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Abstract

The paper deals with a statistical analysis aiming at quantitative characterization in the frequency domain of the strength of one-way effcts and reciprocity between a pair of series in the presence of a third series, suggesting a unified frequency-domain method of statistical estimation and testing for the proposed partial measures of interdependency. In particular, the paper provides an estimation procedure of those measures based on a numerical canonical factorization method of spectral densities and proposes Monte Carlo Wald tests for those measures. The method is applicable to data generated by the stationary multivariate ARMA process.

JEL classification: C12, C13, C32, C53

Key Words: Canonical factorization; Granger non-causality; Measure of one-way effect; Measure of reciprocity; Partial interdependency; Prediction error; Vector ARMA model; Wald statistics.

1 Introduction

Detecting between a pair of time series causal directions and the extent of their effects and also testing non-existence of feedback relation between them constitute major focal points in multivariate time-series analysis since Granger (1963, 69) introduced the celebrated definition of causality in view of prediction improvement. Although the Granger causality concept allows straightforward interpretation as long as it is focused on a pair of time

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series, the presence of a third series incurs interpretative complexities, since the third series may produce such confounding phenomena as spurious or indirect causality; see Granger (1980) and Hsiao (1982). For a proper graphical causal analysis to be conducted for a given set of time-series, we need a general methodology to identify the data generating process of those series as well as to estimate and test the extent of causality between a pair of vector processes in the presence of a third vector process.

It is a merit of time-series interdependency analysis that it can characterize relations by means of such frequency-domain terms as long-run or short-run effects. With respect to interdependency (or causal dependency) analysis in the frequency domain, there are studies by Gel'fand and Yaglom (1959), Granger (1969), Sims (1972), Geweke (1982, 1984) and Hosoya (1991, 2001) and Granger and Lin (1995). Hosoya (1991) introduced the concept of the measure of a series $\{y(t)\}$ one-sidedly effecting another $\{x(t)\}$ which is defined in terms of prediction improvement of $\{x(t)\}$ due to the addition of the past values of the one-way effect component of $\{y(t)\}$, whereas Geweke defined the feedback measure in terms of the improvement due to addition of the past values of $\{y(t)\}$ as a whole. Hosoya's measure has the merit that the equivalence relationship is established between the overall one-way effect measure of $\{y(t)\}$ to $\{x(t)\}$ and the integral of its associate frequency-wise measure; see the proposition E.1 of Appendix A.1. Frequency-domain analyses of causality seem more informative than time-domain counterparts, since it enables us not only to conduct significance testing of the Granger non-causality, but also to measure frequency-wise causal strength and to construct a variety of confidence intervals of those measures; see Hosoya (1997a), Yao and Hosoya (2000) and Hosoya, Yao and Takimoto (2005) for large-sample Wald tests of the simple measures of interdependency and the allied confidence set construction.

To deal with the third series presence problem, Hosoya (2001) characterized, in the frequency domain, the causal effect which one series produces onto another in the presence of a third series, introducing the idea of elimination from the pair of the subject-matter series the one-way effect of third series, and provided representations of the partial measures of interdependency based on the first approach of the two different approaches given respectively in Sections 2 and 3 of Hosoya (1991). The 2001 paper, however, assumed that a canonical factor of the joint spectral density in question is somehow available thanks to the assumed Szegö condition (2.1) given below, and did not discuss how to arrive at numerically such a factor nor how to conduct statistical inference. The contribution of this paper is that, focused on the stationary vector ARMA model, it provides a computationally executable numerical procedure to estimate and test the partial measures of interdependency.

The procedure of this paper is based on the second approach of Hosoya (1991) which turns out to be more suited for the VARMA model. Recent literature suggests that the VARMA model has better forecasting ability than the VAR model as far as macroeconomic time-series are concerned; see, for example, Athanasopoulos and Vahid (2008). To be specific, this paper gives a representation of the joint spectral density matrix of the pair of processes in which the third series one-way effect is eliminated, as well as the accompanying canonical factor matrix in numerically manageable forms respectively, so that the factorization algorithm of Hosoya-Takimoto (2010) for multivariate MA spectral matrices is usefully applied without much computational load. Based on the numrical estimation of the measures, the paper proposes an inference procedure using a type of Monte Carlo Wald test for the purposes of testing the extent of the partial measures of interdependency.

As a precursory study, Breitung and Candelon (2006) proposes a numerically practicable test, proposing an eclectic approach in order to test the null hypothesis of partial one-way effect being equal to zero. Their approach is to estimate an AR-DL (autoregressive distributed-lag) single-equation model involving the distributed lags of the causing series as well as a third-series or its one-way component of a third series and use the frequency-response function generated by the DL coefficient estimates of the causing series for the purpose of testing the frequency-wise non-causality; see Breitung and Candelon (2006, p.369) and also for an allied empirical study Gronwald (2009). While their F-test approach is computationally simpler, it does not include testing a specified non-null value of those measures, nor extends to higher dimensional models; see Remark 3.2.

In contrast to Breitung and Candelon (2006) or Hosoya (1991, 2001) who did not address development of statistical estimation procedures of partial measures of interdependency, this paper evaluates the partial measures of interdependency in the frequency domain exploiting the "full information" of the observed series, in the sense that we construct the partial measures of interdependency based on the the full generating system of the observation time series involved and estimate those measures using the estimate of the parameter of the full system.

