would likewife acquire a greater degree of elafticity; this current would accordingly laft till the elafticity of the internal became equal to that of the external air. And fhould the air of the chamber fuddenly become more denfe, and it's elafticity greater than that of the external air, it would force it's way out, and it's denfity gradually diminifhing, it's elafticity too would diminifh, till it became equal to the external air;

the

the current would then ceaſe, and the air in the chamber would be in equilibrium with the external. Free air, then, is in a ſtate of reſt only when it has the ſame degree of elaſticity with that which ſurrounds it; and as ſoon as that of the one tract becomes more or leſs elaſtic than the adjoining, the equilibrium can no longer ſubſiſt; but if the elaſticity is greater, the air will expand itſelf and ſlide into ſpaces where it is ſmaller: and from this motion of the air reſults the wind.* Hence it comes to paſs that the elaſticity of the air is ſometimes greater, ſometimes leſs in the ſame place; and this variation is indicated by the *barometer*, the deſcription of which merits a particular conſideration. I confine myſelf, at preſent, to theſe qualities of air, it's condenſation and rarefaction, intreating you to recollect, that the more condenſed it is, the greater power of expanſion or elaſticity it acquires; and that on the contrary, the more it is rarefied, the more this quality is diminiſhed. Experimental philoſophers have invented one machine for rarefying of air, and another for condenſing it: the former is called the air-pump, the latter the condenſer. Theſe machines ſerve to perform many curious experiments, with which you are already well acquainted. I reſerve to myſelf, however, the liberty of recapitu-

* The action of the moon upon the atmoſphere, and the motion of the earth's rotation likewiſe produce regular winds. Chains of mountains ſometimes change the direction of winds. Hence we ſee that the known cauſe of currents of air are of three kinds, regular, accidental, and local.—*F. E.*

lating

lating ſome of them, becauſe they are neceſſary to elucidate and explain the nature and properties of air, which, as they powerfully contribute to the preſervation of animals, and the production of plants, preſs upon us the importance of forming a juſt idea of them.

14th May, 1760.

LETTER XI.

Gravity of the Air.

I HAVE endeavoured to demonſtrate, that the air is a fluid, endowed with the particular property of ſuffering compreſſion into a ſmaller ſpace, and of expanding into a greater, when no obſtacle interpoſes. This property of air, known by the name of ſpring, or elaſticity, from it's reſemblance to a ſpring, which it requires an effort to unbend, and which reſumes it's form as ſoon as the effort ceaſes, is accompanied by another, in common to it with all bodies in general, namely, gravity or weight, in virtue of which all bodies tend toward the centre of the earth, and by which they are under the neceſſity of falling down, unleſs ſupported. The learned are very much divided, and very uncertain, reſpecting the primary and mechanical cauſe of this power, but it's exiſtence is indubitable.* Daily experience evinces

* The properties of matter muſt ultimately be referred to the arbitrary appointment of the Author of Nature. There are certain

it. We know even the quantity of it, and can meaſure it exactly. For the weight of a body is nothing elſe but the power which conſtrains it to deſcend; and as the weight of every body may be exactly meaſured, we know perfectly well the effect of gravity, though the cauſe, or that inviſible power which acts upon all bodies, forcing them to deſcend, may be abſolutely unknown to us. It follows, that the more matter any body contains, the heavier it is. Gold and lead are heavier than wood or a feather, as they contain more matter in the ſame bulk, or in the ſame extent. But as air is a very ſubtile and thin ſubſtance, and it's gravity of conſequence very little, this property uſually eſcapes our ſenſes. Experiments, however, may be made, capable of producing full conviction that it poſſeſſes gravity. You have ſeen how the air may be rarefied in a veſſel or a tube; and by means of the air-pump, this rarefaction may be carried ſo far, as almoſt entirely to exhauſt the air, and to leave the receiver ſenſibly a vacuum. Or you may take a tube A B C D, (*plate* I. *fig.* 4.) into which you introduce the piſton, ſo as perfectly to touch the bottom, and to leave no air between the two ſurfaces. To perform the experiment with more advantage, let there be at the bottom of the

tain principles at which the prudent philoſopher will chooſe to ſtop, leſt, by puſhing his reſearches too far, he involve himſelf in greater obſcurity. Thoſe who attempted to account for gravity by mechanical impulſe, committed an egregious overſight; for the queſtion ſtill recurs, What produces this impulſe? No metaphyſical work has ever done ſo much ſervice to philoſophy as Mr. Hume's admirable eſſay on "Neceſſary Connexion."—*F. E.*

tube

tube a little aperture G, through which the air may eſcape, as the piſton is puſhed forward. Let the aperture then be cloſely ſtopped, that not a particle of air may be included between the piſton and the bottom of the tube. Having made this arrangement, draw back the piſton, and the external air not being able to force it's way into the tube, there will remain between the bottom of the tube and the piſton, a perfect vacuum, which may be increaſed at pleaſure, by continuing to draw back the piſton. You may thus exclude the air contained in a veſſel; and ſuch veſſel, reduced to a vacuum, being tried in accurate ſcales, will be found to weigh leſs than when filled with air. Hence we deduce this very important concluſion, That the air contained in an empty veſſel increaſes it's weight, and that the air itſelf poſſeſſes gravity. Were the veſſel large enough to contain 800 pounds weight of water, we might diſcover by this experiment, that the body of air which fills it would weigh nearly one pound. Hence we conclude, that air is 800 times lighter than water. I muſt be underſtood as ſpeaking of the common air which ſurrounds us, and which we breathe; for you know that with the aſſiſtance of art, air may be compreſſed by forcing it into a ſmaller ſpace, and it's gravity thereby increaſed. Were the veſſel which I have mentioned, to be filled with air compreſſed to twice the conſiſtency of common air, it would weigh two pounds more than when empty. Were it filled with air 800 times more compreſſed than common air, it would weigh 800 pounds more than when empty,

that

that is, as much as if it were filled with water. The air, then, poſſeſſing a certain degree of gravity, though in the natural ſtate of this fluid it's gravity is extremely ſmall, it muſt, however, as well as all other bodies, tend toward the centre of the earth, and conſequently preſſes on every thing which oppoſes this tendency. For this reaſon the ſuperior air preſſes downward on the inferior, and this laſt undergoes a compreſſion from the weight of the whole maſs of air which is above it. Hence it comes to paſs, that in theſe regions, the air has a certain degree of compreſſion or denſity, which is the effect of the gravity of the ſuperior air; and that if the ſuperior air had more or leſs gravity, the air which ſurrounds us would likewiſe become more or leſs denſe. It is thus that the air below ſupports the weight of the ſuperior air, and that the more we aſcend, the more it loſes it's denſity and rarefies; ſo that were it poſſible to continue to aſcend, the air would at length be totally loſt, or would become ſo ſubtile and ſo rarefied, as to be no longer perceptible. Were you to deſcend, on the contrary, into a very deep pit, you would find the denſity of the air continually increaſing, from the increaſe of the maſs of air preſſing downward upon it.

17th May, 1760.

LETTER

LETTER XII.

Of the Atmoſphere, and the Barometer.

HAVING demonſtrated that air is a fluid, elaſtic, and poſſeſſed of gravity, I proceed to remark, that the earth is ſurrounded on all ſides by this fluid, and that the ſpace which it fills is called the atmoſphere. It would be abſolutely impoſſible for a perfect vacuum to exiſt on any part of the earth's ſurface; for the air of the adjoining regions, compreſſed by the weight of the ſuperior air, and making inceſſant efforts to dilate, would force itſelf into the empty ſpace and fill it. The atmoſphere, therefore, occupies the whole region which ſurrounds the earth; the inferior air is continually compreſſed by the weight of the ſuperior air, and that until the degree of elaſticity which reſults from this compreſſion, is able to form an equilibrium to the compreſſing power. Then, although this air is compreſſed only in a downward direction, it produces, in virtue of it's elaſticity, efforts to expand itſelf not only downwards, but ſideways alſo. For this reaſon, the air in a chamber is as much compreſſed as the external, which appeared a paradox to certain philoſophers. For they reaſoned thus: In a chamber, the inferior air is compreſſed only by the ſmall quantity of ſuperior air included in that chamber, whereas the external air is compreſſed by the weight of the whole atmoſphere, the height of which is immenſe. But the

the difficulty is at once removed, by the property which air poſſeſſes, of expanding itſelf when compreſſed in all directions. Now the air in the chamber is at firſt reduced, by the external air, to the ſame degree of compreſſion and elaſticity with itſelf; hence, whether I am in my chamber, or in the open air, I feel the ſame compreſſion; being always underſtood, that I mean at the ſame height, or at the ſame diſtance from the centre of the earth. For I have already remarked, that on getting to the ſummit of a high tower, or of a lofty mountain, the compreſſion of the air is leſs, becauſe the weight of the ſuperior air is then diminiſhed. Various phenomena confirm this ſtate of the compreſſion of the air.

Take, for inſtance, (*plate* I. *fig*. 5.) a tube A B, cloſe at the end A, and having filled it with water, or any other fluid, invert it, ſo that the open end B may be undermoſt, and you will find that the fluid does not run out. The elaſticity of the air acting at B, in oppoſition to the fluid, ſupports it in the tube. But if you make an aperture into the tube at A, the fluid immediately deſcends: the air which is admitted by the aperture acts then from above, by it's preſſure upon the water, and forces it downward; which demonſtrates, that while the tube was cloſe at top, it was the external air which ſupported the water in it. And were ſuch a tube to be placed in a receiver, from which the air was extracted by the air-pump, the fluid would inſtantly deſcend. The ancients, to whom this property of air was unknown, alleged, that nature ſupported the water in the tube, from the

the horror which it has of a vacuum. For, ſaid they, were the fluid to deſcend, there muſt be a vacuum at the upper end of the tube, as the air could find no admiſſion into it. According to them, therefore, it was the horror of a vacuum which kept the fluid ſuſpended in the tube. It is now demonſtrated, that it is the force of the air which ſupports the weight of the fluid in the tube; and as this force has a determinate quantity, the effect cannot exceed a certain limit.

It is found by experiment, that if the tube A B is more than 33 feet in length, water will no longer remain ſuſpended in it, but will run out till it comes to the height of 33 feet; the ſpace left a-top will, of courſe, be a real vacuum. The force of the air then cannot ſupport water in the tube at more than the height of 33 feet; and as the ſame force ſupports the whole atmoſphere, it is concluded, that a column of the atmoſphere is of equal weight, the baſis being equal with a column of water 33 feet high. If, inſtead of water, you were to uſe mercury, which is 14 times heavier, the force of the air could ſupport it in the tube at the height of only 28 inches; and if you go beyond that, the mercury deſcends, till it's height correſponds to the preſſure of the atmoſphere, leaving the ſpace a-top in the tube a vacuum. Such a tube cloſe above, and open below, being filled with mercury, forms the inſtrument called the *Barometer*, by means of which it has been diſcovered, that the atmoſphere is not always of equal gravity. For it's real gravity is aſcertained by the barometer, from the height

height of the mercury, which, as it riſes or falls, indicates that the denſity of the air, or the preſſure of the atmoſphere, is increaſing or diminiſhing.

20th May, 1760.

LETTER XIII.

Of Wind-Guns, and the Compreſſion of Air in Gun-Powder.

HAVING explained that remarkable property of air which is denominated compreſſibility, by means of which it is reducible into a ſmaller ſpace, we are enabled to give an account of ſeveral productions of both nature and art. I ſhall begin with an explanation of the wind-gun, though I have no doubt but you are well acquainted with that inſtrument. It's conſtruction is ſimilar to that of the common fuſil; but inſtead of powder, we employ condenſed air to diſcharge the bullet.

In order to comprehend the proceſs of this operation, it muſt be obſerved, that air can be compreſſed only by a force proportional to the degree of condenſation which you wiſh to obtain; in this ſtate, it ſtrives to extend itſelf, and the efforts which it makes are preciſely equal to the force neceſſary to reduce it to the ſize which it actually occupies. The more, then, that the air is condenſed, the more violent are it's efforts to dilate; and if the air is raiſed to a denſity twice as great as when it is free, which is the caſe

caſe when we reduce it to half the ſpace which it occupies in it's natural ſtate, the force with which it endeavours to expand is equal to the preſſure of a column of water 33 feet high. Figure to yourſelf a great caſk of this height, filled with water; this fluid will, undoubtedly, make a ſtrong preſſure on the bottom of the veſſel. If you make a hole in it near the bottom, the water will force itſelf out with conſiderable violence: and on ſtopping the aperture with your finger, you will be abundantly ſenſible of this preſſure of the water. The bottom of the caſk ſuſtains throughout a ſimilar preſſure. Now a veſſel containing air twice as denſe as that of the atmoſphere, muſt undergo preciſely ſuch a preſſure, and if it were not ſufficiently ſtrong to ſuſtain it, would burſt. The ſides, then, of this veſſel muſt be as ſtrong as the bottom of the caſk I have mentioned. If in the ſame veſſel the air were three times as denſe as common air, the force with which it would act upon the ſides muſt be increaſed in the proportion of one more, and would be the ſame which is ſuſtained by the bottom of a caſk full of water, of 66 feet in height. You will eaſily conceive that this force muſt be very great, and that it muſt go on increaſing in the ſame ratio, according to the different degrees of condenſation of the air. This being laid down, there is, at the bottom of the air-gun, a cavity ſtrongly fortified on all ſides, into which the air is more and more compreſſed, in order to reduce it to as high a degree of denſity as the force employed for

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that purpofe can admit. The air confined in this cavity will thereby acquire a prodigious power to force itfelf out: and if an aperture is made, it will fly off with a velocity proportional to that power. Now there is fuch an aperture which terminates in the cavity of the tube into which the ball is put. It is clofely ftopped; but when you wifh to difcharge the piece, you open, for an inftant, the valve which fhuts it; and the air rufhing forth, forces out the ball with all the velocity which we remark in fhooting. Every time you difcharge, the valve is kept open only a fingle moment; a certain quantity of air, therefore, and no more, can efcape, and enough ftill will remain for feveral fhot. But on difcharge, it's denfity and correfponding elafticity diminifh; and for this reafon, the latter difcharges are lefs forcible than the former, till the force is at length entirely exhaufted. Were the valve to remain open any confiderable time, more air would make it's efcape, which would all go to wafte; for this force acts upon the ball only while it is in the barrel of the gun; as foon as it is fhot off, it is ufelefs to leave a paffage for the air. Hence it appears, that were it poffible to carry the condenfation of this fluid a great deal farther, you will produce from the wind-gun the fame effects as from the guns and cannons in common ufe.

