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Theoretical and Experimental Study on Tsunami Induced Instability of Caisson Type Composite Breakwater

by

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Abstract

On Friday, 11 March 2011, the most powerful earthquake induced tsunami ever recorded attacked the northeast coast of Japan. The world's deepest breakwater, Kamaishi breakwater, experienced serious damage during this event. In order to find the failure mechanism and to reproduce the failure process from the recorded tsunami data, this paper applies a fundamental theoretical approach to analyze the stability of caisson under seepage flow. Two types of experiments were performed to investigate the influence of seepage force on the stability of caisson type composite breakwaters. The following main conclusions are drawn: (1) by flow-net graphic solution analysis, it can be concluded that the area of instability in rubble mound on the harbor side enlarges when wave height increases, and the rubble mound becomes unstable due to seepage force when the rubble mound slope angle increases, (2) the experiment results of the bearing capacity test represented that the bearing capacity of rubble mound decreased by about 50% due to horizontal component of seepage flow compared with the condition without seepage flow. In conclusion, it can be said that the seepage flow in the rubble mound beneath caisson should be taken into account as a significant influential factor in the design of caisson type composite breakwater against tsunami.

Keywords: Bearing capacity, Caisson, Breakwater, Earthquake, Experiment, Failure, Pore pressure, Seepage flow, Stability analysis, Tsunami

1. Introduction

The 2011 earthquake off the Pacific coast of Tōhoku¹⁾ was a magnitude 9.0 (M_w) undersea megathrust earthquake off the northeast coast of Japan that occurred at 14:46 JST on Friday, 11

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March 2011. It was the most powerful known earthquake ever to have hit Japan. The earthquake triggered powerful tsunami reached heights of up to 40.5 meters in Miyako in Tōhoku's Iwate Prefecture. Tsunami waves caused catastrophe and damaged the caisson breakwaters along the coast. The Kamaishi Tsunami Protection Breakwater (**Fig. 1**) is 1,950 m long and 63 m in depth, was completed in March 2009 after three decades of construction at a cost of \$1.5 billion. It was once recognized by the Guinness World Records as the world's deepest breakwater. It was heavily damaged by the 2011 Tōhoku earthquake and tsunami. According to the GPS (Global Positioning System) wave height record at Kamaishi port when the tsunami arrived at the coast, a gradually rise in water level occurred between 15:07 JST and 15:15 JST (a duration of 8 minutes) has been observed. Such a long period of high hydraulic head difference made the Kamaishi breakwater experience serious damage during this tsunami attack. The failure mechanism of breakwater was still under studied.

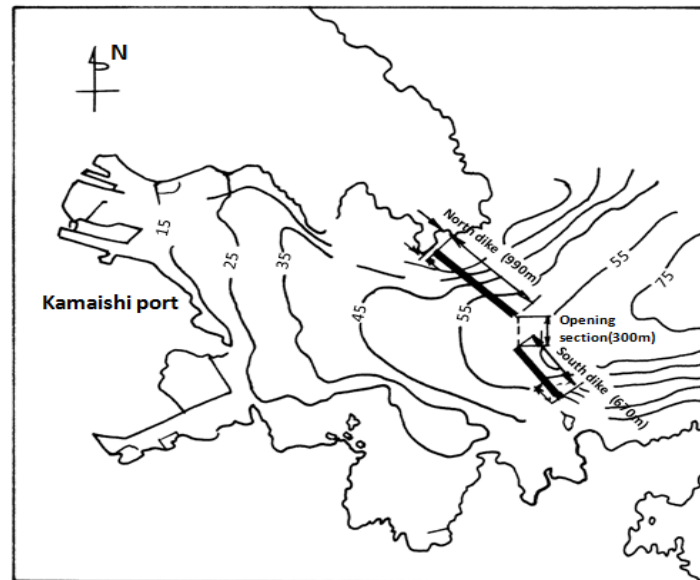


Fig. 1 Kamaishi breakwater layout in Kamaishi port²⁾, Iwate prefecture (after Uezono,1987)

Caisson type composite breakwater consists of a rubble mound foundation and a caisson. It becomes the most common structural type of breakwater in Japan nowadays. The stability of caisson type composite breakwaters is the key issue when it comes to safe design. The stability of caisson breakwaters have been researched by scholars in the last decades, failure mechanisms have been pointed out as scouring, wave dragging, over topping, wave-induced liquefaction on the seabed, etc.. Zen *et al.*³⁾ published a case study of a breakwater damaged by wave-induced liquefaction. Oumeraci *et al.*⁴⁾ analyzed the failure of breakwaters under several wave conditions without tsunami condition. Franco *et al.*⁵⁾ researched composite breakwater instability influenced by over topping. In the other literatures, seepage force's effect on slope stability has focused on riverbanks and dam sites. Dunne⁶⁾ and Crosta *et al.*⁷⁾ investigated some of the different slope failure mechanisms included the instability caused by seepage. Nakamura⁸⁾ investigated slope instability at dam sites.