Spectral density matrix characterizes the frequency-wise dependency between individual series constituting a multivariate time-series. Although it expresses the covariance between the frequency-wise spectrally decomposed factors of respective time-series, it does not characterize the time-lag or time-lead dependencies between the series involved. For the latter analysis, the spectrum knowledge is not enough, but the knowledge of canonical factor matrix of the density matrix and the prediction theory based on the factor are needed. Consequently, canonical factorization of spectral density matrix constitutes a crucial step in the construction of predictors and in the evaluation of the prediction contents, having thus a variety of applications in time series analysis and control; Hosoya and Takimoto (2010) surveys the allied literature. Since the Granger concept is framed on the basis of the prediction error evaluation, the factorization procedure also constitutes an essential step in constructing allied measures. Rational spectrum estimation based on a set of finite observations has a wide range of applications in time-series analysis and it is often conducted based on a time-domain ARMA representation of the data generating process; see for typical examples Hannan-Rissanen (1982) and Hannan-Kavalieris (1984). The ARMA models fitting in the time domain automatically estimates a transfer function (or a canonical frequency-response function) of the data generating process. In case the spectral density to be used does not correspond to the direct observation process but to a derived one, however, a certain factorization algorithm for rational spectra is required. In particular, the construction of the measures introduced by Hosoya (1991, 2001) requires canonical factorization of spectrum which is not necessarily obtained directly from the observation process.

The paper is organized as follows: Section 2 overviews the partial measures of interdependency which consist of the partial measures of one-way effect, reciprocity and association. Theorem 2.1 shows that the frequency-wise measure of reciprocity is constant over the whole frequency domain if the spectral density matrix of pair of the process in interest is canonically factorizable. The procedure presented in Section 2.2 to derive the partial measures is based on the second approach of Hosoya (1991). In view of Theorem 2.2 of the present paper, the second approach turns out to be more suited for representing the partial measures of interdependency for the VARMA model. Section 3 provides an inferential procedure for quantities related to the partial measures and allied confidence set constructions. Based on the VARMA model, Section 3.1 gives a representation of the joint spectral density matrix of the pair of processes in interest, in which third series one-way effect is eliminated. To derive the required canonical factor matrix, we show how the factorization algorithm of Hosoya-Takimoto (2010) for multivariate MA spectral matrices is applied. Based on a canonical factor matrix, we can proceed to the construction of the partial measures. Also the same Section 3.1 shows that the canonical factorization of the general ARMA model is reducible to that of a vector finite-order MA spectrum, thanks to Theorem 2.2. Section 3.2 explains Monte Carlo Wald testing of the measures. Appendix A.1 collects the definitions of the partial interdependency measures used in the paper and also exhibits equations which hold between those measures. Since the proofs can be carried out in parallel to Hosoya (1991, 2001), they are omitted. Appendix A.2 gives the proofs of Theorems 2.1 and 2.2. Appendix A.3 provides an explicit representation of the joint spectral density of the reciprocal components for a pair of series.

The paper uses the following notations and symbols: The sets of all integers and nonnegative integers are denoted respectively by $\mathbb{Z}$, $\mathbb{Z}^{0+}$. For a random-vector x or a pair of random vectors x and y, $Cov(x)$ and $Cov(x, y)$ denote respectively the covariance matrices of x and $vec(x, y)$. The determinant of a square matrix C is written as $\det C$. The identity matrix of order p is written as I_p. If A is a complex-valued matrix, A' and A^* denote respectively the transpose and conjugate transpose matrix of A. Definition (or equivalence) is indicated by $\equiv$. Suppose that a real sequence $c[j],\ j = -a, \cdots, a$ satisfies the condition $c[j] = c[-j],\ c[0] > 0$ and that $c(z) = \sum_{j=-a}^{a} c[j]z^j$ is nonnegative for $z = e^{-i\lambda}$ $(-\pi < \lambda \leq \pi)$. Then there exists a real sequence $b[j]$ $(j = 0, \cdots, a)$ such that $b(z) = \sum_{j=0}^{a} b[j]z^j$ does not have zeros inside the unit circle and the relation $c(z) = \frac{1}{2\pi}b(z)b(z^{-1})$ holds. Such a factorization is said to be canonical and $b(z)$ is said to be a canonical factor of $c(z)$. If $b_0 > 0$, the factorization is unique. See also Appendix A.1 for additional explanation of notations and symbols used in the paper.

2 Partial measures of interdependency

2.1 Elimination of a third-series effect

Let $x(t), y(t), z(t)$ be respectively real random p_1, p_2, p_3-vectors and suppose that the process $\{x(t), y(t), z(t); t \in \mathbb{Z}\}$ jointly constitutes a second-order stationary process. This subsection describes how to derive from a pair of processes $\{x(t), y(t)\}$ the pair $\{u(t), v(t)\}$ which is free from the one-way effect component of the third series $\{z(t)\}$.

