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出版情報：Cryogenics. 50 (2), pp.111-117, 2010-02-01. Elsevier
バージョン：

Uncertainty considerations in AC loss measurement of multifilamentary superconducting wires performed via a pickup coil method

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Abstract. The uncertainty of AC loss measurements for multifilamentary superconducting wires by a pickup coil method is evaluated on the basis of the law of propagation of uncertainty. In this evaluation, the effects of measurement conditions, signal processing, and the division of the AC loss into its components (hysteresis loss and coupling loss) are taken into account as elements of uncertainty. The effect of the measurement conditions is evaluated using theoretical expressions of the two main components. Additionally, the effect of signal processing is considered in accordance with the actual processes of the AC loss measurement using experimental outputs. The main factors that contribute to the uncertainty in the propagation process are discussed. The estimated resultant uncertainties are compared to experimental ones for round robin tests of AC loss measurement of Nb-Ti multifilamentary wires exposed to an alternating transverse magnetic field.

Keywords: multifilament wires (A); liquid helium (B); AC losses (C); electromagnetic phenomena (C)

1. Introduction

In recent years, metrological traceability is often required to ensure the reliability of a result of measurement. A prerequisite of the metrological traceability is that the uncertainty of the measurement result is evaluated and documented [1]. Consequently, in standardization of a measurement method or a test procedure it is becoming important to include in the standard a description of a recommended method of uncertainty evaluation of the measurement result. So far, traditional parameters such as “error” and “accuracy” have been used to express the distribution of observed values around a true value in a set of measurements. However, some problems have been pointed out with regard to the usage of these parameters. One problem is that these parameters have not been used with common meanings internationally. Another problem is that these parameters are defined on the basis of the true value of the measured quantity, and hence there can be no clearly specified way of evaluating these parameters because the true value is never knowable in measurement.

These problems were discussed to reach an international agreement for the reliability of the measurement results by the lead of the International Committee for Weights and Measures (CIPM). In 1993 (corrected and reprinted in 1995), *Guide to the Expression of Uncertainty in Measurement* (GUM) was published by the International Organization for Standardization (ISO) in collaboration with other six international organizations including the International Electrotechnical Commission (IEC). The focus of GUM was the establishment of "general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields--from the shop floor to fundamental research." [2] In the new approach, the uncertainty is defined as a parameter that characterizes the dispersion of the values that could reasonably be attributed to the measurand. Quantitatively, it is expressed either as a standard uncertainty which corresponds in magnitude to a standard deviation, or as an expanded uncertainty which defines a confidence interval having a specific level of confidence. The Technical Committee 90 (TC90) for superconductivity in IEC had a decision in the Kyoto meeting held in 2006 that the concept of uncertainty defined in GUM is to be introduced to new documents for standardization and to the existing ones in their maintenance cycles in order to quantitatively describe various types of dispersion in measurement results by “uncertainty”, instead of the previous parameters of “error”, “accuracy” and so on.

WG9 in IEC/TC90 has an activity in preparing documents to standardize methods to measure AC loss in superconducting wires that is one of the most important specifications for AC use of superconductivity. Since the AC apparatuses using superconducting wires become more popular than before, it is more important to introduce “uncertainty” to the standards of the measurement methods of AC loss in accordance with the decision of IEC/TC90.

International Standard IEC 6788-8 [3] was published as the first edition in April 2003 for AC loss measurement of Cu/Nb-Ti composite superconducting wires with a pickup coil method. The main aim of the present paper is to provide a platform to introduce the concept of uncertainty to the second edition of the International Standard IEC 6788-8. Uncertainties and their propagation in AC loss measurements made by the pickup coil method are evaluated mainly by considering the effects of measurement conditions, signal processing and the division of the AC loss into its components (hysteresis loss and coupling loss). The estimated results are also compared with experimental results from past round robin tests of AC losses in Nb-Ti multifilamentary wires.

2. Procedures of AC loss measurement as standardized in IEC61788-8

The AC loss per cycle in a superconducting wire can be estimated by integrating Poynting's vector $\boldsymbol{E}\times\boldsymbol{H}$ on a closed surface surrounding the wire over a period T of alternating electromagnetic environment, where $\boldsymbol{E}$ is the electric field and $\boldsymbol{H}$ is the magnetic field. The AC loss per cycle [J/m^3] per unit volume of the specimen**,** $\boldsymbol{W}$**,** is given by

$$W = -\frac{1}{V_\mathrm{s}}\int_0^T \mathrm{d}t \int_A \mathrm{d}\boldsymbol{A}\cdot\boldsymbol{E}\times\boldsymbol{H} \tag{1}$$

where V_s is the volume of the specimen surrounded by the surface A. Eq. (1) does not depend on the origins of the AC loss in the specimen, but instead gives a basis for measurement methods for the AC loss. In the pickup coil method, the main experimental equipment to measure the AC loss consists of pickup coils and a coiled specimen, which are arranged in a uniform alternating magnetic field, typically applied by a superconducting magnet in liquid helium. The main pickup and compensation coils are coaxially positioned on the outside and inside of the coiled specimen, respectively. The cross section is shown schematically in Fig. 1. For a sufficiently long coiled specimen exposed to a uniform AC magnetic field parallel to the coil axis, the AC loss can be evaluated with Eq. (1) by using the terminal voltages of the pickup coils in the following. The total interlinkage flux of the applied field in the compensation coil is usually made a little larger than that in the main pickup coil by adjusting the number of turns. The signal from the main pickup coil is counterbalanced against a reduced signal of the compensation coil by means of a compensation circuit in order to remove the major component of the signal from the main pickup coil which is due to the temporal change of the external interlinkage flux. A typical electrical circuit used for AC loss measurement by the pickup coils is shown in Fig. 2.