In general, the caisson type composite breakwater was designed against failure by using wind wave data for reference. The most distinguished difference between wind waves and tsunami is addressed that tsunami has much longer wave periods which can last a few minutes or more. It

makes the large hydraulic head difference generated between the sea side and harbor side of caisson. Hitachi et.al⁹⁾ studied the stability of armored stone considering the tsunami condition as the steady flow. So far, few researches on the past failure events or theoretical analysis have considered the stability of rubble mound beneath caisson influenced by seepage flow. It can be said that under tsunami loading conditions, the seepage flow may play an important role on the stability such as piping phenomena and reduction of bearing capacity in rubble mound beneath caisson.

Therefore, the purpose of this paper is to experimentally reproduce the failure induced by the seepage flow in rubble mound beneath caisson and to explain the failure mechanism based on a simple theoretical approach in order to make clear of the significance of seepage flow in the design process of caisson type composite breakwaters against tsunami.

The section 4 of Kamaishi breakwater south dike, the deepest part in the whole breakwaters, was chosen as the research section. The illustration of it is shown in **Fig. 2**.

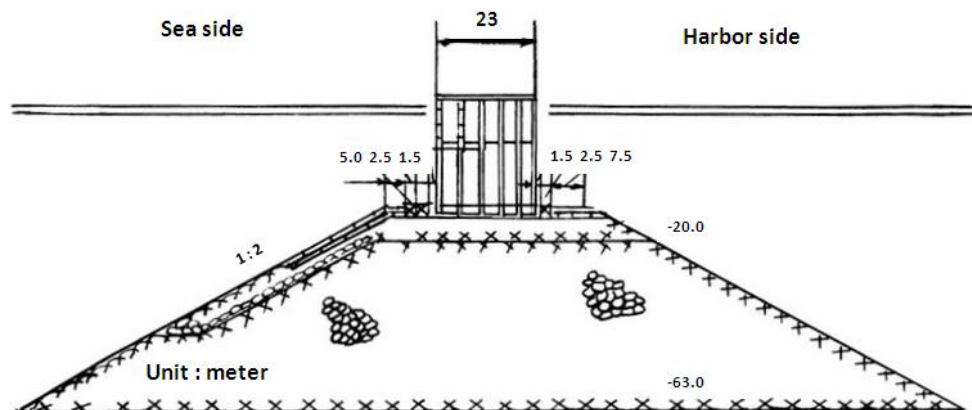


Fig. 2 Cross section of Kamaishi breakwater's southern dike, section 4.¹⁰⁾

From this study, a simplified failure criterion has been proposed to analyze the stability of the mound element under seepage condition. Two kinds of laboratory experiments have been planned to investigate the failure mechanism. **Figure 3** shows the research procedures.

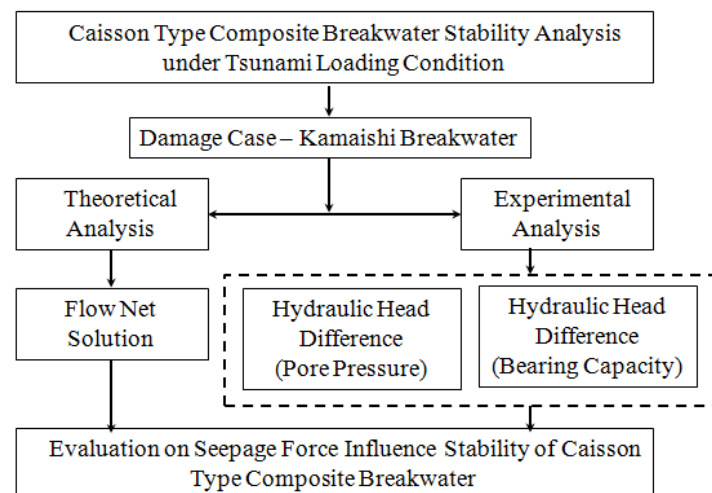


Fig. 3 Flow chart of stability analysis of caisson type composite breakwater.

2. Theoretical Analysis on “Pop-out” Failure

Theoretical analysis is made by using Darcy’s law with the following assumptions: (1) the rubble mound is fully saturated, homogeneous, and isotropic; (2) the flow is in steady state; (3) the fluid is incompressible; and (4) the hydraulic head is constant. Such a phenomenon is described by the Laplace equation:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0 \quad (1)$$

Where H is the hydraulic head and (x, y) are the Cartesian coordinates.

A common method for solving this equations in soil mechanics is the graphical technique of drawing flow nets where contour lines of hydraulic head and the stream function make a curvilinear grid, allowing complex geometries to be solved approximately. Since an equivalent amount of flow is passing through each element, the smallest squares in a flow net are located at points where the flow is concentrated.