Denote by H the Hilbert space generated from the set of the components of $\{x(t), y(t), z(t); t \in \mathbb{Z}\}$ For brevity, we write the sub-Hilbert spaces of H $H\{x(t_1 - j), y(t_2 - j), z(t_3 - j);\ j \in \mathbb{Z}^{0+}\}$ as $H\{x(t_1), y(t_2), z(t_3)\}$ and $H\{x(j); j \in \mathbb{Z}\}$ as $H\{x(\infty)\}$. For a random vector $x(t)$ indexed by t, $\{x(t)\}$ denotes the process $\{x(t); t \in \mathbb{Z}\}$ unless otherwise specified. Translating the linear prediction problems to those of the projections onto Hilbert subspaces is a standard technique in the theory of stationary stochastic processes; see Rozanov(1967) or other related literature of stationary processes. This paper mostly follows the standard notations and the basic framework of the theory, but the concept of the one-way effect requires projections of random vectors on special Hilbert subspaces.

Following the conventional practice of the theory of stationary processes, we identify an information set of variables with the Hilbert subspace generated by the set of variables and thus identify the linear prediction of a variable by means of the information set of predictor variables with the projection of the predicted variable onto the Hilbert space generated by the predictor variables, where the projection is the best linear predictor and the accompanying prediction error is said the residual (or perpendicular) of the projection. The third-effect elimination we propose is the elimination of the one-way effect of the third series from a pair of series in focus. In the case of the three series system $\{x(t), y(t), z(t)\}$, the one-way effect component of $z(t)$ (which is demoted by $z_{0,0,-1}(t)$) is the projection residual of $z(t)$ onto the subspace generated by $\{x(s), y(s), z(s-1),\ -\infty < s \leq t\}$; namely, it is the component of $z(t)$ which is unpredictable by the information set consisting of $x(s), y(s)$ up to time period t and $z(s)$ up to time period $t-1$.

We identify the partial relations of interdependency between $\{x(t)\}$ and $\{y(t)\}$ in the presence of a third series with the corresponding simple relations between $\{u(t)\}$ and $\{v(t)\}$ which are obtained respectively by the projection residuals of $x(t)$ and $y(t)$ onto $H\{z_{0,0,-1}(\infty)\}$ which is equivalent to the projection residuals onto $H\{z_{0,0,-1}(t-1)\}$. Hence the partial measures between $\{x(t)\}$ and $\{y(t)\}$ are defined to be the corresponding simple measures between $\{u(t)\}$ and $\{v(t)\}$. To distinguish an interdependency concept which focuses on a pair of processes alone from the partial version which takes account of a third series, the former concept is said simple in the sequel. Hence the simple causality, in the paper, means the one which does not take third series into account.

Let $f(\lambda)$ be the joint spectral density of the process $w(t) = (x(t)^*, y(t)^*, z(t)^*)^*$, $t \in \mathbb{Z}$ and suppose that $f(\lambda)$ satisfies the Szegö condition

$$\int_{-\pi}^{\pi} \log\det f(\lambda) d\lambda > -\infty, \tag{2.1}$$

then the density has the factorization

$$f(\lambda) = \frac{1}{2\pi} \Lambda(e^{-i\lambda}) \Lambda(e^{-i\lambda})^* \tag{2.2}$$

by means of a $(p_1 + p_2 + p_3) \times (p_1 + p_2 + p_3)$ matrix $\Lambda(z)$ which is analytic and of full rank inside the unit disc. Namely, in (2.2), $\Lambda(e^{-i\lambda})$ is the boundary value of the analytic function

$$\Lambda(z) = \sum_{j=0}^{\infty} \Lambda[j] z^j$$

with the real matrix coefficients $\Lambda[j]$. Such a factorization is said to be canonical in the sequel; see Rozanov (1967, pp.71-77) and Hannan (1970, pp.157-163). Let $\varepsilon(t) \equiv (\varepsilon_1(t)^*, \varepsilon_2(t)^*, \varepsilon_3(t)^*)^* \equiv w_{-1}(t) \equiv (x_{-1,-1,-1}(t)^*, y_{-1,-1,-1}(t)^*, z_{-1,-1,-1}(t)^*)^*$ be the one-step ahead prediction-error of the process $w(t)$ by its past values. Denote the covariance matrix of $\varepsilon(t)$ by $\Sigma^\dagger$ and denote the partition matrix as

$$\Sigma^\dagger = \begin{bmatrix} \Sigma^\dagger_{\cdot\cdot} & \Sigma^\dagger_{\cdot 3} \\ \Sigma^\dagger_{3\cdot} & \Sigma^\dagger_{33} \end{bmatrix}.$$