The effect of artillery is accordingly founded on the fame principle. Gunpowder is only a fubftance, which contains in it's pores an air extremely condenfed.

denfed.* Nature produces here the fame operations which we employ for compreffing the air, but carries the condenfation to a much higher degree. All that is neceffary is to open the little cavities in which this denfe air is confined, that it may have liberty to efcape. This is performed by means of fire, which

* Recent experiments have fomewhat corrected this explanation. Gunpowder, it is well known, is a compofition of fulphur, nitre and charcoal. In the detonation of this fubftance, the heat puts the fulphur and charcoal in a condition to diffolve the acid of the nitre, and to take from it the dephlogiftic air which enters into it's compofition. The atmofpheric *mephites*, which is another principle of this acid, finding itfelf thereby difengaged, begins to expand, and forms a firft elaftic permanent fluid. The firing of the charcoal produces fixed air, which is a fecond elaftic permanent fluid. That of the fulphur produces the vitriolic acid, which is reduced to vapour by the heat of the inflammation *(a)*. Finally, the water which enters into the compofition of the powder, is likewife converted into vapour. Here, then, are four elaftic fluids produced in the progrefs of this operation. To their expanfion the phenomenon of the explofion is to be afcribed. The two laft, brought back to a liquid ftate by being cooled, form the fmoke we perceive after the difcharge.—*F. E.*

(a) This account of the aeriform fluids, extricated by the inflammation of gun-powder, feems very embarraffed. Sulphur is not an effential ingredient in gun-powder; but as it burns at a low heat, it renders the mafs more fufceptible of catching fire. The inflammation of gunpowder is precifely the fame phenomenon with the detonation of charcoal and nitre. That falt is compofed of vegetable alkali and nitrous acid, which confifts of pure and mephitic airs united in a certain proportion. By means of the heat at firft applied, and then rapidly evolved during the procefs of inflammation, the nitrous acid is decompofed; it's mephitic air is expelled, while it's pure air, combining with the charcoal, forms fixed air, which is alfo difcharged. It appears from experiment, that this aerial compound, at the inftant of it's extrication, has upwards of five hundred times the elafticity of common air.—*E. E.*

 burfts

burfts open thefe little envelopes: the air then fuddenly flies off, with incredible velocity, and forces before it bullets and balls in a manner entirely fimilar to that which we have remarked in the cafe of the wind-gun, but with much greater impetuofity. Here, then, are two very furprifing effects produced from the condenfation of air, with this fingle difference, that in the one, it is the work of art; and in the other, that of nature. We fee therefore in this, as in every thing elfe, how infinitely the operations of human fkill are furpaffed by thofe of nature.

24th May, 1760.

LETTER XIV.

The Effect produced by the Heat and Cold on all Bodies, and of the Pyrometer and Thermometer.

BESIDE the properties already mentioned, air has another very remarkable quality, in common to it with all bodies, not excepting fuch as are folid; I mean the change produced on it by heat and cold. It is obfervable, in general, that all bodies, being heated, dilate or increafe in fize. A bar of iron made very hot, is fomewhat longer and thicker than when it is cold. There is an inftrument called the *Pyrometer*, which accurately indicates the flighteft differences of length or fhortnefs, that a bar of iron undergoes, to which it is applied. You know that in

in a watch, fome of the wheels move very flowly, though they communicate motion to others which revolve with confiderable rapidity. By a fimilar mechanifm it is poffible, from a change almoft imperceptible, to produce one very confiderable, as is the cafe of the pyrometer, which I have juft mentioned. It has an index, which runs over a very confiderable fpace, on the flighteft change produced in the length of the body on which the experiment is made. On applying this inftrument to a bar of iron, or any other metal, placed over a burning lamp, the index is immediately put in motion, and fhews that the bar is becoming longer; and, as the heat increafes, the bar likewife increafes in length. But on extinguifhing the lamp, and the bar growing cold again, the index moves in a contrary direction, and thereby fhews that the bar is growing fhorter. The difference, however, is fo flight, that, without the help of this inftrument, it would be difficult to perceive it. Yet this variation is abundantly perceptible in the motion of pendulum time-pieces. The ufe of the pendulum is to regulate the motion. If you lengthen it, the clock goes flower, and if you fhorten it, the clock goes fafter. Now it is remarked, that in very hot weather all clocks lofe time, and proportionably gain it in very cold weather; which clearly demonftrates, that the pendulum is lengthened or fhortened, according to the temperature of the air.

All bodies undergo this alteration, but the quantity differs greatly, according to the nature of the

 fubftance

fubftance of which they are compofed. In fluids, efpecially, this variability is very perceptible. To afcertain it, take a glafs tube, B C, *(plate* I. *fig.* 6*)* joined at the end B to a hollow ball A, and let it be filled with any liquor you pleafe up to M. On heating the ball A, the liquor will rife from M toward C: when it becomes cold again, the liquor will fall toward B. This clearly proves that the fame liquid occupies a greater fpace when it is heated, and a fmaller when cold. It is likewife clear, that this variation muft be more perceptible, when the ball is large, and the tube narrow. For, if the whole mafs of liquor increafes or diminifhes by a thoufandth part, that thoufandth part will occupy, in the tube, a fpace great in proportion to it's narrownefs. Such an inftrument then is exceedingly proper to indicate to us the different degrees of heat and cold; for if the liquor rifes or falls, it is a certain indication that the heat is increafing or diminifhing. This inftrument is called the *Thermometer*, which points out the changes that take place in the temperature of the air, and of the bodies which furround us. It muft not be confounded with the barometer, whofe ufe is to indicate the gravity of the air, or rather the force with which it is compreffed. This caution is the more neceffary, that the barometer and thermometer have a confiderable refemblance: being both glafs tubes filled with mercury; but their conftruction, and the principles on which they are founded are entirely different. This quality of body, extenfion by

heat, and contraction by cold, belongs likewife, in a very fuperior degree, to air. I fhall explain it at greater length in my next letter.*

27th May, 1760.

LETTER XV.

Changes produced in the Atmofphere by Heat and Cold.

HEAT and cold produce the fame effect on air, as on every other body. Air is rarefied by heat, and condenfed by cold. From what I have faid of the elafticity of air, you eafily perceive, that a certain quantity of this fluid is not determined to occupy only a certain fpace, as all other bodies are; but by

* There are three kinds of thermometers in ufe at prefent, that of Reaumur is adopted in France, Switzerland, and Italy; that of Celfius in Sweden and Denmark. In both of thefe, the fcale commences at the freezing point; but the interval, between that and the boiling point, is divided, in the former, into 80 parts, and the latter, into 100. Farenheit's thermometer is ufed in Britain and Holland; the freezing point is marked on it 32, and the boiling 212, the interval containing 180 degrees. The freezing point is very nearly permanent, but the boiling point depends on the preffure of the atmofphere, and near the furface of the earth it varies one degree and fix-tenths for every inch of variation in the height of the barometer. Water has been heated in a clofe veffel to fuch a degree, as to melt lead and tin; and in the receiver of an air-pump, it may be converted into vapour, at the ordinary temperature of the air. Hence the reafon why water boils fo quickly on the fummit of lofty mountains. The boiling point would be at 172° on the heights of the Andes.—*E. E.*

it's nature, it has a perpetual tendency to dilate, and actually does expand itſelf, as long as it meets no obſtacle.

This property of air is denominated elaſticity. When this fluid is confined in a veſſel, it makes efforts in every direction to burſt it; and theſe efforts are greater or leſs in proportion to it's condenſation. Hence we come to this concluſion, that the elaſticity of air is in exact proportion to it's denſity; ſo that when it's denſity is doubled, it's elaſticity is likewiſe doubled; and that, in general, a certain degree of elaſticity correſponds to a proportional degree of denſity. It muſt be remarked, however, that this takes place no longer than while the air preſerves the ſame degree of heat. Whenever it becomes hotter, it acquires greater power of expanſion than what correſponded to it's denſity; and cold produces the oppoſite effect, by diminiſhing it's expanſive power. In order then to determine the elaſticity of a maſs of air, it is not ſufficient to know it's denſity; you muſt likewiſe know it's degree of heat. In order to ſet this in a clear light. Let us ſuppoſe two chambers cloſely ſhut on all ſides, but united by a door of communication; and that the heat in both is equal. In order to this the air in both chambers muſt have the ſame degree of denſity. For were the air more denſe, and conſequently more elaſtic, in the one than in the other, part of it would eſcape from the one, and force it's way into the other, till the denſity in both were the ſame. But let us ſuppoſe that one of the chambers has become hotter than the other, the air thereby

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acquiring a greater elaſticity, would of courſe force itſelf into the other, and reduce that which it found there into a ſmaller ſpace, till the elaſticity in both chambers was brought to the ſame degree. During this change there will be a current of air, through the door, from the chamber which is more, into that which is leſs heated; and when the equilibrium is reſtored, the air will be more rarefied in the warm apartment, and more condenſed in the cold; and yet the elaſticity of both will be the ſame. From this it clearly follows, that two maſſes of air of different denſity, may have the ſame elaſticity, when the one is hotter than the other; and this circumſtance taken into conſideration, it may happen, that with the ſame degree of denſity, they may be endowed with different degrees of elaſticity.

What I have ſaid of two chambers may be applied to two countries; and hence it may be concluded, that when one country becomes warmer than the other, there muſt of neceſſity be a current of air from the one to the other: and from this reſults the wind.

Here, then, is one fruitful ſource of winds, though there are perhaps others, which conſiſt in the different degrees of heat, which prevail in different regions of the earth; and it is demonſtrable, that the whole air which ſurrounds the earth could not be in a ſtate of reſt, unleſs that, univerſally, at equal heights, there were found the ſame degree, not only of denſity, but likewiſe of heat. And ſhould it happen that there were no wind over the whole ſurface of the earth, it might with certainty be concluded, that

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the air would likewife be every where equally denfe and warm at equal heights. Now as this never happens, there muft of neceffity always be winds, at leaft in fome regions. But thefe winds are, for the moft part, to be met with only on the furface of the earth; and the higher you rife, the lefs violent winds are. Winds are hardly perceptible at the fummit of very high mountains;* there perpetual tranquillity reigns; from which it is impoffible to doubt, that at confiderable elevations, the air is always in a ftate of reft. Hence it follows, that in regions remarkably elevated, there univerfally prevails all over the earth, the fame degree of denfity and heat; for were it hotter in one place than in another, the air could not be in a ftate of reft. And, as there is no wind in thefe elevated regions, it muft neceffarily follow, that the degree of heat there muft be univerfally and always the fame; which is a very furprizing paradox, confidering the great variations of heat and cold which we feel on the furface of the earth, during the courfe of a year, and even of one day; without taking into the account the difference of climate, that is, the intolerable heats felt under the equator, and the

* This does not appear perfectly exact. A perpetual current of wind, from eaft to weft, muft be produced by the motion of the earth's rotation. It refults, likewife, from M. d'Alembert's theory of winds. Befides, the attraction of the moon, which is capable of raifing the waters of the globe, undoubtedly communicates fome motion to the atmofphere. Here, then, we have fuperior currents.

When aëroftation is carried to perfection, it will, perhaps, procure us fatisfying information refpecting this article of meteorology.—*F. E.*

dreadful

dreadful cold which ever prevails toward the poles of the earth. Experience itfelf, however, confirms the truth of this aftonifhing fact. The fnow and ice remain equally, fummer and winter, on the mountains of Switzerland, and are equally unchangeable on the Cordeliers, lofty mountains of Peru, fituated under the very equator, and where there perpetually reigns, neverthelefs, a cold as exceffive as that of the polar regions. The height of thefe mountains is not a German mile,* or 24,000 feet. From this it may be, with confidence, concluded, that were it poffible for us to afcend to the height of 24,000 feet, above the earth, we fhould always and univerfally meet with the fame degree of cold, and that cold exceffively fevere.† We fhould remark there no fenfible difference during either fummer or winter, under the equator, or near the poles. At this height, and ftill higher, the ftate of the atmofphere is univerfally, and at all feafons, the fame; and the variations of heat and cold take place near the furface of the earth alone. It is only in thefe inferior regions, that the effect of the rays of the fun becomes perceptible. You have, undoubtedly, fome curiofity to know the reafon of this. It fhall be the fubject of the following letter.

31*ft May*, 1760.

* About 4 3-5ths miles, Englifh.

† M. Charles, in his aërial voyage of the 1ft Dec. 1783, felt this change of temperature in a very fenfible manner; for then, on the furface of the earth, the fluid in the thermometer ftood at 7° above the freezing point, and after about 10 minutes of afcenfion, it had fallen to 5° below it.—*F. E.*

LETTER

LETTER XVI.

The Cold, felt on high Mountains and at great Depths, accounted for.

IT appears very ſurpriſing, that we ſhould feel the ſame degree of cold in all regions, after we have riſen to a certain height, ſay 24,000 feet; conſidering that the variations with reſpect to heat, on the earth, not only in different climates, but in the ſame country, at different ſeaſons of the year, are ſo perceptible. This variety, which takes place at the ſurface of the globe, is undoubtedly occaſioned by the ſun. It appears, at firſt ſight, that his influence muſt be the ſame above and below, eſpecially when we reflect, that a height of 24,000 feet, or a mile, though very great with reſpect to us, and even far beyond the height of the loftieſt mountains, is a mere nothing, compared to the diſtance of the ſun, which is about thirty millions of miles.* This is, therefore, a very important difficulty, which we muſt endeavour to ſolve. For this purpoſe I begin with remarking, that the rays of the ſun do not communicate heat to any bodies, but ſuch as do not grant them a free paſſage. You know that bodies, through which we can diſcern objects, are denominated *tranſparent*, *pellucid*, and *diaphonous*. Theſe bodies are glaſs, cryſtal, diamond,

* Mr. Euler always means German miles, of 4000 fathoms each, or ſomewhat under 4 3-5ths miles Engliſh.—*E. E.*

diamond, water, and ſeveral other liquids, though ſome are more or leſs tranſparent than others. One of theſe tranſparent bodies being expoſed to the ſun, is not heated to ſuch a degree as a body not tranſparent, as wood, iron, &c. Bodies not tranſparent are denominated *opaque*. A burning-glaſs, for example, by tranſmitting the rays of the ſun, ſets on fire opaque bodies, while the glaſs itſelf is not ſenſibly heated. Water expoſed to the ſun becomes ſomewhat warm, only becauſe it is not perfectly tranſparent; when we ſee it conſiderably heated by the ſun at the brink of rivers, it is becauſe the bottom, being an opaque body, is heated by the rays which the water tranſmits. Now, every heated body communicates that heat to all adjoining bodies; the water accordingly derives heat from the bottom. If the water be very deep, ſo that the rays cannot penetrate to the bottom, it has no perceptible heat, though the ſun bears upon it.