The water goes through every element in the flow net and the upward seepage force reduces the effective force on the mound, resulting in lower mound shear strength. This paper adapted the basic concept of the “pop-out” failure which has been proposed by Budhu *et al.*¹¹⁾ in their study on slope stability problem under seepage force influence while following a different definition of the critical factor. The reason is that this case study is an underwater slope in a fully saturated condition but the former cases were slopes with free surfaces. The direction of flow follows the flow-line, and the seepage force direction can be described as the same direction. Acceptability criteria for the damage of a breakwater are represented as the following: to simplify the process of mound element failure by seepage force, a criteria equation is given as:

$$\gamma' \cos \theta < \gamma_w \frac{\Delta H}{N \times a} \quad (2)$$

where γ' is the rubble mound’s effective unit weight, θ is the slope angle, γ_w is the unit weight of water, ΔH is the hydraulic head difference between the seaside and the harbor side, N is the number of equipotential line, and a is the length of the unit square inside the flow net as shown in **Fig. 4**.

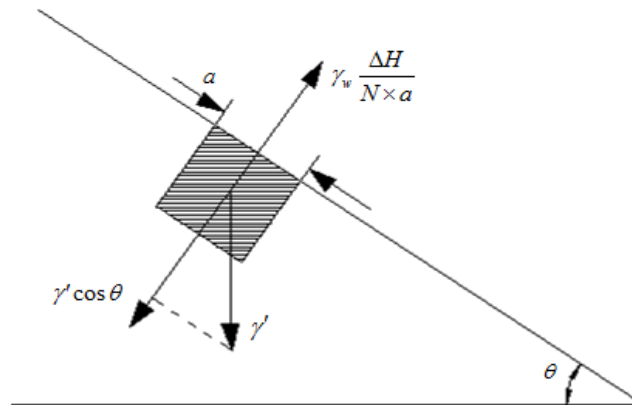


Fig. 4 Force decompose sketch of unite square on surface of rubble mound.

The critical factor of stability is defined as F_a and is shown in Eq. 3

$$F_a = \frac{\gamma' \cos \theta}{\gamma_w \Delta H / (N \times a)} \quad (3)$$

When $F_a < 1$, the unit square inside the rubble mound becomes instable.

Procedure of flow net solution:

- ① Illustrate the flow net;
- ② Get the length of “ a ” of each element along the sloped surface of the rubble mound (at the turning point of the crest, get the direct value of “ a ” instead of adding both lengths);
- ③ Calculate the value of F_a with Equation 3 with known slope angle, effective unit weight and hydraulic head difference;
- ④ Find the location where F_a is less than 1 which can indicate the point of failure on the rubble mound.

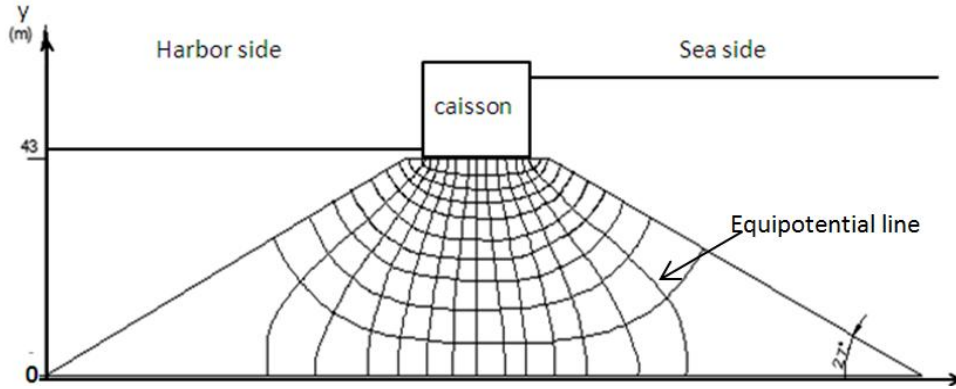


Fig. 5 Flow net illustration of Kamaishi breakwater.

Figure 5 applies the Kamaishi breakwater case data from the tsunami event and uses the theoretical method of calculation in order to find the failure area. The dimension data of Kamaishi breakwater were described as following: the caisson was 26 meters in width and 20 meters in height, slope angle of rubble mound was 27 degrees with the slope ratio equals to 1:2. The height of rubble mound was 43 meters. Hydraulic head difference from sea side towards harbor side was 13.64meters. The calculated data are given as $\gamma_w = 1 \text{ N/m}^3$; $\gamma' = 0.8 \text{ N/m}^3$; $\theta = 27^\circ$; $N = 14$;

$\Delta H = 13\text{m}$ and the value of “ a ” is obtained from **Fig. 5**.

In order to get a deeper understanding of the influence of seepage flow on the stability of the rubble mound, parametric studies were carried out to investigate the relationship between slope angle and the value of “ a ” as well as the relationship between hydraulic head difference and the value of “ a ”. The results were shown in **Fig. 6** and **Fig. 7**. In these two figures, the vertical axis represents the value of calculated F_a . The solid red lines shown in **Fig. 6** and **Fig. 7** imply that when F_a is equal to 1, the values of “ a ” above solid red line were the unstable areas. Therefore, the critical value of F_a can be obtained from the graphs. Major trends of the development of these values are concluded as: the value of F_a decreases when slope angle increases under the condition of constant hydraulic head difference. On the other hand, the stability of rubble decreases when hydraulic head difference increases for the same breakwater.