When the specimen is exposed to a uniform alternating magnetic field H_e in the pickup coil, the AC loss can be measured from,

$$W = -\frac{1}{N_p S_s} \int_0^T U_{p\text{-}c}(t) H_e(t) \, dt = -\frac{t_s}{N_p S_s} \sum_{i=1}^{n_s} U_{p\text{-}c}(t_i) H_e(t_i) \quad (2)$$

which is deduced from Eq. (1) [3]. Here $U_{p\text{-}c}$ is the compensated voltage from the main pickup coil and H_e is the applied magnetic field, N_p is the number of turns in the main pickup coil, and S_s an effective cross-sectional area of the coiled specimen obtained by dividing the total specimen volume by the height of the coiled specimen. The applied field is obtained by substituting the measured terminal voltage U_c of the compensation coil into

$$H_e(t) = \frac{1}{\mu_0 N_c S_c} \int_0^t U_c(t') \, dt' = \frac{t_s}{\mu_0 N_c S_c} \sum_{i=1}^{n} U_c(t_i) \quad (3)$$

with the number of turns N_c and the interlinkage area per turn S_c in the compensation coil. In Eqs. (2) and (3), the right hand sides are an approximate form for digital processing where t_i is a discrete time with a sampling interval t_s and $t = n\, t_s$ (n = 1, 2, . . . , n_s) for the number of sampling times per cycle n_s.

For composite superconducting wires with fine filaments, the AC loss can be divided into hysteresis loss in the individual filaments, coupling loss between the filaments and eddy current loss in the normal conducting parts. In cases where the wires do not have a thick outer normal conducting sheath, the main components are the hysteresis loss and the coupling loss. We can estimate the former part by extrapolation to zero frequency (using the low frequency, linear portion of the curve).

3. Methods of uncertainty evaluation in the AC loss measurement

Uncertainty in the AC loss measurement by the pickup coil method is mainly attributable to effects of measurement conditions, signal processing and division of the AC loss into its components. The effects of measurement conditions are mainly associated with uncertainty in temperature and magnetic field, and the determination of sample dimensions. $U_{\text{p-c}}$ in Eq. (2) is an induced voltage due to the temporal change of magnetic moment in the specimen, which is caused by the external AC magnetic field, and therefore the uncertainties in the measurement conditions will be mainly propagated in the AC loss measurement through $U_{\text{p-c}}$. Nevertheless, since it is difficult to explicitly express $U_{\text{p-c}}$ by the temporal change of the magnetic moment, the effects of the measurement conditions are evaluated with theoretical expressions of two main components in the AC loss, the hysteresis loss W_h and the coupling loss W_c for one cycle and unit volume of the specimen, which are given for an applied alternating magnetic field with practical large amplitude, respectively, as

$$W_h = \frac{4}{3}\mu_0 H_p{}^2\left(2\frac{H_m}{H_p}-1\right) \cong \frac{8}{3}\mu_0 H_p H_m \tag{4}$$

$$W_c = 4\pi^2 \tau \mu_0 H_m{}^2 f \qquad (5)$$

where H_m is the amplitude of applied magnetic field, H_p the penetration field, τ the coupling time constant and $f = 1/T$ the frequency. Since Eqs. (4) and (5) are theoretically obtained from the Maxwell's equations with electromagnetic properties of the multifilamentary superconducting wires [4], [5], we can evaluate the effects of the above measurement conditions on the AC losses by these equations. Eqs. (2) and (3), on the other hand, give a basis to consider the propagation of uncertainty through the measurements of the terminal voltages and the digital integration to calculate the AC loss in the signal processing. The third is an additional one to divide the AC loss into the two components by considering the difference between their frequency dependences. Main results of the relative combined standard uncertainties for the two loss components obtained in these evaluations are summarized in Table 1, which corresponds to a usual uncertainty budget.

4. Uncertainties associated with measurement conditions

As described in Appendix B, important properties of superconductors can be expressed as functions of the dimension D (diameter of a columnar superconductor or twist pitch of a composite wire) and temperature T of the specimen, and the magnetic field H to which the specimen is exposed. Hence the uncertainties of the superconductor properties can be derived from three basic uncertainties: $u_r(D)$, $u_r(T)$, and $u_r(H)$, where the symbol u_r denotes the relative standard uncertainty.

Although these basic uncertainties are mentioned in the first edition of IEC 6788-8, their values are given not in terms of standard uncertainties, but in terms of "accuracies." In the present paper, we regard the "accuracy" as representing a confidence interval corresponding to 95% confidence level, and in addition we assume that the probability distribution in question is approximately a normal distribution. This implies that the accuracy is regarded as a relative expanded uncertainty with a coverage factor of 2. From these considerations, the three basic uncertainties, as well as the uncertainties of superconductor properties derived from these can be evaluated as shown in Table 1(a).

