LECTURES AND ESSAYS

(continued)

INSTRUMENTS USED IN MEASUREMENT.[1]

By *Measurement*, for scientific purposes, is meant the measurement of *quantities*. In each special subject there are quantities to be measured ; and these are very various, as may be seen from the following list of those belonging to geometry and dynamics.

Geometrical Quantities.

Lengths
Areas
Volumes
Angles (plane and solid)
Curvatures (plane and solid)
Strains (elongation, torsion, shear).

Circumstances of Motion.

Time
Velocity
Momentum
Acceleration
Force
Work
Horse-power
Temperature
Heat.

Properties of Bodies.

Mass
Weight
Density
Specific gravity
Elasticity (of form and volume)
Viscosity
Diffusion
Surface tension
Specific heat.

[1] [‘Handbook to Loan Collection of Scientific Apparatus, 1876].

Notwithstanding the very different characters of these quantities, they are all measured by reducing them to the same kind of quantity, and estimating that in the same way. Every quantity is measured by finding a *length* proportional to the quantity, and then measuring this length. This will, perhaps, be better understood if we consider one or two examples.

The measurement of *angles* occurs in a very large majority of scientific instruments. It is always effected by measuring the *length of an arc* upon a graduated circle; the circumference of this circle being divided not into inches or centimetres, but into degrees and parts of a degree—that is, into aliquot parts of the whole circumference.

As a step towards their final measurement, some quantities, of which work is a good instance, are represented in the form of *areas*; and there seems reason to believe that this method is likely to be extended. Instruments for measuring areas are called Planimeters; and one of the simplest of these is Amsler's, consisting of two rods jointed together, the end of one being fixed and that of the other being made to run round the area which is to be measured. The second rod rests on a wheel, which turns as the rod moves; and it is proved by geometry that the area is proportional to the distance through which the wheel turns. Thus the measurement of an area is reduced to the measurement of a length.

Volumes are measured in various ways, but all depending on the same principle. Quantities of earth excavated for engineering purposes are estimated by a rough determination of the shape of the cavity, and the

measurement of its *dimensions*, namely, certain lengths belonging to it. The contents of a vessel are sometimes gauged in the same way; but the more accurate method is to fill it with liquid and then pour the liquid into a cylinder of known section, when the quantity is measured by the height of the liquid in the cylinder, that is, by a length. The volumes of irregular solids are also measured by immersing them in liquid contained in a uniform cylinder, and observing the height to which the liquid rises; that is, by measuring a length. An apparatus for this purpose is called a Stereometer. The liquid must be so chosen that no chemical action takes place between it and the solid immersed, and that it wets the solid, so that no air bubbles adhere to the surface. Thus mercury is used in the case of metals by the Standards Department.

Time is measured for ordinary purposes by the length of the arc traced out by a moving hand on a circular clock-face. For astronomical purposes it is sometimes measured by counting the ticks of a clock which beats seconds, and estimating mentally the fractions of a second; and in cases where the period of an oscillation has to be found, it is determined by counting the number of oscillations in a time sufficient to make the number considerable, and then dividing that time by the number. But by far the most accurate way of measuring time is by means of the line traced by a pencil on a sheet of paper rolled round a revolving cylinder, or a spot of light moving on a sensitive surface. If the pencil is made to move along the length of the cylinder so as to indicate what is happening as time goes along, the time of each event will be found when the cylinder is unrolled by measuring the distance of the mark recording it from the end of the unrolled sheet, provided that the rate at which the cylinder goes round is known. In this way Helmholtz measured the rate of transmission of nerve-disturbance.

A very common case of the measurement of *force* is the barometer, which measures the pressure of the atmosphere per square inch of surface. This is determined by finding the height of the column of mercury which it will support (mercurial barometer), or the strain which it causes in a box from which the air has been taken out (aneroid barometer). The height in the former case may be measured directly, or it may first be converted into the quantity of turning of a needle, and then read off as length of arc on a graduated circle; in the latter case the strain is always indicated by a needle turning on a graduated circle.

The *mass*, and (what is proportional to it) the *weight*, of different bodies at the same place, are measured by means of a balance; and at first sight this mode of measurement seems different from those which we have hitherto considered. For we put the body to be weighed in one scale, and then put known weights into the other until equilibrium is obtained or the scale turns, and then we count the weights. But in a steel-yard the weight is determined directly by means of a length; and in a balance which is accurate enough for scientific purposes, both methods are employed. We get as near as we can with the weights, and then the remainder is measured by a small rider of wire which is moved along the beam, and which determines the weight by its position; that is, by the measurement of a length.