Furthermore, detailed results indicate that the areas of instability occur within 1.2m off the caisson toe on the harbor side along the rubble mound crest. According to Harry's¹²⁾ study on the seepage force induced instability problem, this reasonably small area might be the triggering factor

for the entire failure of rubble mound or it might play an integral role in the global stability when combined with wave forces. Therefore, two types of laboratory experiments were carried out to investigate the influence of seepage on the stability of caisson type breakwaters.

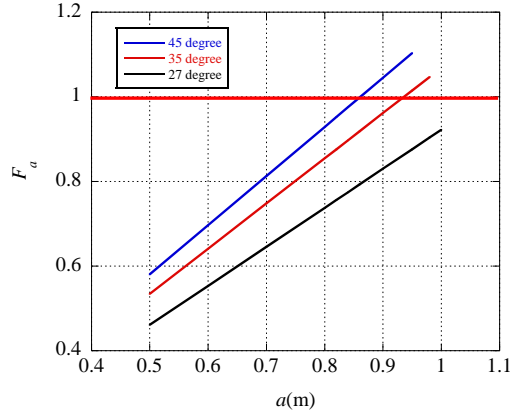


Fig. 6 Relationship between a and F_a with different slope angle (same hydraulic head difference).

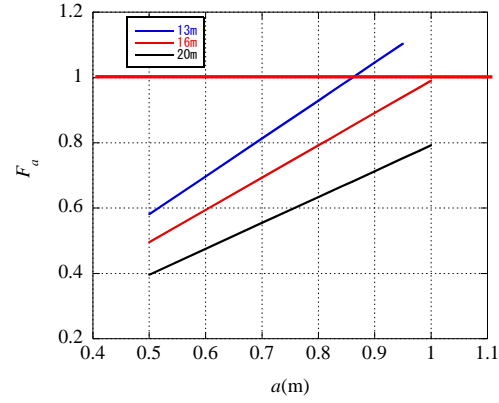


Fig. 7 Relationship between a and F_a with different hydraulic head differences (same slope angle).

3. Experiment on Seepage Flow-induced Instability

3.1 Outlines of experiment

A model for laboratory experiment was established by simulating the Kamaishi breakwater's southern dike, section 4 at a scale of 1/100. The rubble mound material is crushed stones of which D_{min} is 2mm, D_{max} is 35mm and D_{50} is 10mm. The dimensions of caisson are 195mm in height, 185mm in length and 190mm in width. The caisson was filled with sand. The density of sand is 2.03 g/cm^3 . In order to reproduce the effects of hydraulic head difference on caisson type breakwaters during tsunami waves, three pumps were introduced to create hydraulic head difference. The experiment layout was shown in **Fig. 8**. The cross section view of the caisson type composite breakwater located in water tank is shown in **Fig.8** as (a) cross section. A pump system was established to control the hydraulic head difference from the sea side towards the harbor side of the physical model. Legend in **Fig.8** shows the installations of measuring equipment. The plan view of the physical model is presented in Fig.8 as (b) plan view. All the equipments were set on the caisson which was located in the middle of tank. The two side caisson boxes among three caisson boxes were placed to avoid the friction effect along tank walls. All the dimensions are shown in **Fig. 8**.

The objective of the experiment is to observe the failure phenomena when hydraulic head difference is created. Water pressure gauges were embedded in the rubble mound to investigate the variation of pore pressure during the experiment, and wave height sensors were set on both sides of the caisson as shown in **Fig. 9**. The x-y coordinate was used as shown in **Fig.9** and initial point was also given in the graph for later coordinate reference. The experimental data were collected electronically and processed by computer.

It is very important to observe the area of instability along the rubble mound surface when hydraulic head difference is created. Video recording of the experiments is applied.

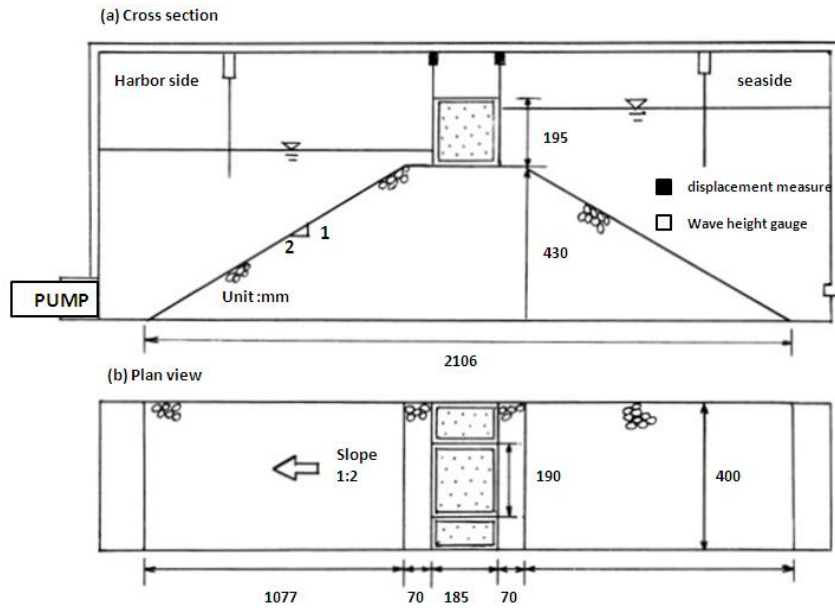


Fig. 8 Layout of experiment model in water tank (script).