As air is a very tranſparent body to a much higher degree than glaſs or water, it follows that it cannot be heated by the ſun, becauſe the rays are freely tranſmitted through it. The heat which we frequently feel in the air is communicated to it by opaque bodies, which the rays of the ſun have heated; and were it poſſible to annihilate all theſe bodies, the air would ſcarcely undergo any change in it's temperature by the rays of the ſun: expoſed to it or not it would be equally cold. But the atmoſphere is not perfectly tranſparent: it is even ſometimes ſo loaded with vapours, that it loſes almoſt entirely it's tranſparency,

transparency, and presents only a thick fog. When the air is in this state, the rays of the sun have a more powerful influence upon it, and heat it immediately.

But these vapours rise to no great height; at the height of 24,000 feet, and beyond, the air is so subtile and so pure, that it is perfectly transparent; and for this reason the rays of the sun cannot immediately produce any effect upon it. This air is likewise too remote from terrestrial bodies to receive a communication of heat from them; they act only upon such as are adjacent. Hence you will easily perceive that the rays of the sun cannot produce any effect in regions of the air very much elevated above the surface of the earth; and that the same degree of cold must always and universally prevail in such regions, as the sun has no influence there, and as the heat of terrestrial bodies cannot be communicated so far. This is nearly the case on the summit of very high mountains, where it is always much colder than on plains and in vallies.*

* There are clouds, however, above these mountains, and in almost as great a quantity as above the plains, which is demonstrated by the snows which cover the highest summits. There are few naturalists who have not been surprised by clouds in their excursions upon the mountains. The heat that is felt when such clouds are formed must be attributed almost entirely to the transmission of the water which found itself dissolved in the air, under the form of elastic fluid, to a liquid state. The heat of the solar rays, intercepted by the cloud, can produce no change in the inferior temperature, as it would have been transmitted from the ground.—*F. E.*

The

The city of Quito, in Peru, is almost under the equator, and were we to form our judgment from it's situation on the globe, we would suppose it oppressed with intolerable heat; the air, however, is abundantly temperate, and differs very little from that of Paris. Quito is situated at a great height above the real surface of the earth. In going to it from the sea shore you have to ascend for several days; it is accordingly built in an elevation equal to that of our highest mountains, though surrounded by others still much higher, called the Cordeliers. This last circumstance would afford a reason for thinking that the air there must be as hot as at the surface of the earth, as it is contiguous on all sides to opaque bodies, on which the rays of the sun fall. The objection is solid; and no solution can be given but this. That the air at Quito, being very elevated, must be much more subtile, and of less gravity than with us; and the barometer, which always stands considerably lower, incontestably proves it.

Air of such a quality is not so susceptible of heat as common air, as it must contain less vapour and other particles which usually float in the atmosphere; and we know by experience that air very much loaded is proportionably susceptible of heat. I must here subjoin another phenomenon no less surprising: In very deep pits, and lower still, if it were still possible to descend, the same degree of heat always and universally prevails, and nearly for the same reason. As the rays of the sun exert their influence only on the surface of the earth, and as the heat which they there

there excite communicates itſelf up and down, this effect at very great depths is almoſt imperceptible. The ſame thing holds reſpecting conſiderable heights. This elucidation will, I flatter myſelf, prove ſatisfactory.*

3d June, 1760.

LETTER

* The reaſon which Profeſſor Euler aſſigns for the cold that prevails in the higher regions of the atmoſphere ſeems plauſible, but will not ſtand an accurate examination. Light is much impaired in it's paſſage through the atmoſphere, and the heat communicated is in every caſe proportional to the quantity of abſorption. It appears, from ſome ingenious experiments of M. Bouguer, that we receive only four-fifths of the rays of a vertical ſun; and when that luminary approaches the horizon, the portion of his light which reaches the ſurface of the earth, is much ſmaller. Thus at an elevation of 20 degrees it is one half; at that of 10 degrees one third; and at that of five degrees one-eighth. Hence the ſun-beams are moſt powerful on the ſummits of lofty mountains, for they ſuffer the greateſt diminution in paſſing through the denſe air of the lower regions. If the air derived it's heat from the ſurface of the earth, thoſe countries would be warmeſt which enjoyed the greateſt quantity of ſun-ſhine. The Britiſh iſlands are ſhrouded in clouds nine months of the year; yet our climate is milder than that of the ſame parallel on the Continent, where the ſky is generally ſerene. The elevated town of Quito, expoſed to a brilliant ſun, enjoys a temperate air; while the Peruvian plains, ſhaded with fleecy clouds, are parched with heat. Were the reaſoning in the text to be admitted, we ſhould conclude that the tops of mountains are warmer than their baſes. To ſay that air, much rarefied, is not ſuſceptible of heat, is a very extraordinary aſſertion, ſince we are acquainted with no ſubſtance whatever that may not be heated. Beſides, a more intenſe cold may be artificially produced than what prevails in the lofty regions of the atmoſphere. We muſt recur to other principles for the true ſolution

LETTER XVII.

Of Light, and the Syſtems of Deſcartes and Newton.

HAVING ſpoken of the rays of the ſun, which are the focus of all the heat and light that we enjoy, you will undoubtedly aſk, What are theſe rays? This is beyond queſtion one of the moſt important inquiries in phyſics, as from it an infinite number

ſolution of the fact. It is indifferent what portion of the air firſt receives the heat; the effect depends entirely on the nature of it's diſtribution. If the atmoſphere were of an uniform denſity throughout, the heat would at all heights be likewiſe the ſame. But as the denſity varies according to the altitude, the diſtribution of heat is affected by that circumſtance, and follows a certain correſponding law. I would gladly develope the principles from which this theory is deduced, but the popular nature of the preſent treatiſe forbids all abſtract diſcuſſion. I ſhall therefore content myſelf with giving a table of the diminution of heat at different altitudes.

Altitude in feet.	Diminution of heat, in degrees, of Farenheit.
3,000	12°
6,000	24½
9,000	38
12,000	53
15,000	68½
18,000	86½
21,000	94½

The diminution of heat, on the aſcent, is not quite ſo great in extenſive continents; for the intercourſe between the rare and the denſe portions of the atmoſphere is, in this caſe, neceſſarily ſlow, and the heat, which is principally formed at the ſurface, will only be partially diſperſed.

It

number of phenomena is derived. Every thing that refpects light, and that renders objects vifible, is clofely connected with this inquiry. The ancient philofophers feem to have taken little intereft in the folution of it. They contented themfelves with faying that the fun is endowed with the quality of fhining, of giving heat and light. But is it not worth while to inquire, Wherein does this quality confift? Do certain portions, inconceivably fmall, of the fun himfelf, or of his fubftance, come down to us?

It is a common miftake to fuppofe, that the fame heat obtains, at a certain depth, in every part of the globe. The fact is, that heat, originally derived from the fun, is communicated very flowly to the matter below the furface, which, therefore, does not feel the viciffitude of feafons, but retains the average temperature of the climate for many ages. Hence the utility of examining the heat of fprings, which is the fame with that of the fubftances through which they flow. The following table exhibits the average heat of places on the level of the fea, computed by the celebrated aftronomer, Profeffor Meyer, for every five degrees of latitude.

Latitude.	Average Temperature.	Latitude.	Average Temperature.
0 —	— 84°	50 —	— 53½°
5 —	— 83½	55 —	— 49
10 —	— 82½	60 —	— 45
15 —	— 80½	65 —	— 41½
20 —	— 78	70 —	— 38
25 —	— 74½	75 —	— 35½
30 —	— 71	80 —	— 33½
35 —	— 67	85 —	— 32½
40 —	— 62½	90 —	— 32
45 —	— 58		

By comparing this table with the preceding, it is eafy to difcover, for any latitude, the altitude of the curve of congelation, or where the average temperature is 32°.—*E. E.*

us? Or is the tranfmiffion fimilar to the found of a bell, which the ear receives? though no part of the fubftance of the bell be feparated from it, as I obferved in explaining the propagation and perception of found.

Defcartes, the firft of modern philofophers, maintained this laft opinion, and having filled the whole univerfe with a fubtile matter compofed of fmall globules, which he calls the fecond element, he fuppofes that the fun is in a ftate of continual agitation, which he tranfmits to thefe globules, and pretends that they again communicate their motion in an inftant to every part of the univerfe. But fince it has been difcovered that the rays of the fun do not reach us inftantaneoufly, and that they take eight minutes to fly through that immenfe diftance,* the opinion of *Defcartes*, which laboured befide under other difficulties, has been given up.

The great *Newton* afterwards embraced the former fyftem, and maintained that the luminous rays are really feparated from the body of the fun, and the particles of light thence emitted with that inconceivable velocity which brings them down to us in about eight minutes. This opinion, which is that of moft modern philofophers, particularly the Englifh, is called

* This important fact was difcovered toward the end of the laft century by Roemer, a learned Dane, of the ancient Academy of Sciences. It was an inequality of the fatellites of Jupiter which led him to it. The caufe of this aberration, difcovered by Bradley in 1728, inconteftably demonftrates the fame phenomenon.—*F. E.*

called *the ſyſtem of emanation;* it being imagined that rays emanate from the ſun and other luminous bodies, as water emanates or ſprings from a fountain.

This opinion appears at firſt ſight very bold, and irreconcileable to reaſon. For were the ſun emitting continually, and in all directions, ſuch floods of luminous matter, with a velocity ſo prodigious, he muſt ſpeedily be exhauſted, or at leaſt ſome alteration muſt, after the lapſe of ſo many ages, be perceptible. This, however, is contradicted by obſervation. It cannot be a matter of doubt, that a fountain which ſhould emit ſtreams of water in all directions, would be exhauſted in proportion to the velocity of the emiſſion; much more the ſun, whoſe rays are emitted with a velocity ſo inconceivable. Let the particles of which rays of light are formed be ſuppoſed as ſubtile as you pleaſe, nothing will be gained: the ſyſtem will ever remain equally untenable. It cannot be affirmed that this emanation is not made in all directions: for, wherever you are placed, the whole ſun is viſible, which proves inconteſtably, that rays from every point of the ſun are emitted toward the ſpot which you occupy. The caſe is very different from that of a fountain, which ſhould emit ſtreams of water in all directions. For one point in the fountain could furniſh only one ſtream directed to a particular ſpot, but every point of the ſun's ſurface muſt emit an infinite number, diffuſing themſelves in all directions. This circumſtance alone infinitely increaſes the expenditure of luminous matter, which the ſun would have to make.

Another difficulty, and which appears equally inſuperable, is, that the ſun is not the only body which emits rays, but that all the ſtars have the ſame quality: and as every where the rays of the ſun muſt be croſſing the rays of the ſtars, their colliſion muſt be violent in the extreme. How muſt their direction be changed by ſuch colliſion! This colliſion muſt take place with reſpect to all luminous bodies, viſible at the ſame time. Each, however, appears diſtinctly, without ſuffering the ſlighteſt derangement from any other: a certain proof that many rays may paſs through the ſame point, without diſturbing each other, which ſeems irreconcileable to the ſyſtem of emanation. Let two fountains be ſet a playing upon each other, and you will immediately perceive their different ſtreams diſturbed and confounded: it muſt of conſequence be concluded, that the motion of the rays of light is very eſſentially different from that of a *jet d'eau*, and in general from all ſubſtances forcibly emitted.

Conſidering afterwards tranſparent bodies through which rays are freely tranſmitted in all directions, the ſupporters of this ſyſtem are under the neceſſity of affirming that theſe bodies contain pores, diſpoſed in ſtraight lines, which iſſue from every point of the ſurface, and proceed in all directions; it being inconceivable how there could be any line through which a ray of the ſun might be tranſmitted with ſuch amazing velocity, and even without the ſlighteſt colliſion. Here then are bodies wonderfully porous, which have the appearance, neverthelefs, of being extremely folid.

Finally, in order to enjoy vifion, the rays muft enter into the eye, and penetrate it's fubftance with the fame velocity. All thefe difficulties, taken together, will, I doubt not, fufficiently convince you, that the fyftem of emanation has in no refpect a foundation in nature; and you will certainly be aftonifhed that it could have been conceived by fo great a man, and embraced by fo many enlightened philofophers. But it is long fince Cicero remarked, that nothing fo abfurd can be imagined as to find no fupporter among philofophers. For my part, I am too little a philofopher to adopt the opinion in queftion.

7th June, 1760.

LETTER XVIII.

Difficulties attending the Syftem of Emanation.

HOWEVER ftrange the doctrine of the celebrated *Newton* may appear, that rays proceed from the fun by a continual emanation, it has, however, been fo generally received, that it requires an effort of courage to call it in queftion. What has chiefly contributed to this is, no doubt, the high reputation of the great Englifh philofopher, who firft difcovered the true laws of the motions of the heavenly bodies: and it is this very difcovery which led him to the fyftem of emanation.

Defcartes, in order to fupport his theory, was under the neceffity of filling the whole fpace of the heavens with a fubtile matter, through which all the celeftial bodies move at perfect liberty. But it is well known that if a body moves in air, it muft meet with a certain degree of refiftance; from which *Newton* concluded, that however fubtile the matter of the heavens may be fuppofed, the planets muft encounter fome refiftance in their motions. But, faid he, this motion is not fubject to any refiftance: the immenfe fpace of the heavens, therefore, contains no matter. A perfect vacuum, then, univerfally prevails. This is one of the leading doctrines of the Newtonian philofophy, that the immenfity of the univerfe contains no matter in the fpaces not occupied by the heavenly bodies. This being laid down, there is between the fun and us, or at leaft from the fun down to the atmofphere of the earth, an abfolute vacuum. In truth, the farther we afcend, the more fubtile we find the air to be; from whence it would apparently follow, that at length the air would be entirely loft. If the fpace between the fun and the earth be an abfolute vacuum, it is impoffible that the rays fhould reach us in the way of communication, as the found of a bell is tranfmitted by means of the air. For if the air, intervening between the bell and our ear, were to be annihilated, we fhould abfolutely hear nothing, let the bell be ftruck ever fo violently.