The AC losses of the specimen are measured while immersed in liquid helium. The temperature difference between the specimen and the liquid helium can be estimated from a typical level of AC loss and the heat transfer coefficient of liquid helium, and is expected to be smaller than 0.001 K. Hence the temperature of the specimen can reasonably be assumed to be equal to that of the liquid helium. The latter temperature can be measured with a thermometer or by converting an observed atmospheric pressure in the cryostat using the phase diagram of helium. The first edition of IEC 6788-8 requires that the accuracy of the temperature thus determined be better than 0.1 K. If this requirement is met, the relative standard uncertainty of the specimen temperature, $u_r(T)$, for a typical level of the liquid helium temperature of 4.2 K, is estimated to be 1.2×10^{-2} at the largest.

The uncertainty of the applied magnetic field can be classified into two components: One associated with the conditions of the AC loss measurement, and the other associated with the signal processing in measuring the magnetic field by the compensation coil based on Eq. (3). The latter component is evaluated later in the next section. The major source of the former uncertainty component is the non-uniformity of the field within the space where the pickup coils are set. The relative accuracy of this component is given in the first edition of IEC 6788-8 as 0.01. The corresponding relative standard uncertainty $u_r(H)$ is then estimated to be 5.0×10^{-3}.

Finally, the relative standard uncertainty of the dimension of the specimen, $u_r(D)$, is evaluated, again from the corresponding accuracy given in the first edition of IEC 6788-8, as 5.0×10^{-3}.

In the above evaluation of the uncertainties, the quantity $u_r(X)$ implies a relative standard uncertainty for the mean $\overline{X}$ of a measurement quantity X, which is usually expressed as $u_r(\overline{X})$. In this study, the former type of expression $u_r(X)$ will be used for simplicity instead of $u_r(\overline{X})$.

5. Propagation of uncertainty due to measurement conditions

The effects of the measurement conditions on the hysteresis loss W_h can be considered with Eq. (4) through the relations for the upper critical field H_{c2}, the pinning force density F_p and the penetration field H_p, Eqs. (B.1), (B.2) and (B.3) in Appendix B. The uncertainties of these superconducting parameters evaluated from three uncertainties are listed in the uncertainty budget. The relative combined standard uncertainties $u_{c,r1}(W_h)$ in relation to the measurement conditions is given by

$$u_{c,r1}(W_h) = \sqrt{u_{c,r}{}^2(H_p) + u_r{}^2(H_m)} = \sqrt{2^2(m-\gamma)^2 u_r{}^2(T) + \{(\gamma-1)^2+1\}u_r{}^2(H) + u_r{}^2(D)} \tag{6}$$

where the relative combined standard uncertainties $u_{c,r}(H_p)$ is related to the basic uncertainties by using the addition rule Eq. (A.6) and the multiplication one Eq. (A.8) for the combined standard uncertainties in the following way,

$$\begin{aligned} u_{c,r}(H_p) &= \sqrt{u_{c,r}{}^2(J_c) + u_r{}^2(D)} = \sqrt{(m-\gamma)^2 u_{c,r}{}^2(H_{c2}) + (\gamma-1)^2 u_r{}^2(H) + u_r{}^2(D)} \\ &= \sqrt{2^2(m-\gamma)^2 u_r{}^2(T) + (\gamma-1)^2 u_r{}^2(H) + u_r{}^2(D)} \end{aligned} \tag{7}$$

The relation (5) of the coupling loss W_c shows that the uncertainty is mainly attributed to those of coupling time constant τ and the amplitude of applied magnetic field H_m. The relative combined standard uncertainties $u_{c,r1}(W_c)$ in relation to the measurement conditions is also given by

$$u_{c,r1}(W_c) = \sqrt{u_{c,r}{}^2(\tau) + 2^2 u_r{}^2(H_m)} = \sqrt{u_r{}^2(\tau) + 2^2 u_r{}^2(H)} = \sqrt{2^3 u_r{}^2(H) + 2^2 u_r{}^2(D)} \tag{8}$$

where the relations (B.4) and (B.5) are considered.

It can be simply seen that the resultant uncertainty of the AC loss is dependent on the component ratio between the hysteresis loss and the coupling loss. In order to divide the AC loss into the major components, the AC losses are measured in a wide range of frequency, where the AC loss W_{lower} at a lower frequency limit is almost corresponding to the hysteresis loss and the AC loss W_{upper} at a upper frequency limit is composed of the two components that are comparable to each other. In this way, the relative combined standard uncertainties, $u_{c,r1}(W_{lower})$ and $u_{c,r1}(W_{upper})$, of the AC loss are given by

$$u_{c,r1}(W_{lower}) = u_{c,r1}(W_h) \tag{9}$$

at the lower frequency limit and

$$u_{c,r1}(W_{upper}) = \sqrt{\alpha^2 u_{c,r1}{}^2(W_h) + (1-\alpha)^2 u_{c,r1}{}^2(W_c)} \tag{10}$$

at the upper frequency limit. Eq. (10) is derived from Eq. (A.6) with the condition of $\overline{W_h} = \alpha \overline{W_{upper}}$ and $\overline{W_c} = (1-\alpha)\overline{W_{upper}}$ at the upper frequency limit, where $\overline{A}$ is a mean of A.

The results of uncertainty evaluation for the measurement condition are listed as a typical example of α = 0.5 with m = 2 and γ = 0.5 for a NbTi conductor in Table 1(a). The main contribution to the resultant uncertainty comes from the measurement temperature for the hysteresis loss and the applied magnetic field for the coupling loss.