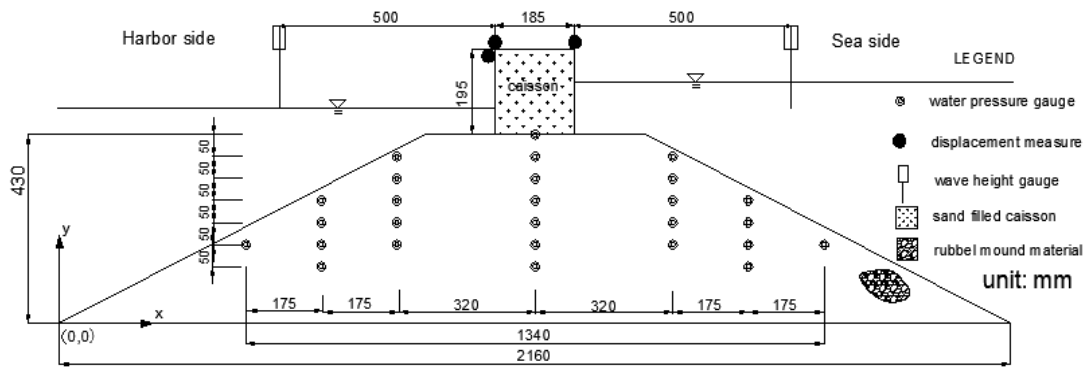


Fig. 9 Experiment equipments settings.

3.2 Results and discussions

Data from the above experiments were processed and utilized for calculation. In Fig.10, only the upper part of rubble mound in Fig.9 where the pore pressure gauges were installed area is demonstrated. The starting of x-axis is set 300mm from the left toe of rubble mound and y-axis is set 130mm from the left toe of rubble mound. From the pore pressure data obtained in the experiment, the hydraulic gradient distribution was illustrated as in Fig. 10 by Tecplot software¹⁴⁾. It represents that the hydraulic head difference generates great pore pressure from the seaside towards the harbor side, which might cause the instability of rubble mound.

In order to understand the influence of seepage force on the stability of the rubble mound, failure criteria are applied to the following analysis. The seepage force F can be decomposed into horizontal and vertical components by the following equations, and the influence of seepage force in each direction can be evaluated by the following equations.

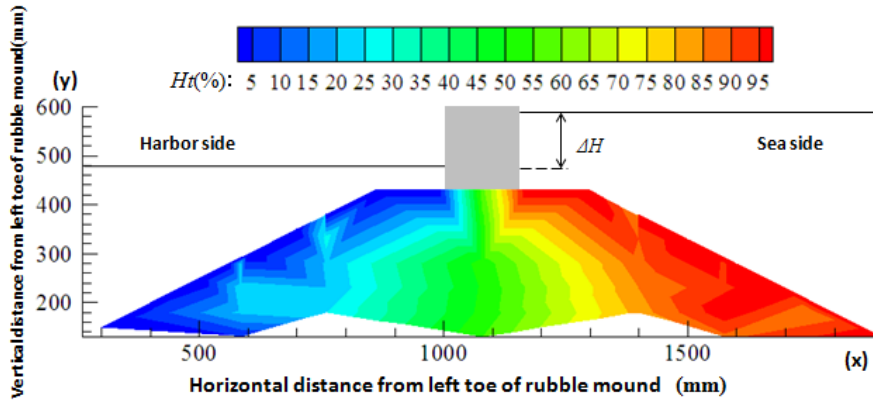


Fig. 10 Pore pressure distribution generated by the hydraulic head difference.

$$\left. \begin{aligned} F &= \sum (i \cdot \gamma_w) A \\ F_x &= \sum (i_x \cdot \gamma_w) A \\ F_y &= \sum (i_y \cdot \gamma_w) A \end{aligned} \right\} \quad (4)$$

Where i is the hydraulic gradient, A is the area of water traveling through, and the shear strength T_f under the caisson can be described as

$$T_f = c' \cdot B + (W' - F_y) \cdot \tan \phi' \quad (5)$$

Where B is the unit width of the rubble mound, c' is cohesion, ϕ' is the friction angle, A is the area of the rubble mound and W' is the effective overburden weight. In this calculation, because of the testing material is cohesion less, the value of c' is set zero.