Then the residual of the projection of $\varepsilon_3(t)$ on the linear space spanned by $\varepsilon_\cdot(t) \equiv (\varepsilon_1(t)^*, \varepsilon_2(t)^*)^*$ is given by $\varepsilon_3(t) - \Sigma^\dagger_{3\cdot} {\Sigma^\dagger_{\cdot\cdot}}^{-1} \varepsilon_\cdot(t)$ and it constitutes the one-way effect component of $z(t)$. For normalizing $Cov(\varepsilon_\cdot(t), \varepsilon_3(t) - \Sigma^\dagger_{3\cdot} {\Sigma^\dagger_{\cdot\cdot}}^{-1} \varepsilon_\cdot(t))$, define

$$\begin{bmatrix} \varepsilon^\dagger_\cdot(t) \\ \varepsilon^\dagger_3(t) \end{bmatrix} = \begin{bmatrix} {\Sigma^\dagger_{\cdot\cdot}}^{-1/2} & 0 \\ 0 & (\Sigma^\dagger_{33:\cdot})^{-1/2} \end{bmatrix} \begin{bmatrix} I_{p_1+p_2} & 0 \\ -\Sigma^\dagger_{3\cdot} {\Sigma^\dagger_{\cdot\cdot}}^{-1} & I_{p_3} \end{bmatrix} \begin{bmatrix} \varepsilon_\cdot(t) \\ \varepsilon_3(t) \end{bmatrix}, \tag{2.3}$$

and define Π a $(p_1 + p_2 + p_3) \times (p_1 + p_2 + p_3)$ matrix by

$$\Pi = \begin{bmatrix} {\Sigma^\dagger_{\cdot\cdot}}^{-1/2} & 0 \\ 0 & (\Sigma^\dagger_{33:\cdot})^{-1/2} \end{bmatrix} \begin{bmatrix} I_{p_1+p_2} & 0 \\ -\Sigma^\dagger_{3\cdot} {\Sigma^\dagger_{\cdot\cdot}}^{-1} & I_{p_3} \end{bmatrix},$$

where $\Sigma^\dagger_{33:\cdot} \equiv \Sigma^\dagger_{33} - \Sigma^\dagger_{3\cdot} {\Sigma^\dagger_{\cdot\cdot}}^{-1} \Sigma^\dagger_{\cdot 3}$. In view of the construction, Π is a lower triangular block matrix

$$\Pi = \begin{bmatrix} \Pi_{\cdot\cdot} & 0 \\ \Pi_{3\cdot} & \Pi_{33} \end{bmatrix}. \tag{2.4}$$

Set $\tilde{\Lambda}(L) = \Lambda(L)\Lambda(0)^{-1}\Pi^{-1}$ and let its partition be

$$\tilde{\Lambda}(z) = \begin{bmatrix} \tilde{\Lambda}_{\cdot\cdot}(z) & \tilde{\Lambda}_{\cdot 3}(z) \\ \tilde{\Lambda}_{3\cdot}(z) & \tilde{\Lambda}_{33}(z) \end{bmatrix}.$$

Then it follows from the relationships

$$\begin{aligned} w(t) &\equiv (x(t)^*, y(t)^*, z(t)^*)^* = \Lambda(L)\Lambda(0)^{-1}\varepsilon(t) = \Lambda(L)\Lambda(0)^{-1}\Pi^{-1}\Pi\varepsilon(t) = \tilde{\Lambda}(L)\varepsilon^\dagger(t) \\ &= \begin{bmatrix} \tilde{\Lambda}_{\cdot\cdot}(z) & \tilde{\Lambda}_{\cdot 3}(z) \\ \tilde{\Lambda}_{3\cdot}(z) & \tilde{\Lambda}_{33}(z) \end{bmatrix} \begin{bmatrix} \varepsilon_{\cdot}^\dagger(t) \\ \varepsilon_3^\dagger(t) \end{bmatrix}, \end{aligned}$$

that

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \tilde{\Lambda}_{\cdot\cdot}(L)\varepsilon_{\cdot}^\dagger(t) + \tilde{\Lambda}_{\cdot 3}(L)\varepsilon_3^\dagger(t). \tag{2.5}$$

Since $\{\varepsilon_{\cdot}^\dagger(t)\}$ and $\{\varepsilon_3^\dagger(t)\}$ are orthogonal, the spectral density of $\{x(t), y(t)\}$ is given in view of (2.5) by

$$f_{\cdot\cdot}(\lambda) = \frac{1}{2\pi}\{\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})^* + \tilde{\Lambda}_{\cdot 3}(e^{-i\lambda})\tilde{\Lambda}_{\cdot 3}(e^{-i\lambda})^*\}.$$

Denote by $\{u(t), v(t)\}$ the joint process of the residuals of the projection of $x(t)$ and $y(t)$ onto $H\{z_{0,0,-1}(\infty)\} = H\{\varepsilon_3^\dagger(\infty)\}$; then it is given in view of (2.5) by

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \tilde{\Lambda}_{\cdot\cdot}(L)\varepsilon_{\cdot}^\dagger(t), \tag{2.6}$$

whence the spectral density of $\{u(t), v(t)\}$ is represented by

$$h(\lambda) = \frac{1}{2\pi}\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})^*. \tag{2.7}$$

In view of the construction, $\tilde{\Lambda}(z)$ is a canonical factor if $\Lambda(z)$ is canonical, but its square diagonal block $\tilde{\Lambda}_{\cdot\cdot}(z)$ in (2.7) is not warranted to be so; see Remark 2.1 below. When the factor given in (2.7) is not canonical, a certain factorization procedure must be implemented to construct the partial measures of interdependency between $\{x(t)\}$ and $\{y(t)\}$, since all the partial measures proposed are constructed based on the knowledge of a canonical factor of $h(\lambda)$.