Having eſtabliſhed, then, a perfect vacuum between the heavenly bodies, there remains no other opinion to be adopted but that of emanation; which obliged *Newton* to maintain, that the ſun and all other luminous bodies emit rays which are always particles, infinitely ſmall, of their maſs, darted from them with incredible force. It muſt be ſuch to a very high degree, in order to impreſs on rays of light that inconceivable velocity with which they come from the ſun to us in the ſpace of eight minutes. But let us ſee whether this theory be conſiſtent with *Newton's* leading doctrine, which requires an abſolute vacuum in the heavens, that the planets may encounter no manner of reſiſtance to their motions. You muſt conclude, on a moment's reflection, that the ſpace in which the heavenly bodies revolve, inſtead of remaining a vacuum, muſt be filled with the rays, not only of the ſun, but likewiſe of all the other ſtars which are continually paſſing through it from every quarter, and in all directions, with incredible rapidity. The heavenly bodies which traverſe theſe ſpaces, inſtead of encountering a vacuum, will meet with the matter of luminous rays in a terrible agitation, which muſt diſturb theſe bodies in their motions much more than if it were in a ſtate of reſt.

Thus *Newton*, apprehenſive leſt a ſubtile matter, ſuch as *Deſcartes* imagined, ſhould diſturb the motions of the planets, had recourſe to a very ſtrange expedient, and quite contradictory to his own intention, as, on his hypotheſis, the planets muſt be expoſed to a derangement infinitely more conſiderable. I have

I have already ſubmitted to you ſeveral other inſuperable objections to the ſyſtem of emanation; and we have now ſeen that the principal and indeed the only reaſon which could induce *Newton* to adopt it, is ſo ſelf-contradictory as wholly to overturn it. All theſe conſiderations united, leave us no room to heſitate about the rejection of this ſtrange ſyſtem of the emanation of light, however reſpectable the authority of the philoſpher who invented it.

Newton was, without doubt, one of the greateſt geniuſes that ever exiſted. His profound knowledge, and his acute penetration into the moſt hidden myſteries of nature, will be a juſt object of admiration to the preſent, and to every future age. But the errors of this great man ſhould ſerve to admoniſh us of the weakneſs of the human underſtanding, which, after having ſoared to the greateſt poſſible heights, is in danger of plunging into manifeſt contradiction.*

10th June, 1760.

LETTER

* The *pious* as well as learned and ingenious Author, in the firſt edition of theſe Letters, ſubjoined to this reflection on *Newton's* doctrine of emanation a ſeries of reflections which do equal honour to his underſtanding and his heart. The French Editor, for what reaſon it does not appear, has thought proper to ſuppreſs them. Could he imagine a philoſophical work diſgraced by a modeſt and not unſeaſonable infuſion of religious ſentiment? Be how it will, the Engliſh Editor felt himſelf obliged to reſtore the paſſage, in preſenting the too long neglected Euler to the Britiſh nation.—— It follows:

"If we are liable to weakneſſes and inconſiſtencies ſo humiliat-"ing,

LETTER XIX.

A different Syftem refpecting the Nature of Rays and of Light, propofed.

YOU have feen that the fyftem of the emanation of the rays of light labours under infuperable difficulties, and that the doctrine of a vacuum for the heavenly bodies to range in, is equally untenable; as the rays of light would completely fill it. Two things, then, muft be admitted: firft, the fpace through which the heavenly bodies move is filled with a fubtile matter; fecondly, rays are not an actual emanation from the fun and other luminous bodies, in virtue of which part of their fubftance is

" ing, in our refearches into the phenomena of this vifible world, " which lies open to the examination of our fenfes, how wretched " muft we have been had God left us to ourfelves with refpect to " things invifible, and which concern our eternal falvation? On " this important article a Revelation was abfolutely neceffary to " us; and we ought to avail ourfelves of it with the moft pro- " found veneration. When it prefents to us things which may " appear inconceivable, we have but to reflect on the imperfection " of human underftanding, which is fo apt to be mifled, even as " to fenfible objects. Whenever I hear a pretended Freethinker " inveighing againft the truths of religion, and even fneering at " it with the moft arrogant felf-fufficiency, I fay to myfelf: poor " weak mortal, how inexpreffibly more noble and fublime are the " fubjects which you treat fo lightly, than thofe refpecting which " the great *Newton* was fo grofsly miftaken! I could wifh your " Highnefs to keep this reflection ever in remembrance: occafions " for making it occur but too frequently."—*E. E.*

violently

violently emitted from them, according to the doctrine of *Newton*.*

That fubtile matter which fills the whole fpace in which the heavenly bodies revolve, is called *Ether*. Of it's extreme fubtilty no doubt can be entertained. In order to form an idea of it, we have only to attend to the nature of air, which, though extremely fubtile, even on the furface of the earth, becomes more and more fo as we afcend; and entirely ceafes, if I may ufe the expreffion, when it comes to be loft in the ether. The ether, then, is likewife a fluid as the air is, but incomparably finer and more fubtile, as we are affured that the heavenly bodies revolve

* The materiality of light is fupported by the moft convincing proofs that phyfics can afford. The inflection, refraction, and reflection of it's rays, fhew manifeftly that, like other bodies, it is fubject to attraction and repulfion; and the fimple application of the doctrine of forces not only explains fatisfactorily the phenomena, but affigns the precife effects with the moft perfect accuracy. The difficulties which feem to attend the theory of emanation vanifh on a clofe inveftigation. So vaft is the tenuity of light, that it utterly exceeds the powers of conception. The moft delicate inftrument has never been certainly put in motion by the impulfe of the accumulated fun-beams. Even on the moft unfavourable fuppofition it appears from calculation that, in the fpace of 385,130,000 Egyptian years (of 360 days) the fun would lofe only the 1,217,420th of his bulk, from the continual efflux of light. On the fame hypothefis the force impreffed upon the earth by each emiffion is fuch as would make it recede only the two billionth part of an inch in an hundred feconds, and it's effect, during a feries of ages, would therefore be altogether infenfible. After ftating numbers of a magnitude fo enormous, it would be fuperfluous to confider the quantity of ftroke which the eye receives.

freely

freely through it, without meeting any perceptible refiftance. It is alfo without doubt poffeffed of elafticity, by means of which it has a tendency to expand itfelf in all directions, and to penetrate into fpaces where there would otherwife be a vacuum; fo that if by fome accident the ether were forced out of any fpace, the furrounding fluid would inftantly rufh in and fill it again.

In virtue of this elafticity, the ether is to be found not only in the regions which are above our atmofphere, but it penetrates the atmofphere univerfally, infinuates itfelf by the pores of all bodies, and paffes irrefiftibly through them. Were you, by the help of the air-pump, to exhauft the air from a receiver, you muft not imagine that you have produced an abfolute vacuum; for the ether, forcing itfelf through the pores of the receiver, completely fills it in an inftant. Having filled a glafs tube of the proper length with mercury, and immerged it, when inverted, in the ciftern, in order to make a barometer, it might be fuppofed that the part of the tube which is higher than the mercury is a vacuum, becaufe the air is completely excluded, as it cannot penetrate the pores of glafs: but this vacuum which is apparent only, is undoubtedly fupplied by the ether, infinuating itfelf without the fmalleft difficulty.

It is by this fubtilty and elafticity of ether that I fhall by and by explain to you the remarkable phenomena of electricity. It is even highly probable that ether has an elafticity much fuperior to that of air, and that many of the phenomena of nature are produced

produced by means of it. For my own part I have no doubt that the compreffion of the air in gunpowder is the effect of the elaftic power of ether. And as we know by experiment that the air in it is condenfed almoft 1000 times more than common air, and that in this ftate it's elafticity is likewife 1000 times greater, the elafticity of the ether muft in this cafe be fo too, and confequently 1000 times greater than that of common air. We fhall then have a juft idea of ether, in confidering it as a fluid in many refpects fimilar to air, with this difference, that ether is incomparably more fubtile and more elaftic.*

Having feen then that the air, by thefe very qualities, is in a proper ftate for receiving the agitations or fhakings of fonorous bodies, and to diffufe them in all directions, as we find in the propagation of found, it is very natural to fuppofe that ether may in the fame circumftances likewife receive agitations in the fame manner, and tranfmit them to the greateft diftances.† As the vibrations of the air produce *found*,

* This, perhaps, is what in modern times they denominate the matter of heat.—*F. E.*

† The hypothefis of an ether is a clumfy attempt to preclude the neceffity of admitting *action at a diftance*. It has been a received maxim, that caufe and effect muft exift in the fame place; but the leaft reflection will convince us that, were this principle true, there could never be any communication of motion. The difficulty is really the fame, to conceive action exerted at the diftance of the thoufandth part of an inch, as at that of a thoufand miles. The particles of matter are far from being in mutual contact, otherwife

found, What will be the effect of thofe of ether? You will undoubtedly guefs at once *light*. It appears in truth abundantly certain, that light is with refpect to ether, what found is with refpect to air; and that the rays of light are nothing elfe but the fhakings or vibrations tranfmitted by the ether, as found confifts in the fhakings or vibrations tranfmitted by the air.

The fun, then, lofes nothing of his fubftance in this cafe, any more than a bell in vibrating; and, in adopting this fyftem, there is no reafon to apprehend that the mafs of this orb fhould ever fuffer any diminution. What I have faid of the fun muft alfo be extended to all luminous bodies, fuch as fire, a wax taper, a candle, &c.

It will, undoubtedly, be objected, that thefe terreftrial luminaries evidently wafte, and that unlefs they are continually fed and kept up, they will be fpeedily extinguifhed; that confequently the fun muft in time be wafted away, and that the parallel of a bell is not accurate. But it is to be confidered, that thefe fires, befides their light, throw out fmoke, and a great deal of exhalation, which muft be carefully diftinguifhed from the rays of light. Now the fmoke and exhalation evidently occafion a confiderable diminution, which muft not be imputed

wife all bodies would have the fame denfity, and be totally incapable of compreffion. Were the univerfe an abfolute *plenum*, motion and animation would for ever ceafe. To afcribe to ether an extreme rarity, and at the fame time to affert that it fills all fpace, and pervades all bodies, is a contradiction in terms. But the hypothefis is fo big with abfurdity, that it deferves not a particular examination. *See note, p.* 41.—*E. E.*

to

to the rays of light; for were it poffible to feparate them from the fmoke and other exhalations, the luminous quality alone would occafion no expenditure. Mercury may, by means of art, be rendered luminous, as you have probably feen, and that without any diminution of it's fubftance, which proves that light alone produces no wafte of luminous bodies. Thus though the fun illuminates the whole world by his rays, he lofes nothing of his own fubftance, his light being only the effect of a certain agitation, or violent concuffion of his minute particles, communicated to the adjoining ether, and thence tranfmitted in all directions by means of this fluid to the remoteft diftances, as a bell when ftruck communicates it's own agitation to the circumambient air. The more we confider this parallel between fonorous and luminous bodies, the more we fhall find it conformable to nature, and juftifiable by experience; whereas the more we attempt to reconcile the phenomena of nature to the fyftem of emanation, the more difficulties we encounter.

14th June, 1760.

LETTER XX.

Of the Propagation of Light.

THE propagation of light in the ether is produced in a manner fimilar to that of found in the air; and juft as the vibrations occafioned in the particles of air conftitutes found, in like manner the vibration

of

of the particles of ether conftitutes light or luminous rays; fo that *light is nothing elfe but an agitation or concuffion of the particles of ether*, which is every where to be found on account of it's extreme fubtilty, in virtue of which it penetrates all bodies.

Thefe bodies, however, modify the rays of light in many different ways, by tranfmitting or ftopping the propagation of the concuffions. Of this I fhall treat at large in the fequel. I confine myfelf at prefent to the propagation of rays in the ether itfelf, which fills the immenfe fpace in which the heavenly bodies revolve. There the propagation takes place in perfect liberty. The firft thing which here prefents itfelf to the mind is the prodigious velocity of the rays of light, which is about 900,000 times more rapid than that of found, though this laft travels no lefs than 1000 feet in a fecond.

This amazing velocity would be fufficient of itfelf to overturn the fyftem of emanation; but in that which I am attempting to eftablifh, it is a natural confequence, from the principles laid down, as I hope to demonftrate. They are the fame with thofe on which is founded the propagation of found in the air, and this depends at once on it's denfity and elafticity. It is evident that if the denfity of air were diminifhed, found would be accelerated, and if the elafticity of the air were increafed, the fame thing would happen. If the denfity of the air diminifhed, and it's elafticity increafed at once, we fhould have a two-fold reafon for the increafe of the velocity of found. Let us conceive, then, the denfity of the air diminifhed,

diminifhed, and it's elafticity increafed, till it's denfity and elafticity became equal to thofe of ether, and we fhould then no longer be furprifed that the velocity of found had become many thoufands of times greater than it actually is. For you will be pleafed to remember, that, according to the firft ideas we formed of ether, this fluid muft be inconceivably rarer, and more elaftic than air. Now both of thefe qualities equally contribute to accelerate the velocity of vibrations. From this explanation, the prodigious velocity of light is fo far from prefenting any thing irreconcileable to reafon, that it rather perfectly harmonizes with the principles laid down; and the parallel between light and found is in this refpect fo firmly eftablifhed, that we may confidently maintain, That if air fhould become as fubtile and as elaftic as ether, the velocity of found would become as rapid as that of light.

The fubtlity of ether, then, and it's great elafticity, are the reafon which we affign for the prodigious velocity of the motion of light; and fo long as the ether preferves this fame degree of fubtlity and elafticity, this velocity muft continue the fame. Now it cannot be doubted that the ether has, through the whole univerfe, the fame fubtlity and the fame elafticity. For were the ether lefs elaftic in one place than in another, it would force itfelf into it till the equilibrium was perfectly reftored. The light of the ftars, therefore, moves with as great velocity as that of the fun; and as the ftars are at a much greater diftance from us than the fun, a much greater quantity

tity of time is requifite to tranfmit their rays to us. However great the diftance of the fun may appear, whofe rays, neverthelefs, reach the furface of our globe in eight minutes, the fixed ftar neareft to us is at leaft 400,000 times more diftant than the fun: a ray of light iffuing from that ftar will employ then 400,000 times eight minutes in travelling to us, that is 53,333 hours, or 2,222 days, or fix years nearly.