Figure 11 and Fig. 12 show the calculation results from Eq. 4. From the plots, it can be concluded that the hydraulic gradient concentration is greater in the horizontal direction than the vertical direction with the same hydraulic head difference of 118mm.

F_{sx} is the factor of safety represented by Equation 6 for evaluating the failure potential inside the rubble mound in the horizontal direction, when the value of F_{sx} exceeds 1, the rubble mound becomes unstable.

$$F_{sx} = T_f / F_x \quad (6)$$

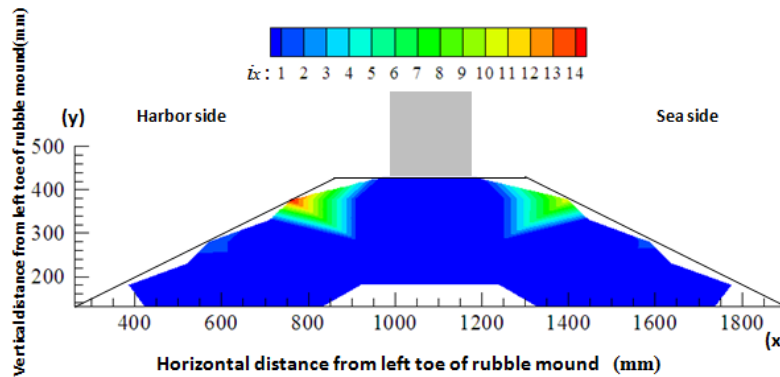


Fig. 11 Hydraulic gradient distribution in the horizontal direction.

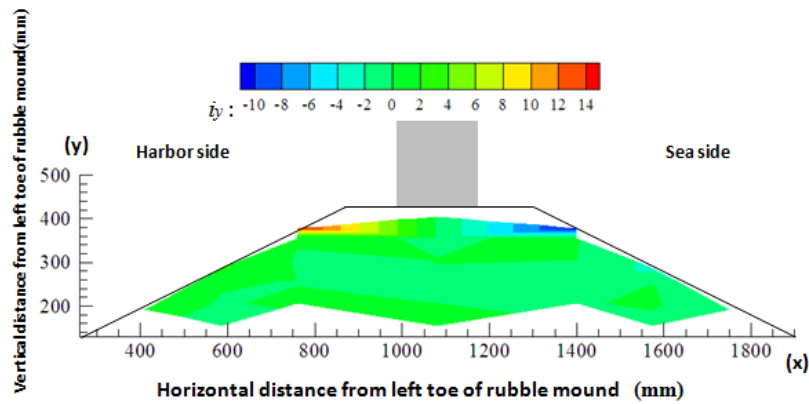


Fig. 12 Hydraulic gradient distribution in the vertical direction.

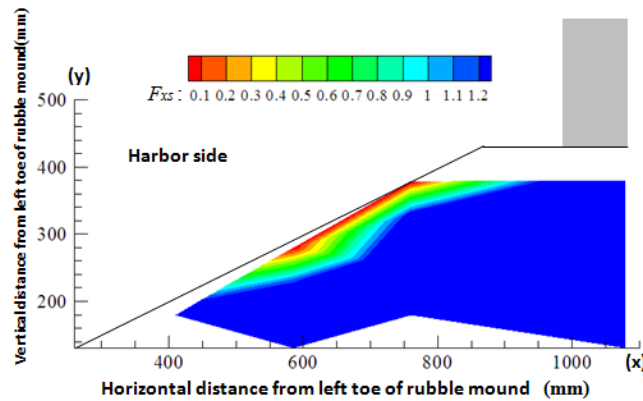


Fig. 13 F_{xs} calculated by failure criteria.

The calculation result is shown in **Fig.13**. It can be seen that when hydraulic head difference is 118mm, the rubble mound slope surface area with depth of 62.3mm has failure potential. This corresponds 6.2 m at full scale. However, in the laboratory modeling tests, the “pop out” failure due to seepage forces is hardly found. Although the hydraulic gradient shows a concentration on the surface of the rubble mound and F_{xs} have been dramatically reduced, it is less likely that the excessive pore-pressure (seepage force) can cause the catastrophic failure at hydraulic head difference of 118mm. This leads to the hypothesis that the combination of seepage force with downstream surface erosion caused by overtopping might be the major cause of caisson type breakwater failure. Detailed investigation and experimentation focused on this failure mechanism should be carried out. Besides the suggestions given above, the lack of failure observed under these wave conditions might be due to the selection of rubble mound material in this experiment. Finer rocks with smaller diameters should be tested in later experiments.

Even though the pore pressure test did not reproduce the failure, the seepage force effects on the stability of caisson type breakwaters still need to be investigated in more detail with respect to the bearing capacity of the rubble mound. The experiments on rubble mound bearing capacity are presented below.