For the measurement of weight in different places a spring-balance has to be used, and the weight is determined by the alteration it produces in the length of the spring; or else the length of the seconds pendulum is measured, from which the force of gravity on a given mass can be calculated. This last is an example of a very common and useful mode of measuring forces called into play by displacement or strain; namely, by measuring the period of the oscillations which they produce.

It seems unnecessary to consider any further examples, as all other quantities are measured by means of some simple geometrical or dynamical quantity which is proportional to them; as temperature by the height of mercury in a thermometer, heat by the quantity of ice it will melt (the volume of the resulting water), electric resistance by the length of a standard wire which has an equivalent resistance. It only remains to show how, when a length has been found proportional to the quantity to be measured, this length itself is measured.

For rough purposes, as for example in measuring the length of a room with a foot-rule, we apply the rule end on end, and count the number of times. For the piece left, we should apply the rule to it and count the number of inches. Or if we wanted a length expressed roughly for scientific purposes, we should describe it in metres or centimetres. But if it has to be expressed with greater accuracy, it must be described in hundredth, or thousandth, or millionth parts of a millimetre; and this is still done by comparing it with a scale.

But in order to estimate a length in terms of these very small quantities, it must be *magnified*; and this is done in three ways. First, geometrically, by what is called a vernier scale. This is a movable scale, which gains on the fixed one by one-tenth in each division. To measure any part of a division, we find how many divisions it takes the vernier to gain so much as that part; this is how many tenths the part is. The quantity to be measured is here geometrically multiplied by ten. Next, optically, by looking at the length and scale with a microscope or telescope. Third, mechanically, by a screw with a disc on its head, on which there is a graduated rim, called a micrometer screw. If the pitch of the screw is one-tenth and the radius of the disc ten times that of the screw, the motion is multiplied by one hundred. The two latter modes are combined together in an instrument called a micrometer-microscope. Another mechanical multiplier is a mirror which turns round and reflects light on a screen at some distance, as in Thomson's reflecting galvanometer.

Properly speaking, however, any description of a length by counting of standard lengths is imperfect and merely approximate. The true way of indicating a length is to draw a straight line which represents it on a fixed scale. And this is done by means of self-recording instruments, which measure lengths from time to time on a cylinder in the manner described above. It is only by this graphical representation of quantities that the laws of their variation become manifest, and that higher branch of measurement becomes possible which determines the nature of the connexion between two simultaneously varying quantities.

INSTRUMENTS ILLUSTRATING KINEMATICS, STATICS, AND DYNAMICS.

Science of Motion.

GEOMETRY teaches us about the sizes, the shapes, and the distances of things; to know sizes and distances we have to measure *lengths*, and to know shapes we have to measure *angles*. The science of *Motion*, which is the subject of the present sketch, tells us about the changes in these sizes, shapes, and distances which take place from time to time. A body is said to move when it changes its place or position; that is to say, when it changes its distance from surrounding objects. And when the parts of a body move relatively to one another, *i.e.* when they alter their distance from one another, the body changes in size, or shape, or both. All these changes are considered in the science of motion.

Kinematics.

The science of motion is divided into two parts: the accurate description of motion, and the investigation of the circumstances under which particular motions take place. The description of motion may again be divided into two parts, namely, that which tells us *what* changes of position take place, and that which tells us *when* and *how fast* they take place. We might, for example, describe the motion of the hands of a clock, and say that they turn round on their axes at the centre of the clock-face in such a way that the minute-hand always moves twelve times as much as the hour-hand; this is the first part of the description of the motion. We might go on to say that when the clock is going correctly, this motion takes place uniformly, so that the minute-hand goes round once in each hour; and this would be the second part of the description. The first part is what was called Kinematics by Ampère: it tells us how the motions of the different parts of a machine depend on each other in consequence of the machinery which connects them. This is clearly an application of geometry alone, and requires no more measurements than those which belong to geometry, namely, measurements of lines and angles. But the name Kinematics is now conveniently made to include the second part also of the description of motion—when and how fast it takes place. This requires in addition the measurement of *time*, with which geometry has nothing to do. The word Kinematic is derived from the Greek *kinēma*, 'motion;' and will therefore serve equally well to bear the restricted sense given it by Ampère, and the more comprehensive sense in which it is now used. And since the principles of this science are those which guide the construction not only of scientific apparatus, but of all instruments and machines, it may be advisable to describe in some detail the chief topics with which it deals.

Dynamics.