Remark 2.1. Suppose that a matrix $\tilde{\Lambda}(z) = \{\tilde{\Lambda}_{ij}(z),\ i, j = 1, 2, 3\}$ is given by

$$\tilde{\Lambda}(z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1.001 & 0.5 \end{bmatrix} z;$$

then all the zeros of $\det \tilde{\Lambda}(z)$ are either on or outside of the unit circle if $-0.499 \le d \le -0.243$. On the other hand, $\det \tilde{\Lambda}_{..}(z) = (1-z)(1-2z)$ has one zero inside the unit circle, where $\tilde{\Lambda}_{..}(z)$ denotes the upper 2×2 diagonal block of $\tilde{\Lambda}(z)$. Consequently, when a partial spectral density is given as $h(\lambda)$ in (2.7), the factor $\tilde{\Lambda}_{..}(e^{-i\lambda})$ on the right-hand side is not guaranteed to be canonical. $\square$

2.2 Defining the partial measures

The partial interdependency measures between $\{x(t)\}$ and $\{y(t)\}$ in the presence of $\{z(t)\}$ are defined to be the corresponding simple measures between $\{u(t)\}$ and $\{v(t)\}$ given in (2.6). Suppose that the spectral density $h(\lambda)$ given in (2.7) satisfies the Szegö condition (b1) so that $h(\lambda)$ has a canonical factorization

$$h(\lambda) = \frac{1}{2\pi}\Gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})^*. \tag{2.8}$$

The equality (2.8) implies that the following time-domain MA representation of the series $\{u(t), v(t)\}$ holds in terms of the one-step ahead prediction error $\epsilon(t) \equiv (\epsilon_1(t)^*, \epsilon_2(t)^*)^* \equiv (u_{-1,-1}(t)^*, v_{-1,-1}(t)^*)^*$; namely,

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \Gamma(L)\Gamma(0)^{-1}\begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{bmatrix}, \tag{2.9}$$

where $E\{\epsilon(t)\} = 0$ and $E\{\epsilon(t)\epsilon(t)^*\} = \Gamma(0)\Gamma(0)^* = \Sigma$. Set, in parallel with (2.3),

$$\begin{aligned} \begin{bmatrix} \epsilon_1^\dagger(t) \\ \epsilon_2^\dagger(t) \end{bmatrix} &\equiv \begin{bmatrix} \Sigma_{11}^{-1/2} & 0 \\ 0 & \Sigma_{22:1}^{-1/2} \end{bmatrix}\begin{bmatrix} I_{p_1} & 0 \\ -\Sigma_{21}\Sigma_{11}^{-1} & I_{p_2} \end{bmatrix}\begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \end{bmatrix} \\ &\equiv \Xi\epsilon(t), \end{aligned} \tag{2.10}$$

whence $E\{\varepsilon^\dagger(t)\varepsilon^\dagger(t)^*\} = I_{p_1+p_2}$. Then we have

$$\begin{aligned} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} &= \Gamma(L)\Gamma(0)^{-1}\Xi^{-1}\Xi\epsilon(t) \\ &\equiv \Gamma^\dagger(L)\epsilon^\dagger(t) \\ &\equiv \begin{bmatrix} \Gamma_{11}^\dagger(L) & \Gamma_{12}^\dagger(L) \\ \Gamma_{21}^\dagger(L) & \Gamma_{22}^\dagger(L) \end{bmatrix}\begin{bmatrix} \epsilon_1^\dagger(t) \\ \epsilon_2^\dagger(t) \end{bmatrix}, \end{aligned} \tag{2.11}$$

where $\{\epsilon_2^\dagger(t)\}$ is the normalized one-way effect component of $v(t)$ to $u(t)$.

The partial frequency-wise measure of one-way effect (FMO) from $\{y(t)\}$ to $\{x(t)\}$ in the presence of $\{z(t)\}$ is defined by

$$PM_{y\to x:z}(\lambda) = \log\det\{I_{p_1} + \Gamma_{11}^{\dagger}(e^{-i\lambda})^{-1}\Gamma_{12}^{\dagger}(e^{-i\lambda})(\Gamma_{11}^{\dagger}(e^{-i\lambda})^{-1}\Gamma_{12}^{\dagger}(e^{-i\lambda}))^{*}\}, \tag{2.12}$$

where the $\Gamma_{ij}^{\dagger}(e^{-i\lambda})$ are defined on (2.11).

Denote by $\ddot{u}_{\cdot,\infty}(t)$ and $\ddot{v}_{\infty,\cdot}(t)$ the projection residuals of $u(t)$ onto $H\{v_{0,-1}(\infty)\}$ and of $v(t)$ onto $H\{u_{0,-1}(\infty)\}$ respectively, and set their joint spectral density matrix as

$$\ddot{h}(\lambda) = \begin{bmatrix} \ddot{h}_{11}(\lambda) & \ddot{h}_{12}(\lambda) \\ \ddot{h}_{21}(\lambda) & \ddot{h}_{22}(\lambda) \end{bmatrix}. \tag{2.13}$$