6. Origins and propagation of uncertainty due to signal processing

In signal processing, Eqs. (2) and (3) are used to evaluate the propagation of uncertainty. In these relations, basic standard uncertainties are related to the turn number N and the cross-sectional area S of the pickup coil, the sampling interval t_s and the terminal voltage U of the pickup coils measured by an q-bit amplifier which are given by

$$u_r(N) = 0.5/\sqrt{3}N \tag{11}$$

$$u_r(S) = 2\,u_r(D) \tag{12}$$

$$u_r(t_s) = (1/2\,f_{clock})/\sqrt{3}\,t_s \tag{13}$$

$$u_r(U) = 1/(2^q\sqrt{3}\,) \tag{14}$$

where f_{clock} is a clock frequency of an A/D converter in the data acquisition computer. Eqs. (11), (13) and (14) indicate that $u_r(N)$, $u_r(t_s)$ and $u_r(U)$ are obtained by a type B evaluation for N, t_s and U, respectively. These basic standard uncertainties are listed in Table 1(b) for a typical set of (N, t_s, f_{clock}, q) = (200 turns, 1/2000 s, 1×10^{6} Hz, 10). From these basic standard uncertainties, the relative combined standard uncertainties, $u_{c,r}(H_e)$ and $u_{c,r2}(W)$, for H_e and W observed in the measurement are obtained as follows;

$$u_{c,r}(H_e) \cong \sqrt{(-1)^2 u_r{}^2(N) + (-1)^2 u_r{}^2(S) + u_r{}^2(t_s) + \frac{1}{n} u_r{}^2(U)} \cong u_r(S) \tag{15}$$

$$u_{c,r2}(W) = \sqrt{(-1)^2 u_r{}^2(N) + (-1)^2 u_r{}^2(S) + u_r{}^2(t_s) + \frac{1}{n_s}[2u_r{}^2(U) + u_{c,r}{}^2(H_e)]} \cong u_r(S) \tag{16}$$

where approximate relations $u_r(N_c) = u_r(N_p) = u_r(N)$, $u_r(S_c) = u_r(S_s) = u_r(S)$ and $u_r(U_c) = u_r(U_{p\text{-}c}) / \sqrt{2} = u_r(U)$ are used. In Eqs. (15) and (16), the last term of the second side is a contribution from the summation part in Eqs. (2) and (3), respectively, where it is assumed that the measurement at a time t_i is independent of that at other time $t_{j\,(\neq i)}$.

The results of uncertainty evaluation for the signal processing are also listed in Table 1(b). Since the contribution of the cross section of pickup coils is dominant in the uncertainties, the resultant uncertainty of the AC loss is almost approximated by that of the cross section in the signal processing.

7. Integration of two effects and division into components

Let us assume that the effects of the measurement conditions and the signal processing on the propagation of uncertainty are integrated by Eq. (A.8). The relative combined standard uncertainty of the AC loss is expressed by

$$u_{\mathrm{c,r}}(W_{\mathrm{lower}}) = \sqrt{u_{\mathrm{c,r1}}{}^2(W_{\mathrm{lower}}) + u_{\mathrm{c,r2}}{}^2(W)} = \sqrt{u_{\mathrm{c,r1}}{}^2(W_{\mathrm{h}}) + u_{\mathrm{c,r2}}{}^2(W)} \quad (17)$$

for the lower frequency limit, and

$$u_{\mathrm{c,r}}(W_{\mathrm{upper}}) = \sqrt{u_{\mathrm{c,r1}}{}^2(W_{\mathrm{upper}}) + u_{\mathrm{c,r2}}{}^2(W)} \quad (18)$$

for the upper frequency limit. In Eqs. (17) and (18), the contribution from the measurement condition is dominant in comparison with that from the signal processing as known in Table 1(a) and (b).

In the final step of the uncertainty evaluation, we divide the AC loss into the main components, the hysteresis loss and the coupling loss. For a fixed amplitude of the applied magnetic field, since the hysteresis loss is obtained as the AC loss at the lower frequency limit, the relative combined standard uncertainty $u_{\mathrm{c,r}}(W_{\mathrm{h}})$ of the hysteresis loss is finally given by

$$u_{\mathrm{c,r}}(W_{\mathrm{h}}) = u_{\mathrm{c,r}}(W_{\mathrm{lower}}) \quad (19)$$

Since the coupling loss is obtained as the difference between the AC loss and the hysteresis loss, $W_\mathrm{c} = W - W_\mathrm{h}$, the relative combined standard uncertainty $u_\mathrm{c,r}(W_\mathrm{c})$ of the coupling loss is evaluated from Eq. (A.6) as

$$u_{\mathrm{c,r}}(W_\mathrm{c}) = \sqrt{\frac{\overline{W}^2}{\overline{W_\mathrm{c}}^2} u_{\mathrm{c,r}}{}^2(W) + \frac{\overline{W_\mathrm{h}}^2}{\overline{W_\mathrm{c}}^2} u_{\mathrm{c,r}}{}^2(W_\mathrm{h})} \tag{20}$$

At the upper frequency limit with the condition of $\overline{W_\mathrm{h}} = \alpha \overline{W_\mathrm{upper}}$ and $\overline{W_\mathrm{c}} = (1-\alpha)\overline{W_\mathrm{upper}}$, Eq. (20) is reduced to

$$u_{\mathrm{c,r}}(W_\mathrm{c}) = \sqrt{\left(\frac{1}{1-\alpha}\right)^2 u_{\mathrm{c,r}}{}^2(W_\mathrm{upper}) + \left(\frac{\alpha}{1-\alpha}\right)^2 u_{\mathrm{c,r}}{}^2(W_\mathrm{lower})} \tag{21}$$

From Eq. (5), the relative combined standard uncertainty $u_\mathrm{c,r}(\tau)$ of the coupling time constant is deduced as

$$u_{\mathrm{c,r}}(\tau) = \sqrt{u_{\mathrm{c,r}}{}^2(W_\mathrm{c}) + 4u_\mathrm{r}{}^2(H)} \tag{22}$$

The propagation of uncertainty evaluated is summarized as a typical example in Table 1(c). In the propagation of uncertainty, the main factors of the rises in uncertainty are the effects of the measurement temperature and the applied magnetic field on the AC loss and the additional effect of dividing the AC loss into the components on the coupling loss and the coupling time constant.