It is then upwards of fix years fince the rays of light iffued from that fixed ftar, the leaft remote and probably the moft brilliant, in order to render it vifible to us, and thefe rays have employed a period fo confiderable to fly through the fpace which feparates us from that ftar. Were God juft now to create a new fixed ftar, at the fame diftance, it could not become vifible to us till more than fix years had elapfed, as it's rays require that length of time to travel this diftance. Had one been created at the beginning of the world a thoufand times more diftant than that which I have mentioned, it could not yet be vifible to us, however brilliant, as 6000 years are not yet elapfed fince the Creation. The firft preacher of the court of Brunfwick, Mr. Jerufalem, has happily introduced this thought in one of his fermons; the paffage runs thus:

" Raife your thoughts from the earth which you " inhabit to all the bodies of the vaft univerfe, which " are fo far above you: launch into the immenfity " of fpace which intervenes between the moft re- " mote which your eyes are able to difcover, and " thofe whofe light, from the moment of creation " till now, has not as yet, perhaps, come down to us. " The immenfity of the kingdom of God juftifies " this reprefentation." (Sermon *on the Heavens, and Eternal Beatitude.)*

I flatter myfelf that thefe reflections will excite a defire of further inftruction refpecting the fyftem of light, from which is derived the theory of colours, and of vifion.

17th June, 1760.

LETTER XXI.

Digreffion, on the Diftances of the Heavenly Bodies, and on the Nature of the Sun, and his Rays.

THE obfervations which I have been making refpecting the time which the light of the ftars employs in making it's progrefs down to us, convey a ftriking idea of the extent and greatnefs of the univerfe. The velocity of found, which flies through the fpace of 1000 feet in a fecond, furnifhes us with nearly the firft ftandard of meafurement. It is about 200 times more rapid than the pace of a man who is a good walker. Now the velocity of the rays of light is 900,000 times ftill more rapid than that of found: thefe rays accordingly perform, every fecond, a courfe of 900 millions of feet, or 37,500 German miles.*

What aftonifhing velocity! Yet the neareft fixed ftar

* More than 170,000 miles Englifh.—*E. E.*

ſtar is ſo remote, that it's rays, notwithſtanding this prodigious velocity, would take more than ſix years in deſcending to us. And were it poſſible for a great noiſe, ſuch as that of the firing of a cannon, iſſuing from that ſtar, to be conveyed to our ears, it would require a period of 5,400,000 years to reach us. And this is applicable only to thoſe ſtars which are the moſt brilliant, and are probably neareſt to us. Thoſe which appear the ſmalleſt are, very probably, ten times ſtill farther remote, and more. A whole century then, at leaſt, muſt elapſe, before the rays of theſe ſtars could poſſibly reach us. How prodigious muſt that diſtance be, which cannot be paſſed through in leſs than 100 years, by a velocity which flies at the rate of 37,500 German miles every ſecond!

Were, then, one of theſe ſtars to be juſt now annihilated, or eclipſed only, we ſhould ſtill continue to ſee it for 100 years to come, as the laſt rays which it emitted could not reach us in leſs time.

The generality of mankind is very far from having any thing like juſt ideas reſpecting the vaſt extent of the univerſe. Many conſider it as a work of little importance, which chance alone might have produced. But what muſt be the aſtoniſhment of one who reflects, on obſerving, that all theſe immenſe bodies are arranged with the moſt conſummate wiſdom, and that the more knowledge we acquire on the ſubject, though it muſt ever be very imperfect, the more we muſt be diſpoſed to admire their order and magnificence?

I return to the great luminous bodies, and particularly

cularly the ſun, which is the principal ſource of the light and heat which we enjoy on the earth. It will be aſked, in the firſt place, Wherein conſiſts the light which the ſun is inceſſantly diffuſing through the whole univerſe, without ever ſuffering the ſmalleſt diminution? The anſwer is obvious, according to the ſyſtem which I have been endeavouring to eſtabliſh. But that of emanation furniſhes no ſatisfactory ſolution. The whole univerſe being filled with that extremely ſubtile and elaſtic fluid, which is called ether, we muſt ſuppoſe, in all the parts of the ſun, an inceſſant agitation, by which every particle is in a conſtant motion of vibration, and this, by communicating itſelf to the circumambient ether, excites in that fluid a ſimilar agitation, and is thence tranſmitted to regions the moſt remote, with the rapidity which I have been deſcribing.

And, to keep up the parallel between ſound and light, the ſun would be in a ſtate ſimilar to that of a bell which ſhould be ringing continually. The particles of the ſun muſt, conſequently, be kept in this inceſſant agitation, to produce in the ether the undulations which we call rays of light. But it is ſtill no eaſy matter to explain, by what power this agitation in the particles of the ſun is conſtantly kept up, as we obſerve, that a match does not long continue burning, but preſently goes out, unleſs it be ſupplied with combuſtible matter. But it muſt be remarked, that as the ſun is a maſs many thouſand times greater than our whole globe, if it is once thoroughly inflamed, it may continue in that ſtate

 for

for ſeveral ages, without ſuffering any ſenſible diminution. Beſides, the caſe is not the ſame with the ſun and our fires and candles, a conſiderable part of whoſe ſubſtance is diſſipated in ſmoke and exhalations, from which a real waſte reſults. Whereas, though perhaps ſome particles may be forced from the ſun in form of ſmoke, they cannot remove to a great diſtance, but ſpeedily fall back into it's maſs, ſo that there cannot be any real expenditure, to occaſion a diminution of his ſubſtance.*

The only thing of which we are ſtill ignorant reſpecting this ſubject, is, the power which inceſſantly maintains all the particles of the ſun in this agitation. But as it contains nothing inconſiſtent with good ſenſe, and as we are under the neceſſity of acknowledging our ignorance of many other things much leſs remote than the ſun, we ought to be ſatiſfied, if our ideas are not involved in contradiction.

21ſt June, 1760.

* The Author is evidently embarraſſed in his explanation of the continual inflammation of the ſun. And though he has ſaid above, that the ſyſtem of emanation was untenable, on account of the frequent and unavoidable colliſion of rays proceeding from different luminous bodies, which muſt diſturb, and even obſtruct the viſion of ſeveral of theſe bodies at once, as he has not explained how two ſounds may be heard at the ſame time, a ſimilar objection might be made to his ſyſtem, which is analogous to the phenomena of ſound.—*F. E.*

LETTER

LETTER XXII.

Elucidations on the Nature of luminous Bodies, and their Difference from opaque Bodies illumined.

THE ſun being a luminous body, whoſe rays are univerſally diffuſed in all directions, you can no longer be at a loſs to account for this wonderful phenomenon, which conſiſts in the ſhaking, or vibration, with which all the particles of the ſun are agitated. The parallel of a bell lends conſiderable aſſiſtance toward the explanation of this fact. But it is obvious, that the vibrations produced by light, muſt be much more vehement and rapid than thoſe produced by ſound, ether being incomparably more ſubtile than air. A feeble agitation not being capable of ſhaking the air ſo as to produce ſound in it, that of a bell, and of all other ſonorous bodies, are too feeble relatively to ether, to produce in it the vibration which conſtitutes light.

You will recollect, that in order to excite a perceptible ſound, more than 30, and leſs than 7552 vibrations muſt be produced in a ſecond; the air being too ſubtile to admit of a ſenſible effect from a ſound conſiſting of leſs than 30 vibrations in a ſecond, but not ſufficiently ſo to receive one of more than 7552 vibrations in the ſecond. A note higher than this could not be at all heard. It is the ſame with reſpect to ether; 7552 vibrations, produced in a ſecond, could not poſſibly act upon it, becauſe of it's greater

ſubtilty,

fubtilty. It requires vibrations much more frequent. An agitation fo rapid could not take place but in the minuteft particles of bodies which elude our fenfes. The light of the fun, then, is produced by a very violent agitation, which affects all his infinitely minute particles, each of which muft fhake many thoufands of times every fecond.

It is a fimilar agitation which likewife produces the light of the fixed ftars, and of all fires, fuch as candles, tapers, torches, &c. which give us light, and fupply the place of the fun during the night. On attentively obferving the flame of a wax-light, you will eafily perceive that, in the minuteft particles, there is a conftant and furprizing agitation; and I do not apprehend that my fyftem is liable on this fide to any contradiction, while that of Newton requires a moft enormous agitation, capable of launching the minuteft particles with the velocity of 37,500* German miles in a fecond.

This, then, is the explanation of the nature of bodies luminous of themfelves: for there are luminous bodies which are not fo immediately, fuch as the moon and the planets, which are fimilar to our globe. We fee the moon only when, and in as far as, fhe is illuminated by the fun; and this is the cafe of all terreftrial bodies, fires excepted, which have a light of their own. But other bodies, which are denominated opaque, become vifible to us only when they are illuminated by fome luminous body.

In a very dark night, or in an apartment, fo clofely

* Upwards of 170,000 Englifh miles.

fhut

fhut on every fide, that no light can find admiffion, to no purpofe will you turn your eyes toward the objects which furround you in the dark: you perceive nothing. But the moment a taper is introduced, you immediately fee, not the taper only, but the other bodies which were before invifible. We have here, then, a very effential difference between luminous and opaque bodies. I have already employed the term *opaque* to denote bodies which are not tranfparent; but it comes to almoft the fame thing, and we muft accommodate ourfelves to the common modes of expreffion, though they are not perfectly accurate. Luminous bodies are vifible by their own light, and never affect our organs of fight more than when the darknefs is otherwife moft profound. Thofe which I here denominate opaque, are rendered vifible to us only by means of a light that is foreign to them. We perceive them not while they remain in darknefs; but as foon as they are expofed to a luminous body, whofe rays ftrike upon them, they become vifible; and they difappear the moment that foreign light is withdrawn. It is not even neceffary, that the rays of a luminous body fhould fall upon them immediately; another opaque body, when well illuminated, produces nearly the fame effect, but in a feebler manner.

The moon is an excellent inftance. We know that the moon is an opaque body; but when fhe is illuminated by the fun, and we fee her during the night, fhe diffufes a feeble light over all opaque bodies, and renders vifible to us thofe which we could not have

perceived

perceived without her affiftance. Placed in the day time in an apartment, whofe afpect is toward the north, and into which, of courfe, the rays of the fun cannot enter, it is, however, perfectly clear, and I am able to diftinguifh every object. What can be the caufe of this clearnefs, but that the whole heaven is illuminated by the fun? What we call the azure fky, and befides, the walls oppofite to my apartment, and the other furrounding objects, are likewife illuminated, either immediately by the fun, or mediately by other opaque bodies, expofed to the action of that focus of light; and the light of all thefe opaque, but illuminated, bodies, as far as it has admiffion into my apartment, renders it luminous, and that in proportion as the windows are high, wide, and well placed. The glafs is little or no interruption, being, as I have already remarked, a tranfparent body, which freely tranfmits the rays of light.

When I completely exclude the light from the apartment, by clofing the window-fhutters, I am reduced to a ftate of darknefs, and difcern no object, unlefs I call for a candle. Here, then, is an effential difference between luminous and opaque bodies; and likewife a very ftriking refemblance, namely, that opaque bodies, when illuminated, illuminate other opaque bodies, and produce, in this refpect, nearly the fame effect as bodies luminous of themfelves. The explanation of this phenomenon has, hitherto, greatly perplexed philofophers, but, I flatter myfelf, that my folution of it has been clear and fatisfactory.

24th June, 1760.

LETTER

LETTER XXIII.

How Opaque Bodies become vifible. Newton's Syftem, of the Reflection of Rays, propofed.

BEFORE I attempt an explanation of the phenomenon of opaque bodies becoming vifible when they are illuminated, it muft be remarked, in general, that we fee nothing but by means of the rays which enter into our eyes. When we look at any object whatever, rays iffuing from every point of that object, and entering into the eye, paint upon it, if I may ufe the expreffion, the image of the object. This is not mere conjecture, but may be demonftrated by experiment. Take, for example, the eye of an ox, or of any animal recently killed, and, after having uncovered the bottom, you find all the objects which were before it painted there. As often then as we fee an object, the image of it is painted on the bottom of our eyes; and this is produced by the rays which proceed from the object to us. I fhall afterwards take occafion to go into a more minute detail on the fubject of vifion, and explain in what manner the images of objects are formed on the bottom of the eye: let this general remark fuffice for the prefent.

As we fee opaque bodies only when they are illuminated, this is a proof, that there muft proceed from every point of thefe bodies rays of light which fubfift only during the illumination. The moment they are placed in the dark thefe rays difappear. They are not proper then to opaque bodies; their origin

origin muſt be ſought in the manner in which other bodies illuminate them. And this is the great queſtion, How illumination alone is capable of producing rays on opaque bodies, or of putting them in nearly the ſame ſtate as luminous bodies are, which, by an agitation in their minuteſt particles, produce rays of light?

The great *Newton*, and other philoſophers, who have examined the ſubject, aſſign *reflection* as the cauſe of this phenomenon: it is, therefore, of the higheſt importance, that you ſhould form a juſt idea of what is called reflection.

This name is given to the repulſion of one body ſtruck againſt another, as may be ſeen in the game of billiards. When the ball is ſtruck againſt the cuſhion or ledge of the billiard table, it recoils again; and this retrograde motion is termed reflection. It is neceſſary, here, to attend to a diſtinction between two caſes. Let us ſuppoſe A B (*plate* I. *fig.* 7.) to be the ledge of a billiard table. The firſt caſe is this: When you play the ball D perpendicularly againſt the ledge, in the direction of D C, perpendicular to A B, and, conſequently, the adjacent angles A C D, and B C D, are right angles: in this caſe, the ball will be driven back, or reflected, in the ſame line D C. The other caſe is, when the ball is played obliquely againſt the ledge, ſuppoſe in the line E C, forming, with A B, an acute angle A C E, this is called the angle of incidence. The ball will, in this caſe, be repelled from the ledge, in the direction of the line C F, ſo that this line ſhall make, on the other

ſide,

ſide, with the ledge B C, an angle B C F, exactly equal to the angle of incidence A C E. This angle, B C F, formed by the line in which the ball recoils, is called the angle of reflection. And this law always takes place when a body in motion meets with an obſtacle.

A cannon ball, ſhot againſt a wall ſufficiently ſtrong to reſiſt it, is reflected conformably to this law. It extends, in like manner, to ſounds, which are frequently reflected from certain bodies; and you know that this reflection of ſound is called echo. It cannot be doubted, that the ſame thing frequently takes place with reſpect to the rays of light. The objects which we ſee in mirrors, are repreſented to us by the reflection of rays, and every well poliſhed ſurface reflects the rays of light which fall upon it. It is undoubtedly certain, therefore, that there are caſes without number in which the rays that fall on certain bodies are reflected; and philoſophers have thence taken occaſion to maintain, that opaque bodies are rendered viſible by means of reflected rays.