4. Experiment on Bearing Capacity

4.1 Outlines of loading test

In order to find out the effects of seepage flow on the bearing capacity of the rubble mound during tsunami, the following tests have been carried out. The relationship between loading pressures and the displacement of caisson was observed in this experiment. The loading procedures were given as follows: firstly put loads in the center of caisson top, and then measured the displacement of caisson under different hydraulic head differences. Hydraulic head differences were set as 0mm, 50mm, 90mm and 118mm. Three pumps were used to adjust the hydraulic head difference from sea side to harbor side. Displacement data were collected for the later calculation and analysis in the following contexts.

4.2 Test results and discussions

When weight was loaded on the caisson, displacements of the caisson were observed, and these displacement data were collected at certain hydraulic head difference. The settlement values have been compared in **Fig. 14**.

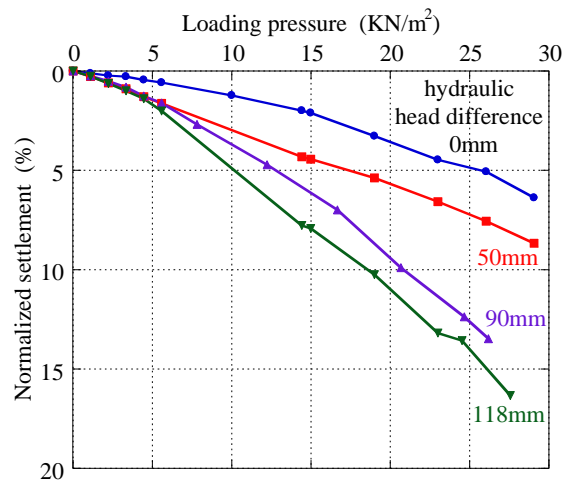


Fig. 14 Relationship between loading pressure and normalized settlement of caisson.

The settlement of caisson due to weight loading shows a trend: when loading increases, the settlement becomes larger. As expected, the larger hydraulic head difference influenced the bearing capacity of the rubble mound. The curves show a clear tendency that the settlement increases about 50% when hydraulic head difference increases from 0mm to 118mm. It is indicated that the displacement of caisson becomes larger when the hydraulic head difference becomes larger. It might trigger entire failure of rubble mound at a certain value of hydraulic gradient. In other words, it is found that the seepage force contributes to reduce the bearing capacity of rubble mound.

Figure 15 shows the relationship between the caisson incline angle and weight loading. When the hydraulic head difference stays constant, the loading value reaches 15 KN/m². The settlement of the caisson develops significantly when the loading increases to 25 KN/m², and the biggest hydraulic head difference (118mm) creates the largest settlement. In other words, the increase of hydraulic head difference has a negative influence on the stability of caisson and with such a big displacement.

The inclinations were also calculated from this experiment and from the results in **Fig. 15**. The deformation of the rubble mound becomes significant when hydraulic head difference increases, in this case to 118mm, and the caisson is prone to be less stable when hydraulic head difference is large.

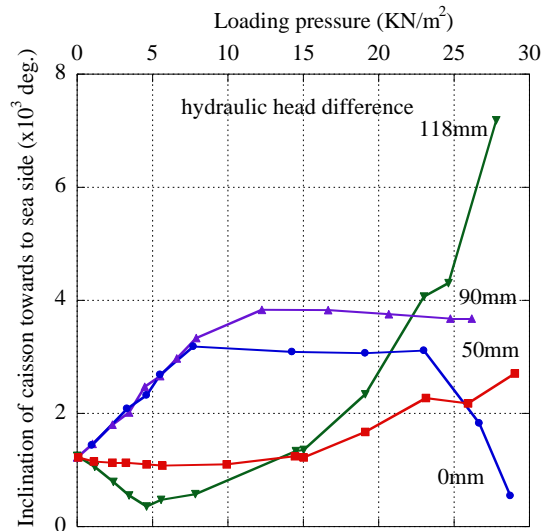


Fig. 15 Relationship between loading pressure and inclination of caisson towards to sea side.

The analysis of the results in **Fig. 14** and **Fig. 15** show that pore pressure generated by seepage flow brings the decreases of effective stress which may cause instability of the caisson. Therefore, it is necessary to calculate the entire stability of rubble mound to analyze the influence of seepage flow on the caisson type breakwaters.

5. Circular Failure Analysis

According to the existing calculation methods¹⁵⁾ for the bearing capacity of rubble mound, the modified Felleinus method was selected to evaluate the circular failure.

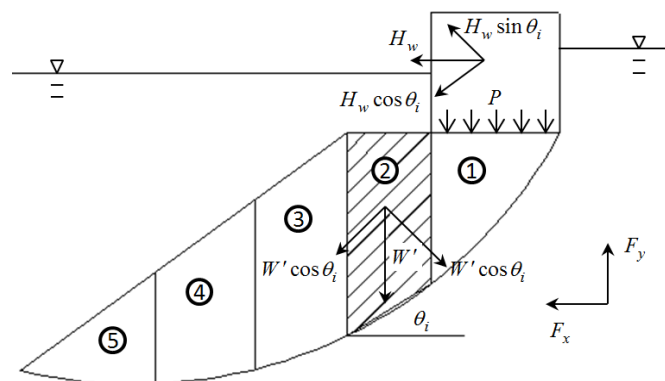


Fig. 16 Model of circular failure analysis.