8. Comparison with COV's in round robin tests

The first edition of IEC 61788-6 was developed on the basis of the round robin tests for Cu/Nb-Ti composite wires [6] and three-component Nb-Ti wires [7]. In the round robin tests, the coefficient of variation (COV), defined as the ratio of the sample standard deviation $s(X)$ of a measurement quantity X to the mean $\overline{X}$, was used as the quantitative statistical description to summarize the interlaboratory comparison. The values of COV obtained in the round robin tests and the relative combined standard uncertainties $u_{c,r}$ evaluated in the present paper are compared in Table 2.

We examine the relationship between COV and $u_{c,r}$. We first note that $u_{c,r}$ is an estimate of the error in AC loss measurement expressed in terms of a relative standard deviation, σ_r. If the measurement error of each laboratory participating in the round robin tests is considered statistically independent, COV can be regarded as another estimate of σ_r. If the number of participating laboratories, n, is relatively small, the reliability in estimating σ_r from COV is accordingly low. Its confidence interval at 95 % level of confidence is given approximately by

$$\frac{\sqrt{n-1}}{\chi_{n-1}(0.025)}\mathrm{COV} \le \sigma_r \le \frac{\sqrt{n-1}}{\chi_{n-1}(0.975)}\mathrm{COV} \tag{23}$$

where $\chi_{n-1}(\alpha)$ is the upper 100α % point of the chi distribution with (n-1) degrees of freedom. The inequality above is derived from the fact that $(n-1)\mathrm{COV}^2/\sigma_r{}^2$ can be approximated by $(n-1)\left(s^2(X)/\mu^2\right)/\sigma_r{}^2$ (μ is the population mean of X), which is distributed according to the chi-square distribution with (n-1) degrees of freedom. The confidence intervals of σ_r given in Eq. (23) are compared with $u_{c,r}$ in Fig. 3. Because the effective degrees of freedom of $u_{c,r}$ can only be poorly estimated, we do not attempt to calculate the confidence intervals of σ_r based on $u_{c,r}$. We might rather think that $u_{c,r}$ is evaluated deliberately and is significantly close to σ_r in comparison with the confidence interval given in Eq. (23). It is observed in Fig. 3 that $u_{c,r}$ falls within the interval determined from COV in each four cases. This means that the $u_{c,r}$ evaluation in the present paper is consistent with the results of the previous round robin tests.

9. Concluding remarks

The uncertainty in AC loss measurements by the pickup coil method has been evaluated by considering the effects of measurement conditions, signal processing, and division of the AC loss into its hysteretic and coupling components. The main contribution from the measurement conditions to the resultant uncertainty comes from the uncertainties in the temperature (which mostly affects the hysteretic component) and the magnetic field (which most strongly affects the coupling components). The uncertainty from signal processing, on the other hand, is dominated by the uncertainty of the area of the pickup coils. The exact way in which these various uncertainties propagate into the hysteretic and coupling components is determined by the way in which the components are separated out from the as-measured total loss. It is shown by statistical analyses that the present uncertainty evaluation is consistent with the previous round-robin-test results of AC loss measurement for NbTi composite wires.

Appendix A. Basic formulae in uncertainty evaluation [2]

In general, components of uncertainty may be categorized according to the method used to evaluate them: Type A evaluation (method of evaluation of uncertainty by the statistical analysis of series of observations.) and Type B evaluation (method of evaluation of uncertainty by means other than the statistical analysis of series of observations). The Type A evaluation is based on repeated measurements in the laboratory in general expressed in the form of Gaussian distributions, and the

Type B one on previous experiments, literature data, manufacturer's information, etc. often provided in the form of rectangular distributions.

In the Type A evaluation, for a measurement quantity (usually called as input quantity) X_A including n independent observations X_k (k = 1, 2, . . . , n), the mean (usually called as input estimate) x_A and the standard uncertainty of the mean $u(x_A)$ are given by

$$x_A = \overline{X_A} = \frac{1}{n}\sum_{k=1}^{n} X_k \tag{A.1}$$

$$u(x_A) = \left[\frac{1}{n(n-1)}\sum_{k=1}^{n}(X_k - x_A)^2\right]^{1/2} \tag{A.2}$$