I ſee juſt now houſes, oppoſite to my windows, which are illuminated by the ſun. According, then, to the opinion of thoſe philoſophers, the rays of the ſun falling on the ſurface of theſe houſes, are reflected from them; they enter into my apartment, and render theſe houſes viſible to me. In the ſame manner, if we believe thoſe philoſophers, the moon and the planets become viſible, and theſe are, unqueſtionably, opaque bodies. The rays of the ſun which

fall

fall on thefe bodies, and illuminate the parts which are expofed to them, are reflected, and are thence tranfmitted to us, juft as if the bodies were luminous of themfelves. According to this opinion, we fee the moon and the planets only by the rays of the fun which they reflect; and you muft frequently have heard it affirmed, that the light of the moon is a reflection of the light of the fun. In the fame manner, fay they, the rays of the fun are reflected by the firft opaque bodies which are expofed to them, on other bodies of the fame nature, and undergo a feries of fimilar reflections, till they are entirely weakened.

But, however plaufible this opinion may at firft fight appear, it involves fo many abfurdities, when clofely examined, that it is abfolutely untenable, which I hope to demonftrate, as a preparation for the true folution of this phenomenon.

28th June, 1760.

LETTER XXIV.

Examination and Refutation of Newton's Syftem.

I AFFIRM, then, that when we fee an opaque body illuminated by the fun, it is impoffible to maintain that it reflects luminous rays, and that, by means of fuch rays, it is rendered vifible to us. The example of a mirror, which, undoubtedly, reflects the rays, and is employed to fupport this opinion, rather

rather confutes it. The mirror, beyond contradiction, fends back the rays which fall upon it; but when thefe reflected rays enter into our eyes, What do they reprefent? You will readily anfwer, that it is not the mirror, but the objects from which they originally proceeded; and the reflection does nothing elfe, but enable us to fee thefe objects in another place. Befides, we fee thofe objects, not on the furface of the mirror, but rather within it, and it may be faid with truth, that the mirror itfelf remains invifible to us.

But, on looking at an opaque body illuminated by the fun, we do not fee in it the image of that glorious orb; we fee only the furface of the bodies, with all the variations to be found on them. We perceive, then, a very effential difference between the rays which are reflected from a mirror, and thofe by means of which opaque bodies are rendered vifible. But there is, befides, another difference equally palpable in the mirror; for on changing the place of the objects, or our own fituation, the appearance will always change, and the rays, reflected from the mirror, will continually reprefent to our eyes other images, correfponding to the nature and pofition of the objects, and to the place where we are ftationed: but, as I have already faid, thefe reflected rays never reprefent to us the mirror itfelf.

Now, let a body be illuminated by the fun, or other bodies, whether luminous or opaque, already illuminated; in whatever manner this body may change it's place, or we change our's relatively to it, it's it's appearance is always the fame; we fee always the fame object, and remark in it no change relative to the different circumftances above mentioned. This furnifhes a new proof, that we do not fee opaque bodies by means of the rays reflected from their furface.*

* This can hardly be deemed a fair ftatement. It is true, that opaque bodies are feen only by reflected light, but it by no means follows, that all the incident light is again reflected. Some bodies are, by their conftitution, difpofed to reflect certain kinds of rays the moft freely, and as the reft are abforbed, the peculiar colour predominates. This colour will, therefore, not be the fame, whatever be the quality of the incident light, but will receive an analogous fhade. For the fame reafon, no fubftance reflects only one fpecies of rays. The elective attractions and repulfions, between the particles of light and a body, are moft remarkable at very minute diftances; and hence the colour is prominent when the furface is rough, for the light, fuffering then a partial repulfion only, gains a nearer approach. I cannot imagine how Mr. Euler would explain thefe facts on his own principles.

It is in a polifhed furface only, that the furrounding bodies can be feen by reflection, for diftinct vifion requires the rays, proceeding from different points, to be tranfmitted with regularity. No fubftance is, indeed, perfectly fmooth, but the different repulfions, exerted by the fuperficial particles, may balance each other, and produce an uniform effect, at the diftance where the reflection takes place. Mr. Euler's principles would lead to the conclufion, that polifh is not at all neceffary to a mirror. Echo is formed from furfaces which are very uneven, fince the air is heaped on the obftacle, and the principal reflux of the undulation commences at a fenfible diftance from it. The fame obtains in water, though in a lefs degree; and is, in general, more remarkable, in proportion to the rarefaction of the fluid. How wonderful, then, in that refpect, muft ether be, which is fuppofed to be the moft fubtle of all fluids? We might expect the walls of a houfe to reflect the moft enchanting picture of the landfcape in front.—*E. E.*

An objection will, perhaps, be ftarted, drawn from the dove's neck, and certain kinds of ftuff, which prefent different objects, according as our point of view changes. But this in no refpect weakens my conclufion with regard to ordinary opaque bodies, which are not fubject to this change. The objection only proves, that thefe fingular objects are endowed with certain qualities: as, for example, that their minuter particles are finely polifhed, and that a real reflection takes place, befide the ufual and ordinary manner in which bodies are rendered vifible to us.

Now, it is eafy to comprehend, that this reflection muft be clearly diftinguifhed from the manner in which ordinary opaque bodies are illuminated.

Finally, the rays reflected from a mirror always reprefent to us, likewife, the colours of the bodies from which they originally proceed, and the mirror, which reflects, makes no change in this refpect. One opaque body illuminated by any other body, in whatever manner, always prefents the fame colours; and every body may be faid to have it's proper colour. This circumftance abfolutely overturns the opinion of all thofe who maintain, that we fee opaque bodies by means of the rays which their furface reflects.

Putting together all the reafons which I have now fubmitted to your confideration, there can be no hefitation in pronouncing, that this opinion is totally untenable in philofophy, or rather, in phyfics. I cannot, however, flatter myfelf with the hope, that philofophers, wedded to opinions once adopted, ſhould yield to theſe reaſons. But the naturaliſt, who is more nearly related to the mathematician, will have leſs difficulty in reſigning an opinion, overthrown by reaſons ſo convincing. You will again recollect what Cicero has ſaid on this ſubject: That nothing ſo abſurd can be conceived, as not to be ſupported by ſome philoſopher. In fact, however ſtrange the ſyſtem which I have been refuting may appear to you, it has, hitherto, been propagated and defended with much warmth.

It is impoſſible to ſay, to what a degree the difficulties and contradictions which I have been endeavouring to expoſe, were unknown to, or overlooked by, the partiſans of this ſyſtem. The great *Newton* himſelf ſtrongly felt their force: but as he reſted in a very untenable idea reſpecting the propagation of light, it is not to be wondered at, that he ſhould overlook theſe great difficulties; and, in general, depth of underſtanding does not always prevent a man from falling into abſurdity, in ſupporting an opinion once embraced.

But if this ſyſtem, that opaque bodies are rendered viſible by reflected rays, be falſe, ſay it's partiſans, What then is the true one? They even think it impoſſible to imagine another explanation of this phenomenon. It is, beſides, rather hard and humiliating for a philoſopher to acknowledge ignorance of any ſubject whatever. He would rather maintain the groſſeſt abſurdities; eſpecially if he poſſeſſes the ſecret of involving them in myſterious terms, which no one is capable of comprehending. For in this caſe,

caſe, the vulgar are the more diſpoſed to admire the learned; taking it for granted, that what is obſcurity to others, is perfectly clear to them. We ought always to exerciſe a little miſtruſt, when very ſublime knowledge is pretended to, knowledge too ſublime to be rendered intelligible. I hope I ſhall be able to explain the phenomenon in queſtion, in ſuch a way as to remove every difficulty.

1ſt July, 1760.

LETTER XXV.

A different Explanation of the Manner in which opaque Bodies illuminated become viſible.

ALL the phenomena of opaque bodies, which I have unfolded in the preceding letter, inconteſtably demonſtrate, that, when we ſee an opaque body illuminated, it is not by rays reflected from it's ſurface, that it becomes viſible, but becauſe it's minuter particles are in an agitation ſimilar to that of the minuter particles of luminous bodies; with this difference, however, that the agitation in opaque bodies is far from being ſo ſtrong as in bodies luminous of themſelves; for an opaque body, however much illuminated, never makes on the eye an impreſſion ſo lively as luminous bodies do.

As we ſee the opaque bodies themſelves, but by no means the images of the luminous bodies which

enlighten

enlighten them, as muſt be the caſe, if we ſaw them by the reflection of their ſurface; it muſt follow, that the rays emitted by opaque bodies are proper to them, juſt as the rays of a luminous body are peculiar to itſelf. As long as an opaque body is illuminated, the minuter particles of it's ſurface are in a ſtate of agitation proper to produce, in the ether, a motion of vibration, ſuch as is neceſſary for forming rays, and for painting in our eyes the image of the body from which they proceed. For this effect, rays muſt be diffuſed, from every point of the ſurface, in all directions; as experience evidently confirms. For, from whatever ſide we look at an opaque body, we ſee it equally in all it's points; from which it follows, that every point emits rays in all directions. This circumſtance eſſentially diſtinguiſhes theſe rays from ſuch as are reflected, whoſe direction is always determined by that of the rays of incidence; ſo that if the incident rays proceed from one ſingle quarter, ſay the ſun, the reflected rays can follow only one ſingle direction.

It muſt be admitted, then, that when an opaque body is illuminated, all the particles on it's ſurface are put in a certain agitation, which produces rays, as is the caſe with bodies luminous of themſelves. This agitation, likewiſe, is ſtronger, in proportion as the light of the illuminating body is more intenſe. Thus the ſame body, expoſed to the ſun, is agitated much more violently, than if, in a room, it were illuminated only by day-light, or in the night-time, by a taper, or by the moon. In the firſt caſe, it's image

image is painted with much greater vivacity on the bottom of the eye, than in the others, eſpecially the laſt; the light of the moon being ſcarcely ſufficient to enable us to diſtinguiſh, or to read, writing of a large ſize. And when the opaque body is conveyed into a cloſe room, or into the dark, nothing is then to be ſeen; a certain proof, that the agitation in it's parts has entirely ceaſed, and that they are now in a ſtate of reſt.

In this, therefore, conſiſts the nature of opaque bodies; their particles are, of themſelves, at reſt, or, at leaſt, deſtitute of the agitation neceſſary to produce light. But theſe ſame particles are ſo diſpoſed, that when illuminated, or ſtruck with rays of light, they are immediately put into a certain agitation, or motion of vibration, proper to produce rays; and the more intenſe the light is, which illuminates theſe bodies, the more violent alſo is this agitation. As long as an opaque body is illuminated, it is in the ſame ſtate as luminous bodies; it's particles are agitated in the ſame manner, and are capable of exciting, of themſelves, rays in the ether; with this difference, that the agitation kept up in luminous bodies by an intrinſic force, ſubſiſts always of itſelf; whereas, in opaque bodies, this agitation is only momentaneous, and produced by the motion of the light which illuminates them.

This explanation is conſiſtent with every phenomenon, and labours under none of the difficulties which determined us to abandon the other, namely, that founded on reflection. Whoever will take the trouble condidly to weigh all thefe reafons, muft admit their force. But a very great difficulty ftill remains to be folved: How comes it that illumination, fimply, can put the particles of an opaque body into an agitation capable of producing rays; and that this agitation fhould always continue nearly the fame, whatever difference there may be in the illumination?

I acknowledge, that were it impoffible to anfwer this queftion, it would be a great defect in my theory, though it would not amount to a complete refutation, for it contains nothing contradictory. Suppofing I were ignorant, how illumination produces an agitation in the particles of opaque bodies, this would only prove that the theory is incomplete: and till it is demonftrated to be abfolutely impoffible that illumination fhould produce this effect, my fyftem muft fubfift. But I fhall endeavour to fupply this defect, by fhewing you, how illumination agitates the minuteft particles of bodies.

5th July, 1760.

LETTER XXVI.

Continuation of the fame Subject.

I HAVE undertaken to fhew how the illumination of an opaque body muft produce in it's minuteft particles, an agitation proper to excite the rays of light, which render that fame opaque body vifible. The

The parallel between found and light, which differ only in refpect of lefs and more, light being the fame thing relatively to ether that found is relatively to air, this parallel, I fay, will enable me to fulfil my engagement. Luminous bodies muft be compared to mufical inftruments actually in a ftate of vibration. It is a matter of indifference whether this be the effect of an intrinfic or of a foreign power: it is fufficient for my purpofe that found is emitted. Opaque bodies, as long as they are not illuminated, muft be compared to mufical inftruments not in ufe, or, if you will, to ftrings which emit no found till they are touched.

The queftion, then, being transferred from light to found, is refolved into this, Whether it be poffible for the ftring of an inftrument, in a ftate of reft, when brought within the fphere of activity of the found of inftruments in a ftate of vibration, to receive, in certain circumftances, fome agitation, and emit found, without being touched? Now this is confirmed by daily experience. If you take the trouble, during a concert, to attend to a particular ftring in proper tune, you will obferve that ftring fometimes to tremble without having been touched, and it will emit the fame found as if it had been immediately put into vibration. This experiment will fucceed ftill better, if the inftruments ftrike the fame note with the ftring. Confider attentively the ftrings of a harpfichord not played upon, while a violin ftrikes the note *a*, for example, and you will obferve on the harpfichord the ftring of the fame note begin fenfibly to tremble, and even to emit found, without having been touched; fome other chords will likewife be agitated, particularly thofe which are diftant an octave, a fifth, and even a third, provided the inftrument be perfectly in tune.

This phenomenon is well known to muficians, and Mr. Rameau, one of the moft celebrated French compofers, eftablifhed his principles of harmony upon it. He maintains, That octaves, fifths, and thirds, muft be confidered as confonances, becaufe one chord is agitated by the found only of another chord, which is in unifon, or an octave, a fifth, or a third, from the firft. But it muft be admitted that the principles of harmony are fo well eftablifhed by the fimplicity of the relations which founds have to each other, that they have no need of a new confirmation. In truth the phenomenon obferved by Mr. Rameau is a very natural confequence from the principles of harmony.

To render this more fenfible, let us attend to two chords wound up to unifon; on ftriking the one, the other will begin of itfelf to tremble, and will emit it's found. The reafon is abundantly clear; for as a chord communicates to the air by it's trembling a motion of vibration fimilar to it's own, the air, agitated by this motion of vibration, muft reciprocally make the chord tremble, provided that by it's degree of tenfion, it be fufceptible of this motion. The air being put into vibration, ftrikes the chord ever fo little at every reverberation, and the repetition of ftrokes foon impreffes on the chord a fenfible motion;

tion; becaufe the vibrations to which it is difpofed by it's tenfion accord with thofe of the air. If the number of vibrations in the air is the half, or the third, or any other whofe relation is fufficiently fimple, the chord does not receive a new impulfe at every vibration, as in the preceding cafe, but only at the fecond, or the third, or the fourth, which will continue to increafe it's tremulous motion, but lefs than in the firft cafe.