That part of the science which tells us about the circumstances under which particular motions take place is called *Dynamics.* It is found that the change of motion in a body depends on the position and state of surrounding bodies, according to certain simple laws;

when considered as so depending on surrounding bodies, the rate of change in the quantity of motion is called *force.* Hence the name Dynamic, from the Greek *dynamis*, 'force.' The word *force* is here used in a technical sense, peculiar to the science of motion; the connexion of this meaning with the meaning which the word has in ordinary discourse will be explained further on.

Statics and Kinetics.

Dynamics are again divided into two branches: the study of those circumstances in which it is possible for a body to remain at rest is called Statics, and the study of the circumstances of actual motion is called Kinetics. The simplest part of Statics, the doctrine of the Lever, was successfully studied before any other part of the science of motion, namely by Archimedes, who proved that when a lever with unequal arms is balanced by weights at the ends of it, these weights are inversely proportional to the arms. But no real progress could be made in determining the conditions of rest, until the laws of actual motion had been studied.

Translation of Rigid Bodies.

Returning, then, to the description of motion, or Kinematics, we must first of all classify the different changes of position, of size, and of shape, with which we have to deal. We call a body *rigid* when it changes only its position, and not its size or shape, during the time in which we consider it. It is probable that every actual body is constantly undergoing slight changes of size and shape, even when we cannot perceive them; but in Kinematics, as in most other matters, there is a great convenience in talking about only one thing at a time. So we first of all investigate changes of position on the assumption that there are no changes of size and shape; or, in technical phrase, we treat of the motion of rigid bodies. Here an important distinction is made between motion in which the body merely travels from one place to another, and motion in which it also turns round. Thus the wheels of a locomotive engine not only travel along the line, but are constantly turning round; while the coupling-bar which joins two wheels on the same side remains always horizontal, though its changes of position are considerably complicated. A change of place in which there is no rotation is called a *translation*. In a rotation the different parts of the body are moving different ways, but in a translation all parts move in the same way. Consequently, in describing a translation we need only specify the motion of any one particle of the moving body; where by a *particle* is meant a piece of matter so small that there is no need to take account of the differences between its parts, which may therefore be treated for purposes of calculation as a point.

We are thus brought down to the very simple problem of describing the motion of a point. Of this there are certain cases which have received a great deal of attention on account of their frequent occurrence in nature; such as Parabolic Motion, Simple Harmonic Motion, Elliptic Motion. We propose to say a few words in explanation of each of these.

Parabolic Motion.

The motion of a *projectile*, that is to say, of a body thrown in any direction and falling under the influence of gravity, was investigated by Galileo; and this is the first problem of Kinetics that was ever solved. We must confine ourselves here to a description of the motion, without considering the way in which it depends on the circumstance of the presence of the earth at a certain distance from the moving body. Galileo found that the path of such a body, or the curve which it traces out, is a parabola; a curve which may be described as the shadow of a circle cast on a horizontal table by a candle which is just level with the highest point of the circle.

It is convenient to consider separately the vertical and the horizontal motion, for in accordance with a law subsequently stated in a general form by Newton, these two take place in complete independence of one another. So far as its horizontal motion is concerned, the projectile moves uniformly, as if it were sliding on perfectly smooth ice; and, so far as its vertical motion is concerned, it moves as if it were falling down straight. The nature of this vertical motion may be described in two ways, each of which implies the other. First, a falling body moves faster and faster as it goes down; and the rate at which it is going at any moment is strictly proportional to the number of seconds which has elapsed since it started. Thus its downward velocity is continually being added to at a uniform rate. Secondly, the whole distance fallen from the starting-point is proportional to the *square* of the number of seconds elapsed ; thus, in three seconds a body will fall nine times as far as it will fall in one second. The latter of these statements was experimentally proved by Galileo ; not, however, in the case of bodies falling vertically, which move too quickly for the time to be conveniently measured, but in the case of bodies falling down inclined planes, the law of which he at first assumed, and afterwards proved to be identical with that of the other. The former statement, that the velocity increases uniformly, is directly tested by an apparatus known as Attwood's machine, consisting essentially of a pulley, over which a string is hung with equal weights attached to its ends. A small bar of metal is laid on one of the weights, which begins to descend and pull the other one up ; after a measured time the bar is lifted off, and then, both sides pulling equally, the motion goes on at the rate which had been acquired at that instant. The distance travelled in one second is then measured, and gives the velocity ; this is found to be proportional to the time of falling with the bar on.