In the Type B evaluation, for a measurement quantity X_B with lower and upper limits of a_- and a_+ and uniform distribution between them, the best estimate x_B is $(a_- + a_+)/2$ and the standard uncertainty $u(x_B)$ is given by

$$u(x_B) = \frac{a_+ - a_-}{2\sqrt{3}} \tag{A.3}$$

Let us consider an output estimate y expressed as a function of N input estimates x_i (i = 1, 2, . . . , N), which are independent of each other, by

$$y = f(x_1, x_2, \ldots, x_N) \tag{A.4}$$

In a simple case where y is a sum of x_i multiplied by constants a_i,

$$y = a_1 x_1 + a_2 x_2 + \ldots + a_N x_N \tag{A.5}$$

the combined standard uncertainty $u_c(y)$ of y is related to the standard uncertainty $u(x_i)$ of x_i by the addition rule

$$u_c{}^2(y) = a_1{}^2 u^2(x_1) + a_2{}^2 u^2(x_2) + \ldots + a_N{}^2 u^2(x_N) \tag{A.6}$$

In the second case where y is a product of x_i raised to powers a, b, .. p, multiplied by a constant A,

$$y = Ax_1^{\,a} x_2^{\,b} \ldots x_N^{\,p} \tag{A.7}$$

the relative combined standard uncertainty $u_{c,r}(y)$ of y is also related to the relative standard uncertainty $u_r(x_i)$ of x_i by the multiplication rule

$$u_{c,r}^{\,2}(y) = a^2 u_r^{\,2}(x_1) + b^2 u_r^{\,2}(x_2) + \ldots + p^2 u_r^{\,2}(x_N) \tag{A.8}$$

In Eq. (A.8), $u_{c,r}(y)$ and $u_r(x_i)$ are defined by $u_c(y)/|y|$ and $u(x_i)/|x_i|$, where $|y|$ and $|x_i|$ are the absolute values of y and x_i, respectively.

Appendix B. Basic properties of superconductors

Basic properties of superconductors used to evaluate uncertainties in the text are summarized as a simplified case in the following Eq. (B.1) to Eq. (B.4) [4], [8-10], and a usual property of the magneto resistive effect in normal conducting metal is given in Eq. (B.5).

$$H_{c2} = H_{c2}(0)\left(1 - \frac{T^2}{T_c^{\,2}}\right) \tag{B.1}$$

$$F_p = J_c B = A H_{c2}^{\;m} \left(\frac{B}{\mu_0 H_{c2}}\right)^{\gamma} \tag{B.2}$$

$$H_p = \frac{2}{\pi} J_c \frac{d_f}{2} \tag{B.3}$$

$$\tau = \frac{1}{2}\left(\frac{L_s}{2\pi}\right)^2 \frac{\mu_0}{\rho} \tag{B.4}$$

$$\frac{\Delta\rho}{\rho} \propto H^2 \tag{B.5}$$

Variables from Eq. (B.1) to Eq. (B.5) are defined in Table B1.

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Table 1(a) Effect of measurement conditions on propagation of relative uncertainty

Table 1(b) Effect of signal processing in measurement on propagation of relative uncertainty

Table 1(c) Propagation of relative uncertainty by integrating two effects and dividing into components

Table 2 COV in the round robin tests and evaluated relative combined standard uncertainty $u_{c,r}$.

Table B1 Vocabulary

Table 1(a)

Components of measurement conditions	Relative uncertainty
Uncertainty sources	
Dimensions: $u_r(D)$	5.0×10^{-3}
Magnetic field: $u_r(H)$	5.0×10^{-3}
Temperature: $u_r(T)$	1.2×10^{-2}
Relative combined standard uncertainties of superconducting parameters	
Penetration field from Eq. (B.3): $u_{c,r}(H_p)=\sqrt{u_{c,r}{}^2(J_c)+u_r{}^2(D)}$	2.4×10^{-2}
Critical current density from Eq. (B.2)*: $u_{c,r}(J_c)=\sqrt{(m-\gamma)^2u_{c,r}{}^2(H_{c2})+(\gamma-1)^2u_r{}^2(H)}$	3.6×10^{-2}
Upper critical field from Eq. (B.1): $u_{c,r}(H_{c2}) = 2\,u_r(T)$	3.6×10^{-2}
Coupling time constant from Eq. (B.5): $u_{c,r}(\tau)=\sqrt{2^2u_r{}^2(D)+u_r{}^2(\rho)}$	0.7×10^{-2}
Magneto-resistive effect from Eq. (B.4): $u_{c,r}(\rho) = 2\,u_r(H)$	1.2×10^{-2}
Relative combined standard uncertainties of AC losses	
AC loss at lower frequency limit: $u_{c,r1}(W_{lower}) = u_{c,r1}(W_h)$	3.6×10^{-2}
AC loss at upper frequency limit with $\alpha = 0.5$: $u_{c,r1}(W_{upper})=\sqrt{\alpha^2u_{c,r1}{}^2(W_h)+(1-\alpha)^2u_{c,r1}{}^2(W_c)}$	1.6×10^{-2}
Hysteresis loss: $u_{c,r1}(W_h)=\sqrt{u_{c,r}{}^2(H_p)+u_r{}^2(H)}$	3.6×10^{-2}
Coupling loss: $u_{c,r1}(W_c)=\sqrt{u_{c,r}{}^2(\tau)+2^2u_r{}^2(H)}$	2.0×10^{-2}

* The conditions of $m = 2$ and $\gamma = 0.5$ are used in Eqs. (6) and (7).