But if the vibrations of the air have not any fimple relation with that which correfponds to the chord, the agitation of that fluid will produce no effect whatever upon it; the vibrations of the chord, if there be any, not correfponding to thofe of the fluid, the following impulfions of the air deftroy for the moft part the effect which the firft might have produced; and this is completely confirmed by experience. Thus when a chord is fhaken by a found, that found muft, in order to it's being perceptible, be precifely the fame with that of the chord. Other founds which have a confonance with that of the chord, will produce, it is true, a fimilar but lefs fenfible effect, and diffonances will produce none at all. This phenomenon takes place not only in mufical ftrings, but in all fonorous bodies whatever. One bell will refound by the noife only of another bell which is in unifon with it, or at the diftance of an octave, a fifth, or a third.

The inftance of a perfon who could break glaffes by his voice farther confirms what I have advanced. When a glafs was prefented to him, by ftriking it he found

found out the note; he then began to ſquall in uniſon, and the glaſs immediately caught the vibration; proceeding to give to his voice all the force he was able, always preſerving the uniſon, the vibration of the glaſs became at length ſo violent, that it broke. It is confirmed, then, by experience that a chord and every other ſonorous body is put into vibration by it's kindred ſound. The ſame phenomenon muſt take place with regard to opaque bodies, of which the minuter particles may be put into a ſtate of agitation by illumination only: which is the queſtion I propoſed to ſolve. The following letter will contain a more ample diſcuſſion of it.

8th July, 1760.

LETTER XXVII.

Concluſion: Clearneſs and Colour of opaque Bodies illumined.

AFTER what has juſt been ſubmitted to your conſideration, you will no longer be ſurpriſed that an opaque body is capable of receiving, from illumination alone, an agitation in it's particles ſimilar to that of the particles of luminous bodies, and which gives them the property of producing rays that render them viſible. Thus the great objection to my explanation of the viſibility of opaque bodies is happily removed; while the other theory, founded on the reflection of rays, has to encounter difficulties

culties which grow in proportion as you attempt to make a more direct application of them to known phenomena.

It is then an eſtabliſhed truth, that the particles of the ſurfaces of all bodies which we ſee, undergo an agitation ſimilar to that of a chord in vibration, but their vibrations are much more rapid; whether it be that this agitation is the effect of an intrinſic force, as in bodies luminous of themſelves, or whether it be produced by the rays of light which fall upon the bodies, that is to ſay by illumination, as is the caſe in opaque bodies. It is falſe, then, that the moon being an opaque body, reflects the rays of the ſun, and that, by means of this reflected light, ſhe is rendered viſible to us, as is commonly underſtood. But the rays of the ſun, falling on the ſurface of the moon, excite in it's particles a concuſſion, from which reſult the rays of the moon; and theſe, entering into our eyes, paint it's image there; it is the ſame with the other planets, and with all opaque bodies. This agitation of opaque bodies, when illumined, laſts only during the illumination which is the cauſe of it: and as ſoon as an opaque body ceaſes to be illumined, it ceaſes to be viſible.

But is it not poſſible that this agitation, once impreſſed on the particles of an opaque body, may be for ſome time kept up, as we ſee that a ſtring once ſtruck, frequently continues to vibrate, though no new impreſſion be made upon it? I do not pretend to deny the fact: I even believe that we have examples of it in thoſe ſubſtances which Mr. Margraff preſented

presented to you, and which, once illumined, preserve their light for some time, though conveyed into a dark room. This, however, is an extraordinary case; the vibration of the minuter particles disappearing in all other bodies, with the illumination which occasioned it. But this explanation, which thus far is perfectly self-consistent, leads me forward to researches of still greater importance.

It is undoubtedly certain, that we find an infinite difference between the particles of opaque bodies, according to the variety of the bodies themselves. Some will be more susceptible of vibrations, and others less, and others finally not at all so. This difference in bodies occurs but too evidently. One, whose particles easily receive the impression of the rays which strike it, appears to us brilliant; another, on the contrary, in which the rays scarcely produce any agitation, cannot appear luminous. Among several bodies, equally illumined, you will always remark a great difference, some being more brilliant than others. But there is besides another and a very remarkable difference between the particles of opaque bodies, respecting the number of vibrations which each of them, being agitated, will make in a certain time.

I have already observed, that this number must always be very great, and that the subtilty of ether is such as to require many thousands in a second. But the difference here may be endless; if some particles, for example, should make 10,000 vibrations in a second, and others 11,000, 12,000, 13,000, according to

to the smallness, the tension, and the elasticity of each, as in the case of musical chords, in which the number of vibrations given in a second may be varied without end; and thence it is I have deduced the difference of high and low notes. As this difference is essential in sounds, and as the ear is affected by it in a manner so particular as to render it the foundation of the whole theory of music, it cannot be called in question that a similar difference in the frequency of the vibrations of rays of light must produce a variation as particular in vision. If, for example, a particle makes 10,000 vibrations in a second, and produces rays of the same species, the rays which enter into the eye will strike the nerves of that organ 10,000 times in a second; and this effect, as well as the sensation, must be totally different from those produced by a different particle which should make more or less vibrations in a second. There will be in vision a difference similar to that which the ear perceives on hearing sharp or flat notes.

You will no doubt be desirous to know into what this difference in vision is to be resolved; and what different sensations correspond to the number, greater or less, of the vibrations produced in every body during a second? I have the honour of informing you, That diversity of colours is occasioned by this difference; and that difference of colour is to the organ of vision what sharp or flat sounds are to the ear. We have resolved, therefore, without going after it, the important enquiry respecting the nature of colours, which has long employed the attention of the greatest

greateſt philoſophers. Some of them have called it a modification of light abſolutely unknown to us. *Deſcartes* maintains, that colours are only a certain mixture of light and ſhade. *Newton* accounts for difference of colour by tracing it up to the rays of the ſun; which, according to him, are a real emanation, whoſe matter may be more or leſs ſubtile; and thence ſettles the rays of all the colours, as red, yellow, green, blue, violet, &c.

But as this ſyſtem falls to pieces of itſelf, all that has been ſaid reſpecting colours conveys no information; and you are now clearly ſenſible, that the nature of each colour conſiſts in the number of vibrations produced in a certain time by the particles which preſent them to the eye.

12th July, 1760.

LETTER XXXVIII.

Nature of Colours in particular.

THE ignorance which prevailed reſpecting the true nature of colours, has occaſioned frequent and violent diſputes among philoſophers; each of whom made an attempt to ſhine, by maintaining a peculiar opinion on the ſubject. The ſyſtem which made colours to reſide in the bodies themſelves, appeared to them too vulgar and too little worthy of a philoſopher, who ought always to ſoar above the multitude. Becauſe the clown imagines that one body

body is red, another blue, and another green, the philoſopher could not diſtinguiſh himſelf better than by maintaining the contrary; and he accordingly affirms that there is nothing real in colours, and that there is nothing in bodies relative to them.

The Newtonians make colours to conſiſt in rays only; which they diſtinguiſh into red, yellow, green, blue, and violet; and they tell us that a body appears of ſuch and ſuch a colour when it reflects rays of that ſpecies. Others, to whom this opinion ſeemed abſurd, pretend that colours exiſt only in ourſelves. This is an admirable way to conceal ignorance; the vulgar might otherwiſe believe that the ſcholar was not better acquainted with the nature of colours than themſelves. But you will readily perceive that theſe affected refinements are mere cavil. Every ſimple colour (in order to diſtinguiſh from compound colours) depends on a certain number of vibrations, which are performed in a certain time; ſo that this number of vibrations, made in a ſecond, determines the red colour, another the yellow, another the green, another the blue, and another the violet, which are the ſimple colours repreſented to us in the rainbow.

If, then, the particles of the ſurface of certain bodies are diſpoſed in ſuch a manner, that being agitated, they make in a ſecond as many vibrations as are neceſſary to produce, for example, the red colour, I call ſuch a body red, juſt as the clown does; and I ſee nothing like a reaſon for deviating from the common mode of expreſſion. And rays which make

make ſuch a number of vibrations in a ſecond, may, with equal propriety be denominated red rays; and finally, when the optic nerve is affected by theſe ſame rays, and receives from them a number of impulſions, ſenſibly equal, in a ſecond, we receive the ſenſation of the red colour. Here every thing is clear; and I ſee no neceſſity for introducing dark and myſterious phraſes, which really mean nothing.

The parallel between ſound and light is ſo perfect, that it hits even in the minuteſt circumſtances. When I produced the phenomenon of a muſical chord, which may be excited into vibration by the reſonance only of certain ſounds, you will pleaſe to recollect, that the one which gives the uniſon of the chord in queſtion is the moſt proper to ſhake it, and that other ſounds affect it only in proportion as they are in conſonance with it. It is exactly the ſame as to light and colours; for the different colours correſpond to the different muſical ſounds.

In order to diſplay this phenomenon, which completely confirms my aſſertion, let a dark room be provided; make a ſmall aperture in one of the ſhutters; before which, at ſome diſtance, place a body of a certain colour, ſay a piece of red cloth, ſo that, when it is illumined, it's rays may enter by the aperture into the darkened room. The rays thus tranſmitted into the room will be red, all other light being excluded: and if you hold on the inſide of the room, oppoſite to the aperture, a piece of cloth of the ſame colour, it will be perfectly illumined, and its red colour appear very brilliant; but if you ſub-

ſtitute in it's place a piece of green cloth, it will remain obſcure, and you will hardly ſee any thing of it's colour. If you place on the outſide, before the aperture, a piece of green cloth, that within the chamber will be perfectly illumined by the rays of the firſt, and it's green colour appear very lively. The ſame holds good as to all other colours; and I do not imagine that a more convincing demonſtration of the truth of my ſyſtem can be demanded.

We learn from it, that, in order to illuminate a body of a certain colour, it is neceſſary that the rays which fall upon it ſhould have the ſame colour; thoſe of a different colour not being capable of agitating the particles of that body. This is farther confirmed by a well known experiment. When the ſpirit of wine is ſet on fire in a room, you know that the flame of ſpirit of wine is blue, that it produces only blue rays, and that every perſon in the room appears very pale, their faces, though painted ever ſo deep, have the aſpect of death. The reaſon is evident; the blue rays, not being capable of exciting, or putting in motion the red colour of the face, you ſee on it only a feeble and bluiſh colour: but if one of the company is dreſſed in blue, ſuch dreſs will appear uncommonly brilliant. Now the rays of the ſun, thoſe of a wax taper, or of a common candle, illuminate all bodies almoſt equally; from whence it is concluded, that the rays of the ſun contain all colours at once, though he himſelf appears yellowiſh.

In truth, when you admit into a dark room the rays of all the ſimple colours, red, yellow, green, blue, and violet, in nearly equal quantities, and blend them, they reprefent a whitifh colour. The fame experiment is made with various powders, coloured in like manner; on being mixed together, a whitifh colour is the refult. Hence it is concluded that white is nothing lefs than a fimple colour; but that it is rather a compound of all the fimple colours; accordingly we fee that white is adapted to the reception of all colours. As to black, it is not properly a colour. Every body is black when it's particles are fuch that they can receive no motion of vibration, or when it cannot produce rays. The want of rays, therefore, produces the fenfation of that colour; and the more particles there are found in any body not fufceptible of any motion of vibration on it's furface, the more blackifh and obfcure it appears.

15th July, 1760.

LETTER XXXIX.

Tranfparency of Bodies relative to the Tranfmiffion of Rays.

I HAVE already remarked, that there are bodies, fuch as glafs, water, and efpecially air, which tranfmit the rays of light, and, on account of this property, are denominated pellucid or diaphonous. The ether, however, is the medium in which the rays of light are formed, to which this property moft intimately appertains; and other tranfparent bodies

are endowed with it only by means of the ether which they contain, and with which they are fo blended, that the agitations excited by the light may be communicated farther without being interrupted in their progrefs. But this tranfmiffion is never performed fo freely as in the pure ether, though it always lofes fomething; and this in proportion as the tranfparent body is more or lefs grofs. The groffnefs may even become fo confiderable, that the light fhall be wholly loft in it; and then the body is no longer tranfparent. Thus, though glafs be a tranfparent body, a great lump of glafs feveral feet thick is not fo. In like manner, however pure the water of a river may be, you cannot fee the bottom where it is very deep, though you can very eafily where it is fhallow.*

Tranfparency, then, is a property of bodies rela-

* The common diftinction of bodies into opaque and tranfparent is inaccurate, for every body has a certain degree of tranfparency. All fubftances abforb light in it's paffage, but in fome this abforption is prodigious, and the quantity of light which penetrates through a certain thicknefs is fo exceedingly minute as to elude our powers of perception. When the thicknefs is much diminifhed, the light becomes fenfible, even in the cafe of bodies that are ufually termed opaque; thus we can fee through a plate of ivory and a leaf of gold. The different properties of fubftances with refpect to the tranfmiffion of light, feems to depend on the greater or lefs regularity of the difpofition of their elementary particles, and on their proximity or diftance from each other; as thefe circumftances augment or diminifh the chance of a ray's paffing within the limit of abforption. Whatever be the intenfity of the incident light, the fame proportion of it is, in a given body, tranfmittted through the fame thicknefs.—*E. E.*

tive only to their thicknefs; and when this property is afcribed to glafs, to water, &c. it muft always be underftood with this reftriction, that thefe bodies are not too grofs; and that to every fpecies there is a certain meafure of thicknefs beyond which the body ceafes to be tranfparent. There is not one opaque body, on the contrary, which may not itfelf become tranfparent, if reduced to a plate extremely fine. Thus, though gold is not tranfparent, gold leaf is fo; and on examining the minuter particles of all bodies with a microfcrope, they are found to be tranfparent. It may then be with truth affirmed, that all bodies are tranfparent when reduced to a certain degree of finenefs; and that no one is fo when too grofs.

In common language we denominate tranfparent the bodies which preferve this quality to a certain degree of thicknefs, though they lofe it when they go beyond that bound. But with refpect to ether, it is of it's own nature perfectly tranfparent, and it's extent diminifhes not this quality in the fmalleft degree. The prodigious diftance of the fixed ftars prevents not their rays from being tranfmitted to us. But though our air appears to be of a perfect tranfparency, if it extended as far as the moon, that tranfparency would be entirely loft, and would prevent every ray of the fun, and of the other heavenly bodies, from penetrating to us. We fhould then be involved in Egyptian darknefs.

