Morse Reductions for Quiver Complexes and Persistent Homology on the Finite-Type Commutative Ladder Quivers

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論 文 名　：Morse Reductions for Quiver Complexes and Persistent Homology on the Finite-Type Commutative Ladder Quivers
（箙複体に対する離散モース縮約と有限型の可換梯子型パーシステント加群についての研究）

区　　分　：甲

論　文　内　容　の　要　旨

Persistent homology is a tool in topological data analysis for studying the robust topological features of data. The persistence diagram provides a compact way to summarize the presence, scale, and persistence of these features. Originally, persistent homology was defined for filtrations, and its algebraic structure explained as a graded module over a graded polynomial ring. The persistence diagram is provided by an indecomposable decomposition of the persistent homology module into the so-called interval modules. Each interval summand then tracks the lifespan of a homology generator through the filtration.

An insight of Carlsson and de Silva in their paper on zigzag persistence is that the algebraic foundations of the persistent homology of filtrations can be rephrased in terms of representations of quivers, in particular of A_n-type quivers, through which they developed the theory of zigzag persistence. In this thesis, we extend the algebraic analysis of persistent homology by using bound quivers and their representations, and by applying Auslander-Reiten theory.

A quiver is a directed graph, which we assume to be finite, acyclic, and connected. We give a definition of a quiver complex as a diagram of (simplicial) complexes and inclusions indexed by a quiver. This generalizes filtrations and zigzag complexes – settings where persistence analysis is already well developed. We then define the persistent homology of a quiver complex, which we show to be a representation of its quiver bound by commutativity relations. Motivated by applications, for example to studying the structure of amorphous glass, we focus on the so-called commutative ladder quivers $CL_n(\tau)$. In particular, we show that $CL_3(\tau)$ with orientation $\tau = fb$ can be used to study simultaneously robust and common features.

In this direction, we show that the commutative ladder quivers $CL_n(\tau)$ with length $n \leq 4$ are representation-finite by computing their Auslander-Reiten (AR) quivers. Their AR quivers are given in the Appendix. We show that there is a close relationship between classical persistence diagrams and the AR quivers of the A_n-type quivers. By this relationship, we provide a generalization of the definition of persistence diagrams.

In the representation-finite commutative ladder case, we show how to visualize our generalized persistence diagrams. By definition, the AR quiver has set of vertices all the isomorphism classes of the indecomposable representations. In our setting of $CL_n(\tau)$ with $n \leq 4$, this is a finite set, so that the domains of the persistence diagrams are finite. Thus we can easily visualize the persistence diagrams in these cases. Moreover, some methods and examples for the interpretation of the persistence diagrams in these cases are provided.

In a related but slightly different direction, we have the following. A paper by Mischaikow and Nanda uses discrete Morse theory for computing the persistent homology of filtrations efficiently. We extend these ideas to our setting of quiver complexes. In particular, given a quiver complex X and an acyclic matching for X, we show that there is an associated Morse quiver complex A with the property that X and A have isomorphic persistent homology. The Morse quiver complex A tends to be smaller in size, so that computing the persistent homology from A instead of X tends to be less costly. An algorithm to compute an acyclic matching for a quiver complex X and the associated Morse quiver complex A is given, by modification of an existing algorithm.

The computation of a persistence diagram follows from the computation of an indecomposable decomposition of a representation. While there exist general methods for this computation, we give an algorithm that uses only elementary linear algebra via changes of bases in the case of $CL_n(\tau)$ with $n = 3$. In the final chapter, we explore a reformulation of representations of $CL_n(\tau)$ via so-called matrix problems. By this link, we suggest an algorithm for computing their indecomposable decompositions via certain permissible operations on matrix problems. This will be expanded upon in a future work.