Table 1(b)

Components of signal processing in measurement	relative uncertainty
Uncertainty sources	
Turn number of pickup coils: $u_r(N) = 0.5/(\sqrt{3}N)$	1.5×10^{-3}
Cross-sectional area: $u_r(S) = 2u_r(D)$	1.0×10^{-2}
Sampling interval: $u_r(t_s) = 1/(2\sqrt{3}f_{clock}t_s)$	5.7×10^{-4}
Terminal voltage of pickup coils by a q-bit amplifier with $q = 10$: $u_r(U) = 1/(2^q\sqrt{3})$	5.6×10^{-4}
Relative combined standard uncertainties of experimental outputs	
Processing magnetic field from Eq. (3)**: $u_{c,r}(H_{es}) = \sqrt{u_r^2(N) + u_r^2(S) + u_r^2(t_s) + u_r^2(U)/n}$	1.0×10^{-2}
Processing total AC loss from Eq. (2): $u_{c,r2}(W) = \sqrt{u_r^2(N) + u_r^2(S) + u_r^2(t_s) + [2u_r^2(U) + u_{c,r}^2(H_{es})]/n_s}$	1.0×10^{-2}

**The condition of $n = 1$ is used in Eq. (15) as an upper limit of estimation.

Table 1(c)

Propagation of relative uncertainty	relative uncertainty
Integrating two effects	
At lower frequency limit: $u_{\mathrm{c,r}}(W_{\mathrm{lower}}) = \sqrt{u_{\mathrm{c,r1}}{}^2(W_{\mathrm{lower}}) + u_{\mathrm{c,r2}}{}^2(W)}$	3.8×10^{-2}
At upper frequency limit: $u_{\mathrm{c,r}}(W_{\mathrm{upper}}) = \sqrt{u_{\mathrm{c,r1}}{}^2(W_{\mathrm{upper}}) + u_{\mathrm{c,r2}}{}^2(W)}$	2.2×10^{-2}
Dividing into loss components	
Hysteresis loss: $u_{\mathrm{c,r}}(W_{\mathrm{h}}) = u_{\mathrm{c,r}}(W_{\mathrm{lower}})$	3.8×10^{-2}
Coupling loss: $u_{\mathrm{c,r}}(W_{\mathrm{c}}) = \sqrt{\left(\frac{1}{1-\alpha}\right)^2 u_{\mathrm{c,r}}{}^2(W_{\mathrm{upper}}) + \left(\frac{\alpha}{1-\alpha}\right)^2 u_{\mathrm{c,r}}{}^2(W_{\mathrm{lower}})}$	5.4×10^{-2}
Coupling time constant: $u_{\mathrm{c,r}}(\tau) = \sqrt{u_{\mathrm{c,r}}{}^2(W_{\mathrm{c}}) + 2^2\, u_{\mathrm{r}}{}^2(H)}$	5.5×10^{-2}

Table 2

Round robin test	*Cu/Nb-Ti composite wire* [3]	*Three-component wire* [4]
Number of laboratories	4	2
Amplitude of magnetic field	1 T	0.5 T
Frequency range	0.005 Hz – 1 Hz	0.01 Hz – 20 Hz
Loss ratio α	0.53	0.12
Hysteresis loss		
COV	1.5×10^{-2}	1.8×10^{-2}
$u_{c,r}$	3.8×10^{-2}	3.8×10^{-2}
Coupling loss		
COV	1.1×10^{-1}	1.0×10^{-3}
$u_{c,r}$	6.2×10^{-2}	2.1×10^{-2}

Table B1

Items	Variables
Upper critical field	H_{c2}
Critical temperature	T_c
Pinning force density	F_p
Scaling parameters of F_p	m, γ
Critical current density	J_c
Penetration field	H_p
Diameter of a columnar superconductor	d_f
Electric resistivity	ρ
Changing part in ρ	$\Delta\rho$
Coupling time constant	τ
Twist pitch	L_s