The second statement, that the space passed over is proportional to the square of the number of seconds elapsed, is verified by Morin's machine, which consists of a vertical cylinder which revolves uniformly while a body falling down at the side marks it with a pencil. The curve thus described is a record of the distance the body had fallen at every moment of time.

Fluxions.

This investigation of Galileo's was in more than one aspect the foundation of dynamical science ; but not the least important of these aspects is the proof that either

of the two ways of stating the law of falling bodies involves the other. Given that the distance fallen is proportional to the square of the time, to show that the velocity is proportional to the time itself ; this is a particular case of the problem. Given where a body is at every instant, to find how fast it is going at every instant. The solution of this problem was given by Newton's Method of Fluxions. When a quantity changes from time to time, its *rate* of change is called the *fluxion* of the quantity. In the case of a moving body the quantity to be considered is the distance which the body has travelled ; the fluxion of this distance is the rate at which the body is going. Newton's method solves the problem, Given *how big* a quantity is at any time, to find its fluxion at any time. The method has been called on the Continent, and lately also in England, the Differential Calculus ; because the difference between two values of the varying quantity is mentioned in one of the processes that may be used for calculating its fluxion. The inverse problem, Given that the velocity is proportional to the time elapsed, to find the distance fallen, is a particular case of the general problem, Given how fast a body is going at every instant, to find where it is at any instant ; or, Given the fluxion of a quantity, to find the quantity itself. The answer to this is given by Newton's Inverse Method of Fluxions ; which is also called the Integral Calculus, because in one of the processes which may be used for calculating the quantity, it is regarded as a whole (integer) made up of a number of small parts. The method of Fluxions, then, or Differential and Integral Calculus, takes its start from Galileo's study of parabolic motion.

Harmonic Motion.

The ancients, regarding the circle as the most perfect of figures, believed that circular motion was not only *simple*, that is, not made up by putting together other motions, but also *perfect*, in the sense that when once set up in perfect bodies it would maintain itself without external interference. The moderns, who know nothing about perfection except as something to be aimed at, but never reached, in practical work, have been forced to reject both of these doctrines. The second of them, indeed, belongs to Kinetics, and will again be mentioned under that head. But as a matter of Kinematics it has been found necessary to treat the uniform motion of a point round a circle as compounded of two oscillations. To take again the example of a clock, the extreme point of the minute-hand describes a circle uniformly; but if we consider separately its vertical position and its horizontal position, we shall see that it not only oscillates up and down, but at the same time swings from side to side, each in the same period of one hour. If we suppose a button to move up and down in a slit between the figures XII and VI, in such a way as to be always at the same height as the end of the minute-hand, this button will have only one of the two oscillations which are combined in the motion of that point; and the other oscillation would be exhibited by a button constrained to move in a similar manner between the figures III and IX, so as always to be either vertically above or vertically below the extreme point of the minute-hand. The laws of these two motions are identical, but they are so timed that each is at its extreme position when the other is crossing the centre. An oscillation of this kind is called a *simple harmonic motion*: the name is due to Sir William Thomson, and was given on account of the intimate connexion between the laws of such motions and the theory of vibrating strings. Indeed, the harmonic motion, simple or compound, is the most universal of all forms; it is exemplified not only in the motion of every particle of a vibrating solid, such as the string of a piano or violin, a tuning-fork, or the membrane of a drum, but in those minute excursions of particles of air which carry sound from one place to another, in the waves and tides of the sea, and in the amazingly rapid tremor of the luminiferous ether which, in its varying action on different bodies, makes itself known as light or radiant heat or chemical action. Simple harmonic motions differ from one another in three respects; in the extent or *amplitude* of the swing, which is measured by the distance from the middle point to either extreme; in the *period* or interval of time between two successive passages through an extreme position; and in the time of starting, or *epoch*, as it is called, which is named by saying what particular stage of the vibration was being executed at a certain instant of time. One of the most astonishing and fruitful theorems of mathematical science is this; that every *periodic* motion whatever, that is to say, every motion which exactly repeats itself again and again at definite intervals of time, is a compound of simple harmonic motions, whose periods are successively smaller and smaller aliquot parts of the original period, and whose amplitudes (after a certain number of them) are less and less as their periods are more rapid. The 'harmonic' tones of a string, which are always heard along with the fundamental tone, are a particular case of these constituents. The theorem was given by Fourier in connexion with the flow of heat, but its applications are innumerable, and extend over the whole range of physical science.