See Appendix A.3 for a concrete representation of $\ddot{h}(\lambda)$. Then the partial measure of reciprocity at frequency λ between $\{x(t)\}$ and $\{y(t)\}$ in the presence of the third series $\{z(t)\}$ is defined by

$$PM_{x.y:z}(\lambda) \equiv M_{u.v}(\lambda) \equiv \log\left[\frac{\det\ddot{h}_{11}(\lambda)\det\ddot{h}_{22}(\lambda)}{\det\ddot{h}(\lambda)}\right].$$

Theorem 2.1. Suppose that the spectral density matrix $\{u(t), v(t)\}$ has the canonical factorization (2.8) and set $\Sigma = \Gamma(0)\Gamma(0)^*$ and $\ddot{\sigma}^2 = \det\Sigma_{11}\det\Sigma_{22}/\det\Sigma$; then we have

$$PM_{x.y:z}(\lambda) = \log\ddot{\sigma}^2; \tag{2.14}$$

namely, the partial frequency-wise measure of reciprocity (FMR) is constant over the whole frequency domain.

Evidently, the quantity $\ddot{\sigma}^2$ defined in Theorem 2.1 is a constant not less than 1; Geweke (1982) calls the quantity $\det\Sigma_{11}\det\Sigma_{22}/\det\Sigma$ the measure of instantaneous feedback. The proof of Theorem 2.1 is given in Appendix A.2. See also D.4 in Appendix A.1 for the definitions of the overall and frequency-wise measures of association.

The following Theorem 2.2 has a useful application for the ARMA model. The representations of the measures of interdependency are much simplified thanks to it. Let $\gamma(z)$ be a scalar-valued analytic function with a expansion with real coefficients defined on the complex plane such that $\gamma(0) = 1$ and which has no zeros inside the unit circle. Suppose then that the spectral density matrix $k(\lambda)$ of the process $\{u(t), v(t)\}$ is expressed as

$$k(\lambda) = |\gamma(e^{-i\lambda})|^2 h(\lambda), \tag{2.15}$$

Moreover, suppose that $h(\lambda) = \Gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})^*$ for a canonical factor $\Gamma(z)$, so that we have a canonical factorization

$$k(\lambda) = \gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})\{\gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})\}^*. \tag{2.16}$$

In this special case, we have:

Theorem 2.2. Suppose $\{u(t), v(t)\}$ has the spectral density $k(\lambda)$ given in (2.15), for which the canonical factorization (2.16) holds. Then the $M_{v\to u}(\lambda), M_{u\to v}(\lambda)$ and $M_{u.v}(\lambda)$ are the same as the corresponding measures for the spectral density $h(\lambda)$.

Remark 2.2. Breitung and Candelon (2006, p.364) directly derive $\epsilon^{\dagger}(t)$ in (2.10) by multiplying the Cholesky factor matrix of the inverse of the covariance matrix of $\varepsilon(t)$. In the case of their bivariate model where $u(t)$ and $v(t)$ are scalar-valued, if the orthogonalization is done by the lower triangle Cholesky matrix, the one-way effect component is automatically derived, since then orthogonalization is conducted by eliminating the effect of $\varepsilon_2(t)$ from $\varepsilon_1(t)$ via the projection. In general, however, when $u(t)$ and $v(t)$ are vector-valued, arbitrary orthogonalization of $\varepsilon_1(t)$ and $\varepsilon_2(t)$ does not necessarily produce the one-way effect measure. □

The Sims' version of non-causality in the presence of the third series $\{z(t)\}$ is characterized as this: A necessary and sufficient condition for $\{y(t)\}$ not to cause partially $\{x(t)\}$ is that $y(t)$ is expressed as

$$y(t) = y^{(1)}(t) + y^{(2)}(t),$$

where $y^{(1)}(t)$ is the projection of $y(t)$ onto $H\{x(t), z_{0,0,-1}(t)\}$ and $y^{(2)}(t)$ is orthogonal to $H\{x(\infty), z_{0,0,-1}(\infty)\}$. Moreover, a necessary and sufficient condition for $\{y(t)\}$ not to cause partially $\{x(t)\}$ in the presence of $\{z(t)\}$ is $PM_{y\to x:z} = 0$; see for allied literature Sims (1972), Hosoya (1977) and Hosoya (2001).

3 Inference based on the ARMA model

Focusing specifically on the stationary vector ARMA process, Section 3.1 shows in concrete steps how the partial measures of interdependency introduced in Section 2 are numerically evaluated. Section 3.2 discusses statistical inference on those measures based on the standard asymptotic theory of the Whittle quasi-likelihood inference for the stationary multivariate ARMA processes. The point is the use of simulation-based estmation of the covariance matrix of the measure-related statistics.

3.1 The stationary ARMA model

Suppose that the process $\{x(t), y(t), z(t)\}$ is a stationary multivariate ARMA process which is generated by

$$A(L)\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = B(L)\varepsilon(t), \quad t \in \mathbb{Z}, \tag{3.1}$$

where $x(t), y(t), z(t)$ are respectively p_1, p_2, p_3-vectors, $A(L)$ and $B(L)$ are a-th and b-th order polynomials of the lag operator L and $A[0] = B[0] = I_{p_1+p_2+p_3}$; namely, we have $A(L) = \sum_{j=0}^{a} A[j]L^j$ and $B(L) = \sum_{j=0}^{b} B[j]L^j$.