The reafon of it is evident, and we remark the fame thing in found, whofe refemblance to light is confirmed

confirmed in every refpect. Air is the moft proper medium for the propagation of found; but the agitations excited in the air are capable of fhaking alfo the particles of all bodies; and thefe again putting in motion the interior particles, finally tranfmit the vibration through the fubftance of all bodies, unlefs they be too thick. There are bodies, then, which, relatively to found, are the fame thing which tranfparent bodies are relatively to light; and all bodies have this property with relation to found, provided they are not too thick. When you are in your apartment, you can hear almoft every thing that paffes in the ante-chamber, though the doors are clofely fhut, becaufe the agitation of the air in the ante-chamber communicates itfelf to the partitions, and penetrates through them into the inner apartment with fome lofs, however. Were the partition removed, you would undoubtedly hear more diftinctly. Now the thicker the walls are, the more of it's force does the found lofe in piercing through them: and the walls may be made fo thick that nothing could be heard from without, unlefs it were fome terrible noife, fuch as a difcharge of cannon.

This leads me forward to a new remark; that very powerful founds may be heard through walls which are impenetrable to founds more feeble; and, confequently, in order to form a judgment whether a wall is capable of tranfmitting founds, it is neceffary to take into the account not only the thicknefs of the wall, but likewife the ftrengh of the found. If the found is very feeble, a very thin wall is fufficient to ſtop it, though a louder could find an eaſy tranſmiſſion. The ſame thing holds as to bodies which are permeable only to a very ſtrong light. Objects not very brilliant are inviſible through a glaſs blackened with ſmoke, but the rays of the ſun force themſelves through it, and it tranſmits perfectly well the image of that luminary. Aſtronomers employ this method to obſerve him; for without ſuch precaution he would dazzle the eye. And when you happen to be in a dark room, with an aperture in the ſhutter expoſed to the ſun, in vain will you attempt to exclude the light by oppoſing your hand to the aperture; the rays of the ſun will force themſelves through.

It is perceivable at the ſame time that the light of the ſun loſes much of it's luſtre in paſſing through a body which, relatively to other objects, is not itſelf tranſparent. But a very ſtrong light may loſe much of it's luſtre, before it is entirely extinguiſhed, while a feebler light is loſt at once. A piece of very thick glaſs, then, will not be tranſparent, with reſpect to objects leſs brilliant, though the ſun may be viſible through it.

Theſe remarks on tranſparent bodies lead me to the theory of refraction, of which you have frequently heard, and which I ſhall endeavour to place in it's proper light.

18th July, 1760.

LETTER

LETTER XXX.

Of the Tranſmiſſion of Rays of Light, though tranſparent Mediums, and their Refraction.

As long as light moves in the ſame medium, whether it be ether, air, or any other tranſparent body, the propagation proceeds in ſtraight lines, denominated rays, as they proceed from the luminous point, in all directions, as the radii of a circle, or a globe, iſſue from the centre. In the ſyſtem of emanation, the particles darted from luminous bodies move in ſtraight lines; the ſame thing holds, in that which I have had the honour of propoſing, in which the agitations are communicated in ſtraight lines, as the ſound of a bell is tranſmitted in a ſtraight line, by which alſo we judge from what quarter the ſound comes; the rays in both ſyſtems, then, are repreſented by ſtraight lines, as long as they paſs through the ſame tranſparent medium; but they may undergo ſome inflection, in paſſing from one to another; and this inflection is called the *refraction* of the rays of light, the knowledge of which is neceſſary to account for many phenomena. I proceed, therefore, to lay down the principles, in conformity to which, refraction takes place.*

* The quantity of refraction is not proportional to the denſity of the medium. Sir Iſaac Newton remarks, that inflammable ſubſtances, though ſpecifically lighter than water, produce a much greater refraction: and it was this analogy which ſuggeſted to him, that diamonds belong to the ſame claſs; a conjecture which has been verified within theſe few years.—*E. E.*

I 4 It

It is an invariable law, that, when a ray, ſuch as E C *(plate* I. *fig* 8.) falls perpendicularly on the ſurface A B of another medium, it continues it's progreſs in the ſame ſtraight line extended, as C F; it will, in this caſe, undergo no inflection or refraction. If, then, E C is a ray of the ſun, falling perpendicularly on the ſurface A B of water, or of glaſs, it will enter it in the ſame direction, and continues it's progreſs in the line C F, which is, likewiſe, perpendicular to the ſurface A B, ſo that E F ſhall be in one and the ſame ſtraight line. This is the only caſe in which there is no refraction. But as often as the ray falls not perpendicularly on the ſurface of another tranſparent body, it does not purſue it's progreſs in the ſame ſtraight line; it recedes leſs or more from it, and undergoes a refraction.

Let P C *(plate* I. *fig.* 9.) be a ray, falling obliquely on the ſurface A B, of another tranſparent medium. On entering into this medium, it will not continue it's progreſs in the direction of the ſtraight line C Q, which is the line P C produced; but will recede from it, in the direction of the line C R, or C S. It will undergo, then, at the point C, an inflection, which we call refraction, which depends partly on the difference of the two mediums, and partly on the obliquity of the direction of the ray P C.

In order to comprehend the laws of this *inflection*, it is neceſſary to explain certain terms employed in treating this ſubject.

1ſt. The ſurface A B, which ſeparates the two mediums, that from which the ray comes, and that into

 which

which it enters, is called the *refringent ſurface*. 2dly. The ray P C, which falls upon it, is called the incident ray; and, 3dly, the ray C R, or C S, which purſues, in the other medium, a courſe different from C Q, is called, the *broken*, or *refracted ray*. And, having drawn through the ſurface A B, the perpendicular line E C F, we call, 4thly, the angle P C E, formed by the incident ray P C, with the perpendicular E C, *the angle of incidence*; and, 5thly, the angle R C F, or S C F, formed by the refracted ray C R or C S, with the perpendicular C F, is called the *angle of refraction*.

Therefore, becauſe of the inflection, which the ray of light undergoes, the angle of refraction is not equal to the angle of incidence P C E; for producing the line P C to Q, the angles P C E and F C Q being vertical, are equal to each other;* as you will eaſily recollect. The angle Q C F, then, is equal to the angle of incidence P C E; therefore, the angle of refraction R C F or S C F, is greater or leſs. There are, then, only two caſes which can exiſt; the one, in which the refracted ray being C R, the angle of refraction R C F, is leſs than the angle of incidence P C E; and the other, in which the refracted ray being C S, the angle of refraction is greater than the angle of incidence P C E. In the former caſe, we ſay, that the ray C R approaches the perpendicular C F; and in the other, that the refracted ray C S, recedes or deviates from the perpendicular.

It is neceſſary, then, to enquire, In what caſes the

* Euclid's Elements, Book I. Prop. 15.

one

one or the other of thefe changes will take place? And we fhall find, that this phenomenon depends on the difference of the denfity of the two mediums; or becaufe the rays are tranfmitted with more or lefs difficulty through each of them. To prove this, it muft be recollected, that ether is of all mediums the moft rare, and that through which rays are tranfmitted, without the flighteft refiftance. After it, the other common tranfparent mediums are thus arranged: air, water, glafs; thus glafs is a medium more denfe than water; water than air; and air than ether.

This being laid down, we have only to attend to thefe two general rules: 1ft. When rays pafs from a medium lefs denfe into one which is more fo, the refracted ray approaches the more to the perpendicular. This is the cafe, in which the incident ray being P C, the refracted ray is C R. 2dly. When the rays pafs from a medium more denfe to one lefs fo, the refracted ray recedes from the perpendicular. This is the cafe, in which the incident ray being P C, the refracted ray is C S. Now, this inflection is greater or lefs, according as the two mediums differ in refpect of denfity. Thus, rays, in paffing from air into glafs, undergo a greater refraction, than when they pafs from air into water; in both cafes, however, the refracted rays approach the perpendicular. In like manner, rays paffing from glafs into air, undergo a greater refraction than when they pafs from water into air; but in thefe cafes, the refracted ray recedes from the perpendicular.

Finally,

Finally, it muft likewife be remarked, that the difference between the angle of incidence and the angle of refraction is fo much greater, as the angle of incidence is greater, or, as the incident ray recedes farther from the perpendicular, the greater will be the inflection or refraction of the ray. A relation of all thefe angles exifts, and is determinable by geometry; but it is not now neceffary to enter into the detail. What has been already faid, is fufficient for underftanding what I have farther to propofe on the fubject.

22d *July*, 1760.

LETTER XXXI.

Refraction of Rays of different Colours.

YOU have feen, that when a ray of light paffes obliquely from one tranfparent medium to another, it undergoes an inflection, which is called refraction, and that the refraction depends on the obliquity of the incidence, and the denfity of the mediums. I muft now call upon you to remark, That diverfity of colours occafions, likewife, a fmall variety in the refraction. This arifes, undoubtedly, from hence, that the rays which excite in us the fenfations of different colours, perform unequal numbers of vibrations in the fame times, and that they differ among themfelves, in the fame manner as fharper or flatter founds do. Thus, it is obfervable,

that

that rays of red undergo the leaſt inflection or refraction; after them come the orange; the yellow, the green, the blue and the violet, follow in order; ſo that violet-coloured rays undergo the greateſt refraction; it being always underſtood, that the obliquity of the incidence, and the denſity of the mediums are the ſame. Hence, it is concluded, that rays of different colours have not the ſame refrangibility; that the red are the leaſt refrangible, and the violet-coloured the moſt ſo.

If then, P C, (*plate* I. *fig.* 10.) is a ray paſſing, for example, from air into glaſs; the angle of incidence being P C E, the refracted ray will approach the perpendicular C F; and if the ray be red, the refracted ray will be in the direction C—*red*; if it be orange, the refracted ray will be C—*orange*, and ſo of the reſt, as may be ſeen in the figure. All theſe rays deviate from the line C Q, which is P C produced, toward the perpendicular C F; but the red ray deviates the leaſt from C Q, or undergoes the leaſt inflection, and the violet recedes the fartheſt from C Q, and undergoes the greateſt inflection.

Now, if P C is a ray of the ſun, it produces, at once, all the coloured rays indicated in the figure; and if a piece of white paper is placed to receive them, you will, in effect, ſee all theſe colours; hence, it is affirmed, that every ray of the ſun contains, at once, all the ſimple colours. The ſame thing happens, if P C is a ray of white, or if it proceeds from a white body. We ſee all the colours produced from it by refraction, whence it is concluded,

that

that white is an aſſemblage of all the ſimple colours, as we ſhewed formerly. In truth, we have only to collect all theſe coloured rays into a ſingle point, and the colour of white will be the reſult.

It is thus we diſcover what are the ſimple colours. Refraction determines them inconteſtibly. In following the order which it preſents, they are theſe: 1. red, 2. orange, 3. yellow, 4. green, 5. blue, 6. violet. But it muſt not be imagined, that there are but ſix: for as difference of colours ariſes from the number of vibrations which rays perform in one and the ſame time, or rather the undulations which produce them: it is clear, that the intermediate numbers equally give ſimple colours.* But we want names, by which to deſign theſe colours; for be-

* This remark, that the number of primitive colours much exceeds ſix, is very juſt. The colours of the rainbow, or of the ſpectrum, formed by a priſm, paſs into each other by inſenſible ſhades, ſo that it is impoſſible to define their boundaries. There is reaſon to ſuſpect, that, even the great *Newton* was, in this inſtance, miſled, by a predilection for the number ſeven, which during many ages, has been regarded with a ſort of myſtical veneration. The correſpondence, which he obſerved, between the diviſions of the ſpectrum, and thoſe of the monochord, and which ſo many authors have ſince repeated, is wholly ideal; for the proportions, between the extent of the different colours are, in a great meaſure, determined by the peculiar quality of the refracting mediums. Thus a priſm of glaſs, in which alkali predominates, forms a ſpectrum, extremely unlike that formed by one of glaſs, compoſed principally of lead. Were a perſon to reckon only the moſt conſpicuous of the primitive colours, he would, moſt probably, ſelect the number ſix, for the indigo can hardly be diſtinguiſhed.—*E. E.*

tween

tween yellow and green, we evidently perceive intermediate colours, for which we have no ſeparate names.

In conformity to the ſame laws, are produced the colours viſible in the rainbow. The rays of the ſun, in paſſing through the drops of water which float through the air, are, by them, reflected and refracted, and the refraction decompounds them into the ſimple colours. You muſt, undoubtedly, have remarked, that theſe colours follow each other, in the ſame order, in the rainbow, the red, orange, yellow, green, blue, and violet; but we diſcover in it, alſo, all the intermediate colours, as ſhades of one colour to another, and had we more names to diſtinguiſh theſe degrees, we might find more of them from the one extremity to the other. A more copious language may, perhaps, enable another nation actually to reckon up a greater number of different colours; and another, it may be, cannot reckon up ſo many; if, for example, it wants a term to expreſs what we call orange. Some to theſe add purple, which we perceive at the extremity of the red, but which others comprehend under the ſame name with red.

C.	D.	E.	F.	G.	A.	B.
Purple.	Red.	Orange.	Yellow.	Green.	Blue.	Violet.

Theſe colours may be compared to the notes of an octave, as I have done here, becauſe the relations of colours, as well as thoſe of ſounds, may be expreſſed by numbers. There is even an appearance, that by ſtraining the violet a little more, you may come round to a new purple, juſt as in riſing from ſound to ſound, on going beyond B, you come round to *c*, which is the octave above C. And, as in muſic, we give to theſe two notes the ſame name, becauſe of their reſemblance, the ſame thing takes place in colours, which, after having riſen through the intervals of an octave, reſume the ſame names: or, if you will, two colours, like two ſounds, in which the number of vibrations in the one, is preciſely the double of the other, paſs for the ſame, and bear the ſame name.

On this principle it was, that father *Caſtel*, in France, contrived a ſpecies of muſic of colours. He conſtructed a harpſichord, of which every key diſplayed a ſubſtance of a certain colour, and he pretended, that this harpſichord, if ſkilfully touched, would preſent a moſt agreeable ſpectacle to the eye. He gave it the name of the *ocular* harpſichord, and you muſt, undoubtedly, have heard it talked of. For my part, painting rather ſeems to be that to the eye, which muſic is to the ear; and I greatly doubt, whether the repreſentation of ſeveral ſhreds of cloth, of different colours, could be very agreeable.

27th July, 1760.

LETTER