

# Dynamic Programming Revisited for Aircraft Flight Trajectory Optimization

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## 論 文 内 容 の 要 旨

Establishment of a trajectory optimization technology for the practical use has been demanded in the various research fields of aeronautical and astronautical engineering. Researchers and engineers, on one hand, aspire for a numerical optimization method such that provides a solution accurate enough to be applied to the practical usages with a reasonable computational time and endures multiple analyses with the various design conditions. On the other hand, most conventional optimal control theory generally requires special skills in formulating the problem and computing the requisite quantities with the given design conditions. Furthermore, solving optimal control problems requires large amount of engineering resources to select the optimization tools, prepare data such as models and objective function, and set adjustable parameters in each tool.

In the optimal control theory which originates from the Brachistochrone problem solved by Johann Bernoulli in 1696, the calculus of variations is cited as a typical indirect method. Euler-Lagrange differential equations introduced from the variational principle and Pontryagin's maximum principle are solved as two-point boundary value problem. This method is capable to produce an exact solution; nevertheless, special techniques are required to solve the two-point boundary value problem and it's extremely difficult to get the solution with the highly nonlinear conditions. Moreover, it's also tough to implement a new formulation and parameter adjustments for every change of design conditions and consequently it consumes much development cost. In the meanwhile, direct methods, in which a finite number of parameters constitute the trajectory and are solved as a parameter optimization problem, have become often used with the assistance of processing capability of highly-sophisticated recent computers. In the various approaches, the gradient based method with generalized optimization tools is expected as a versatile optimization method in a point of practical applications; however, the unfavorable characteristics still remains such that a good convergence performance cannot be guaranteed in the iterative calculation process if the complicated models and inequality constraints are added.

For these reasons, a more easy to handle numerical optimization method is desired to satisfy the researchers' and engineers' demands. This research focuses on dynamic programming (DP) as a promising numerical optimization method and demonstrates its feasibility and versatility in the optimal control problems through practical application examples. DP is an optimization theory based on the Hamilton-Jacobi-Bellman (HJB) partial differential equation and is categorized as the direct method because the objective system is expressed by the quantized grid points of state variables and solved as a combinatorial optimization problem for those points. The calculation algorithm founded on the principle of optimality is decisive; therefore, a global optimality is guaranteed with the predictable computational time. Moreover, it's easy to handle inequality constraints on both state variables and control variables. The easiness to adapt for changing design conditions, models and parameters is also a great advantage of dynamic programming. Other optimization methods which have these many significant advantages at

the same time do not exist. On the other hand, DP's capability still remains not fully used due to the two major challenging drawbacks, the "Curse of dimensionality" and the "Menace of the expanding grid". The former one is the problem such that computational time and memory increase exponentially according to the number of state variables and number of their divisions. The latter one arises if the number of control variables is fewer than the number of state variables. These difficulties have prevented dynamic programming from being used in practical problems. However, DP's scope of application to practical usage is expected to be enlarged by the increasing amount of memory and processing capabilities of recent computers. The limit of the application area of DP should be identified to enhance the potential of the method.

The main objective of this research is to demonstrate the potential and versatility of dynamic programming by exploring its scope of application to the practical optimal control problems. Two novel methods, Piecewise Linear Approximation Dynamic Programming (PLA-DP) and Moving Search space Dynamic Programming (MS-DP) are introduced to alleviate the major drawbacks of dynamic programming and are applied to the practical problems. The validity and availability of the PLA-DP method are demonstrated by applying it to a passenger aircraft's fuel minimal trajectory design problem. In this example, the optimal control problem which has three state variables and two control variables has been successfully solved by PLA-DP method. The smooth optimal trajectory which is more preferable in the actual flight operation and is expected to benefit the developments for Next-Generation Flight Management System (NG-FMS) could be obtained by considering the aircraft dynamics explicitly in the governing equation. As another practical application example, a flight trajectory optimization tool developed with using MS-DP method is applied to the Air Traffic Management (ATM) research. The potential benefits of current Japanese airspace, which are the key issues to consider the Japanese future ideal ATM system, are quantitatively evaluated in the example by using the optimization tool and the passenger aircraft's actual position data recorded by Secondary Surveillance Radar (SSR). The statistical analysis results for 256 flights inbound to Tokyo International Airport have indicated possible savings for fuel consumption and flight time quantitatively. The results obtained by the application example have revealed potential benefits which may be achieved by improving the current air traffic management system. The results have also contributed to encourage further research into the Japanese future ATM modernization program. The MS-DP method has made a remarkable contribution to reduce computational burden in a large-scale optimization problem for multiple aircraft.

This research focuses on dynamic programming and explores its potential into the practical usages. The utilization of dynamic programming has been limited by the technical challenges for a long period even if it has many favorable advantages the other method does not have. This research has proposed two promising and powerful methods which may resolve those challenges, and has made a great contribution to enlarge DP's scope of application by solving practical trajectory optimization problems which include actual model, data and constraints given by various design requirements. The remarkable potential of dynamic programming has been indicated by the proposed methods and valuable new findings through the practical application examples.