Moreover suppose that all the zeros of $\det A(z)$ are outside of the unit circle, $\det B(z)$ has the zeros either on or outside of the unit circle and does not share any common zeros. Moreover, suppose that that the innovation $\{\varepsilon(t)\}$ is a *i.i.d.* white noise process with mean 0 and covariance matrix $\Sigma^\dagger$. Because of the zero conditions of $A(z)$ and $B(z)$, the joint spectral density $f(\lambda)$ of the process (3.1) satisfies the Szegö condition (2.1), whence it has a canonical factorization

$$f(\lambda) = \frac{1}{2\pi}\Lambda(e^{-i\lambda})\Lambda(e^{-i\lambda})^*. \tag{3.2}$$

In view of the zero conditions of $A(z)$ and $B(z)$, a version of the canonical factor $\Lambda(z)$ is given by

$$\Lambda(z) = A(z)^{-1}B(z)\Sigma^{\dagger\frac{1}{2}} = (\det A(z))^{-1}A^\sharp(z)B(z)\Sigma^{\dagger\frac{1}{2}} \equiv (\det A(z))^{-1}C(z),$$

where $\Sigma^{\dagger\frac{1}{2}}$ is the Cholesky factor of $\Sigma^\dagger$ satisfying $\Sigma^\dagger = \Sigma^{\dagger\frac{1}{2}}(\Sigma^{\dagger\frac{1}{2}})^*$; $A^\sharp(z)$ denotes the adjugate matrix (transposed cofactor matrix) of $A(z)$ and $C(z)(\equiv A^\sharp(z)B(z)\Sigma^{\dagger\frac{1}{2}})$ is a finite-order real matrix-coefficient polynomial such that

$$C(z) = \sum_{j=0}^{\bar{a}} C[j]z^j, \ \bar{a} \equiv (p_1 + p_2 + p_3 - 1)a + b.$$

As in the previous section, denote the projection residuals of $x(t)$ and $y(t)$ onto $H\{z_{0,0,-1}(\infty)\}$ respectively by $u(t)$ and $v(t)$, and denote the joint spectral density matrix of $\{u(t), v(t)\}$ by $h(\lambda)$. Now set

$$\tilde{\Lambda}(z) = C(z) \begin{bmatrix} \Sigma_{\cdot\cdot}^{\dagger -1/2} & 0 \\ 0 & (\Sigma_{33:\cdot}^{\dagger})^{-1/2} \end{bmatrix} \begin{bmatrix} I_{p_1+p_2} & 0 \\ -\Sigma_{3\cdot}^{\dagger}\Sigma_{\cdot\cdot}^{\dagger -1} & I_{p_3} \end{bmatrix} \tag{3.3}$$

and let $\tilde{\Lambda}_{\cdot\cdot}(z)$ be the $(p_1+p_2) \times (p_1+p_2)$ upper diagonal block of $\tilde{\Lambda}(z)$. It follows from (2.7) that the spectral density $h(\lambda)$ of $\{u(t), v(t)\}$ is given by

$$h(\lambda) = \frac{1}{2\pi} |\det A(e^{-i\lambda})|^{-2} \tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda}) \tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})^*.$$

In view of Theorem 2.2, all the interdependency measures between $\{u(t)\}$ and $\{v(t)\}$ are derived assuming that the joint spectral density is given as $k(\lambda) = \frac{1}{2\pi}\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})^*$. Although $\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})$ may not be canonical, since $k(\lambda)$ is a MA spectrum, Hosoya and Takimoto (2010)'s algorithm can be used in such non-canonical cases to produce a canonical factor $\Gamma(e^{-i\lambda})$ which satisfies

$$k(\lambda) = \frac{1}{2\pi}\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})\tilde{\Lambda}_{\cdot\cdot}(e^{-i\lambda})^* = \frac{1}{2\pi}\Gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})^*. \tag{3.4}$$

Consequently, all the measures of interdependency introduced in Section 2.2 and also in Appendix A.1 are able to be computed using the factor $\Gamma(z)$ given in (3.4).

3.2 Inferential procedure

Based on a finite set of observations $\{x(t), y(t), z(t);\ t = 1, \cdots, T\}$ and the VARMA modeling (3.1) of the data generating process, we are able to conduct statistical inference on the partial measures of interdependency introduced in Section 2. Denote the whole model parameter by θ ; namely, set

$$\theta \equiv vec\{A[1], \cdots, A[a], B[1], \cdots, B[b], v(\Sigma^{\dagger})\}$$

where $v(\Sigma^{\dagger})$ denotes the $(p_1+p_2+p_3)\times(p_1+p_2+p_3+1)/2$ vector obtained from $vec(\Sigma^{\dagger})$ by eliminating all the supradiagonal elements of the $(p_1+p_2+p_3)\times(p_1+p_2+p_3)$ matrix $\Sigma^{\dagger}$. Let $G(\theta)$ be an m-vector whose components are respectively certain distinct quantities related to the partial measures of interdependency.