The laws of combination of harmonic motions have been illustrated by some ingenious apparatus of Messrs. Tisley and Spiller, and by a machine invented by Mr. Donkin; but the most important practical application of these laws is to be found in Sir W. Thomson's Tidal Clock, and in a more elaborate machine which draws curves predicting the height of the tide at a given port for all times of the day and night with as much accuracy as can be obtained by direct observation. One special combination is worthy of notice. The union of a vertical vibration with a horizontal one of half the period gives rise to that figure of 8 which M. Marey has observed by his beautiful methods in the motion of the tip of a bird's or insect's wing.

Elliptic Motion.

The motion of the sun and moon relative to the earth was at first described by a combination of circular motions; and this was the immortal achievement of the Greek astronomers Hipparchus and Ptolemy. Indeed, in so far as these motions are periodic, it follows from Fourier's theorem mentioned above that this mode of description is mathematically sufficient to represent them; and astronomical tables are to this day calculated by a method which practically comes to the same thing. But this representation is not the simplest that can be found; it requires theoretically an infinite

number of component motions, and gives no information about the way in which these are connected with one another. We owe to Kepler the accurate and complete description of planetary or elliptic motion. His investigation applied in the first instance to the orbit of the planet Mars about the sun, but it was found true of the orbits of all planets about the sun, and of the moon about the earth. The path of the moving body in each of these motions is an ellipse, or oval shadow of a circle, a curve having various properties in relation to two internal points or foci, which replace as it were the one centre of a circle. In the case of the ellipse described by a planet, the sun is in one of these foci; in the case of the moon, the earth is in one focus. So much for the geometrical description of the motion. Kepler further observed that a line drawn from the sun to a planet, or from the earth to the moon, and supposed to move round with the moving body, would sweep out equal areas in equal times. These two laws, called Kepler's first and second laws, complete the kinematic description of elliptic motion; but to obtain formulæ fit for computation, it was necessary to calculate from these laws the various harmonic components of the motion to and from the sun, and round it; this calculation has much occupied the attention of mathematicians.

The laws of rotatory motion of rigid bodies are somewhat difficult to describe without mathematical symbols, but they are thoroughly known. Examples of them are given by the apparatus called a gyroscope, and the motion of the earth; and an application of the former to prove the nature of the latter, made by

Foucault, is one of the most beautiful experiments belonging entirely to dynamics.

Rotation.

Next in simplicity after the *translation* of a rigid body, come two kinds of motion which are at first sight very different, but between which a closer observation discovers very striking analogies. These are the motion of rotation about a fixed point, and the motion of sliding on a fixed plane. The first of these is most easily produced in practice by what is well known as a ball-and-socket joint; that is to say, a body ending in a portion of a spherical surface which can move about in a spherical cavity of the same size. The centre of the spherical surface is then a fixed point, and the motion is reduced to the sliding of one sphere inside another. In the same way, if we consider, for instance, the motion of a flat-iron on an ironing-board, we may see that this is not a pure translation, for the iron is frequently turned round as well as carried about; but the motion may be described as the sliding of one plane upon another. Thus in each case the matter to be studied is the sliding of one surface on another which it exactly fits. For two surfaces to fit one another exactly, in all positions, they must be either both spheres of the same size, or both planes; and the latter case is really included under the former, for a plane may be regarded as a sphere whose radius has increased without limit. Thus, if a piece of ice be made to slide about on the frozen surface of a perfectly smooth pond, it is really rotating about a fixed point at the centre of the earth; for the frozen surface may be regarded as part of an

enormous sphere, having that point for centre. And yet the motion cannot be practically distinguished from that of sliding on a plane.

In this latter case it is found that, excepting in the case of a pure translation, there is at every instant a certain point which is at rest, and about which as a centre the body is turning. This point is called the instantaneous centre of rotation; it travels about as the motion goes on, but at any instant its position is perfectly definite. From this fact follows a very important consequence; namely that every possible motion of a plane sliding on a plane may be produced by the *rolling* of a curve in one plane upon a curve in the other. The point of contact of the two curves at any instant is the instantaneous centre at that instant. The problems to be considered in this subject are thus of two kinds: Given the curves of rolling to find the path described by any point of the moving plane; and, Given the paths described by *two* points of the moving plane (enough to determine the motion) to find the curves of rolling and the paths of all other points. An important case of the first problem is that in which one circle rolls on another, either inside or outside; the curves described by points in the moving plane are used for the teeth of wheels. To the second problem belongs the valuable and now rapidly increasing theory of *link-work*, which, starting from the wonderful discovery of an exact parallel motion by M. Peaucellier, has received an immense and most unexpected development at the hands of Professor Sylvester, Mr. Hart, and Mr. A. B. Kempe.

Passing now to the spherical form of this motion,