Figure Captions

Figure 1 Cross-sectional view of concentric pickup coils and coiled specimen

Figure 2 Electric circuit to calculate AC loss by a pickup coil method

Figure 3 Comparison of the values of COV and $u_{c,r}$

Figure 1

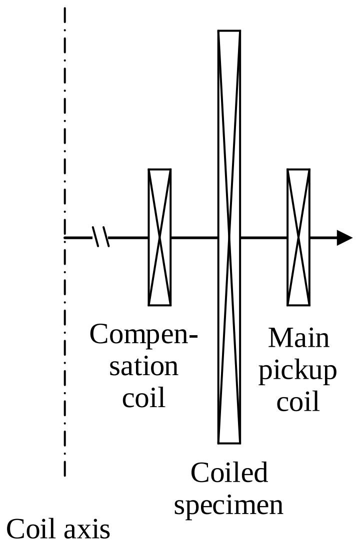
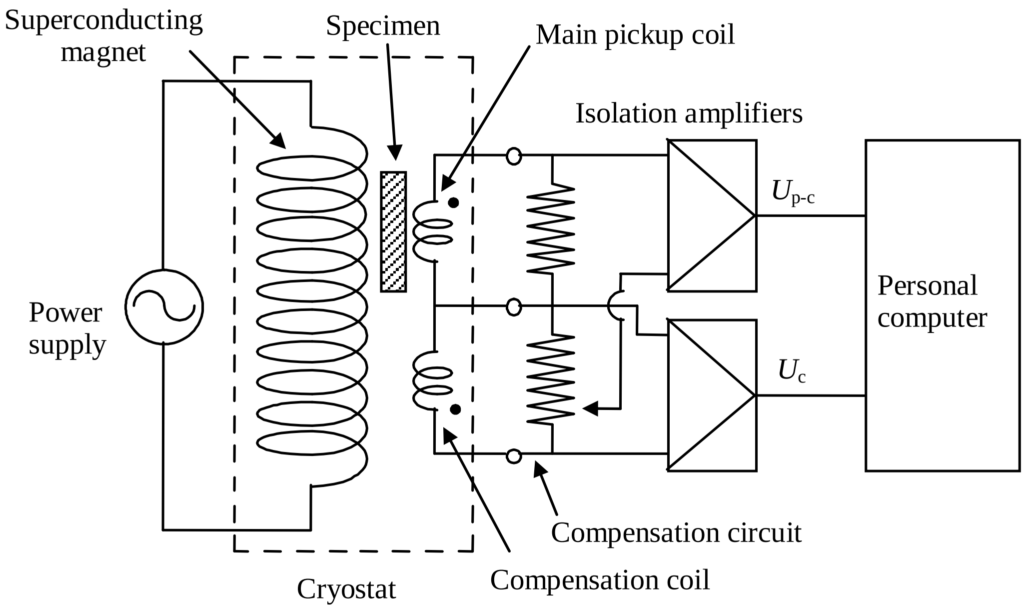
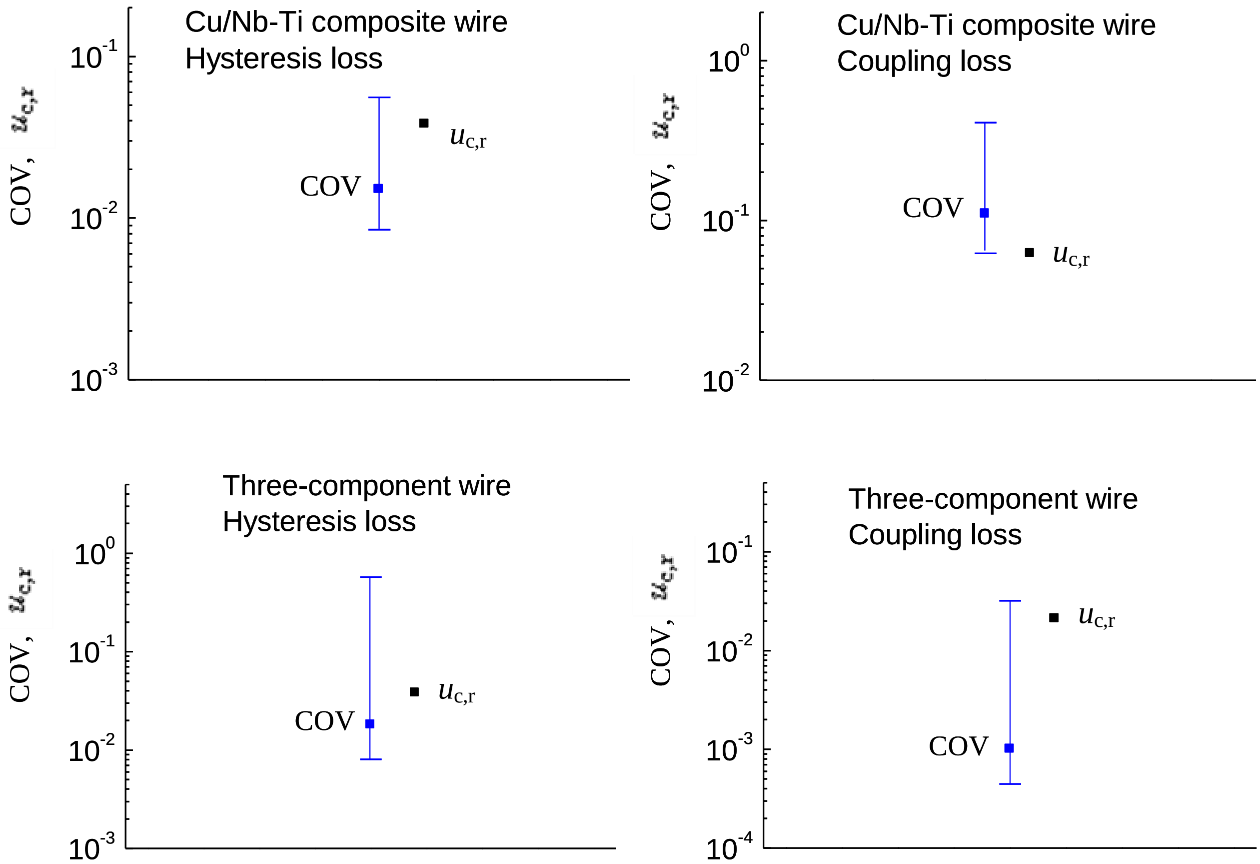

Compen-
sation
coil
Main
pickup
coil
Coiled
specimen
Coil axis

Figure 2



Superconducting magnet
Specimen
Main pickup coil
Isolation amplifiers
$U_{p\text{-}c}$
Personal computer
Power supply
U_c
Compensation circuit
Compensation coil
Cryostat

Figure 3



Cu/Nb-Ti composite wire
Hysteresis loss
COV
$u_{c,r}$
COV, $u_{c,r}$
10^{-1}
10^{-2}
10^{-3}
Cu/Nb-Ti composite wire
Coupling loss
COV
$u_{c,r}$
COV, $u_{c,r}$
10^{0}
10^{-1}
10^{-2}
Three-component wire
Hysteresis loss
COV
$u_{c,r}$
COV, $u_{c,r}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
Three-component wire
Coupling loss
COV
$u_{c,r}$
COV, $u_{c,r}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}