The graphical viewing of computed details has led to a greater understanding of the method,

as illustrated in **Fig. 16**. It shows the calculation of characteristics of the slip surface on the rubble mound with mathematical equations given as follows:

$$\begin{aligned}
 R_i &= \left[(P+W') \cos \theta_i - H_w \sin \theta_i - F_x \sin \theta_i - F_y \cos \theta_i \right] \tan \phi' \\
 S_i &= (P+W') \sin \theta_i + H_w \cos \theta_i + F_x \cos \theta_i - F_y \sin \theta_i \\
 F_s &= \sum_{i=1}^n R_i / \sum_{i=1}^n S_i
 \end{aligned} \tag{7}$$

where P is the weight of the caisson itself, H_w is the horizontal water static load, F_x and F_y are the decomposed seepage force in the horizontal and vertical components respectively, the subscript i is the number of each slice of potential failure, θ_i is the angle of slice slip surface towards the horizontal direction, W' is the effective weight of the rubble mound, R_i is the resistance force and S_i is the shear strength. In these equations, when F_s is larger than one, the caisson is stable.

In order to get a clear understanding of the seepage force influence on caisson stability, further calculations were done and the results are shown in **Fig.17**. The locations of slip surface indicate the influence of seepage flow in rubble mound. By comparing with three conditions, the results show that the slip surface which induced by static horizontal force and seepage force is the shallowest line among three lines, the shear outlet of slip surface moved from the bottom of rubble mound towards the top of rubble mound significantly compared with conditions with seepage flow and without seepage flow in rubble mound.

6. Conclusions

The paper applies a fundamental theoretical approach and experimental approach to analyze the stability of caisson type composite breakwaters under tsunami condition.

Five conclusions are drawn from the study: (1) by flow-net graphic solution analysis, it can be concluded that when wave height increases, the area of instability in the rubble mound enlarges and when the rubble mound slope angle increases, the rubble mound becomes unstable due to seepage force, (2) A failure criteria is applied to analysis the stability of the rubble mound beneath caisson and seepage force acts in horizontal direction is emphasized in the data analysis, (3) pore pressure data were collected from experiment and calculated based on the proposed failure criteria. The results indicate the unstable area along the rubble mound, although failure was not observed in the real experiment, (4) the experiment results of the bearing capacity show the bearing capacity of rubble mound decreased by about 50% when affected by the horizontal seepage force in comparison with the condition without seepage flow, (5) from the slip surface calculation by the modified Felleinus method, the potential slip surface in the rubble mound under developed shallower when influenced by seepage force and horizontal loading from hydraulic head difference on sea side.

In conclusion, it can be said that when consider the composite breakwater design against tsunami, the seepage force should be taken into account as a significant influential factor.

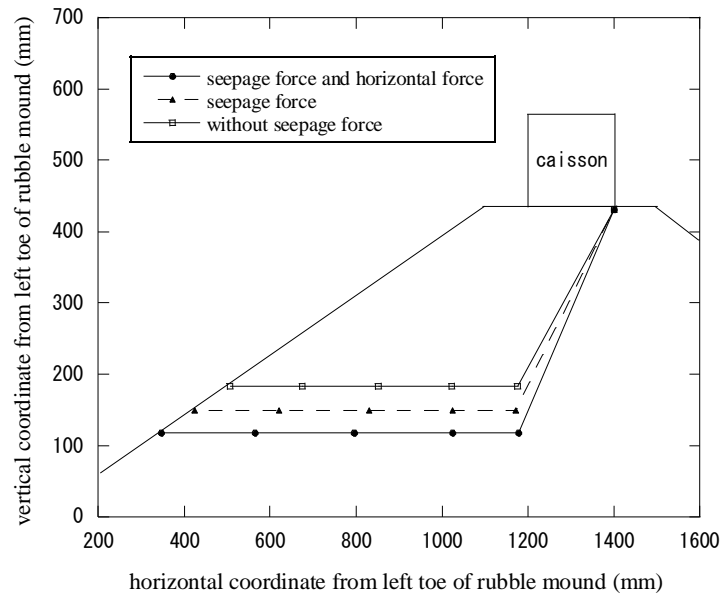


Fig.17 Comparison of slip surface calculation of rubble mound under tsunami loading and without seepage condition.

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Nomenclature

a	Unit length of rubble element on flownet
c'	Cohesion
F_a	Critical factor of stability
F_x	Seepage force decomposed in the horizontal direction
F_y	Seepage force decomposed in the vertical direction
F_{xs}	Factor of safety in x direction
H	Hydraulic head
H_w	Static water loading
ΔH	Hydraulic head difference
i	Hydraulic gradient
N	Number of equipotential line
R_i	Resistance force of unit slice
S_i	Shear strength of unit slice
γ'	Effective unit weight
θ	Slope angle of rubble mound
θ_i	Angle of slice slip surface towards the horizontal direction

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