Takimoto and Hosoya (2004, 2006) provide a relevant parameter estimation procedure, in contrast to conventional nonrestrictive estimation procedures for the VARMA model parameter which do not necessarily produce estimates satisfying the zero conditions of $\det A(z)$ and $\det B(z)$. Modifying the maximum Whittle likelihood estimation, Takimoto and Hosoya's afore-mentioned papers provide a three-step root-modification procedure which produces coefficient estimates warranting the stationarity and invertibility conditions. The procedure is essentially carried out as follows:

Step 1. By fitting a sufficiently higher order VAR process and applying the ordinary least-square method in the time domain, obtain an estimate of the unobservable disturbance terms as the regression residual series. In the case the DGP is VAR process, this step is skipped.

Step 2. Substituting the disturbances in the MA part by the corresponding residuals obtained in Step 1, estimate VARMA model by the time-domain least square method, selecting the lag-orders of the model by means of an information criterion.

Step 3. Determine the estimate $\hat{\theta}$ of the model parameter by maximizing the Whittle likelihood endowed with a penalty function of the zero conditions. The maximizing algorithm is a quasi-Newton iteration method, using the parameter values obtained in Step 2 as the initial value of the iteration.

By setting the penalty asymptotically negligible, the conventional asymptotic normality holds for $\sqrt{T}(\hat{\theta}-\theta)$ under standard regularity conditions for the VARMA model (3.1); see for those conditions Hannan-Rissanen (1982), Hannan-Kavalieris (1984) and Hosoya (1997b), for example. The modified maximum Whittle likelihood estimate for observation size T determined by these steps is denote by $\hat{\theta}$ in the sequel. Incidentally, it is interesting to note that, in contrast to our Step 3, all the estimation procedures proposed by Hannan-Rissanen (1982), Hannan-Kavalieris (1984) and Johansen (1991) have no built-in checking step to prevent the root-conditions being violated.

Our test approach classifies the one-way effect tests into two classes according to the types of the null hypothesis and proposes different test statistics for respective classes. In particular, a new test for the hypothesis of partial non-causality is provided. Consider first the case in which the null hypothesis does not involve the Granger non-causality hypothesis. Since all the partial measures are non-negative, testing them being equal to zero constitutes a boundary value test. For such tests, the direct use of the stochastic expansion of the estimates is not pertinent since the Jacobian matrix is not of full rank. Suppose specifically that $G_i(\theta,\lambda), i = 1,\cdots,m$, are different kinds of scalar-valued measures exhibited in D.1 through D.4 in Appendix A.1 and let $G(\theta,\lambda)$ be a m-vector such that $G(\theta,\lambda) = (G_1(\theta,\lambda),\ldots,G_m(\theta,\lambda))^*$ and $G_i(\theta,\lambda) > 0$. By the stochastic expansion, we have

$$\sqrt{T}\{G(\hat{\theta},\lambda) - G(\theta,\lambda)\} = (D_\theta G(\theta,\lambda))\sqrt{T}(\hat{\theta} - \theta) + o_p(1),$$

where $D_\theta G(\theta,\lambda)$ is the $m \times n_\theta$ Jacobian matrix of $G(\theta,\lambda)$ evaluated at θ; n_θ denotes the size of the vector θ. Suppose that $\sqrt{T}(\hat{\theta} - \theta)$ is asymptotically normally distributed with mean 0 and covariance matrix $\Psi(\theta)$; see Hosoya (1997b) for a set of milder conditions of the asymptotic normality. Then $\sqrt{T}\{G(\hat{\theta,\lambda}) - G(\theta,\lambda)\}$ is asymptotically normally distributed with mean 0 and the $m \times m$ asymptotic covariance matrix

$$H(\theta,\lambda) = D_\theta G(\theta,\lambda)\Psi(\theta)D_\theta G(\theta,\lambda)^*. \tag{3.5}$$

Assume that the vector $G(\theta,\lambda)$ of measures of interdependency is chosen so that $\mathrm{rank}H(\theta,\lambda) = m$ in a neighborhood of the true θ; then the Wald statistic

$$W^{(m)}(\lambda) \equiv T\{G(\hat{\theta},\lambda) - G(\theta,\lambda)\}^* H(\hat{\theta},\lambda)^{-1}\{G(\hat{\theta},\lambda) - G(\theta,\lambda)\} \tag{3.6}$$

is asymptotically χ^2-distributed with m degrees of freedom if θ is the true value. Let G_0 be a given m vector, then the null hypothesis $G(\theta,\lambda) = G_0$ can be tested by the test statistic

$$W^{(m)}(\lambda) \equiv T\{G(\hat{\theta},\lambda) - G_0\}^* H(\hat{\theta},\lambda)^{-1}\{G(\hat{\theta},\lambda) - G_0\},$$

where G_0 is a vector of positive components. Also a confidence set for $G(\theta,\lambda)$ is able to constructed by means of the statistic $W^{(m)}(\lambda)$.

There are several alternative procedures available to estimate the asymptotic covariance matrix $H(\theta,\lambda)$. For example, we might use the asymptotic covariance matrix formula given by Yao and Hosoya (2000) which is based on the numerical differentiation for $D_\theta G(\hat{\theta},\lambda)$ and evaluation of $\Psi(\hat{\theta})$ in the case of the cointegrated VAR model, but the formula becomes much more complex computationally for the general ARMA model set-up.

An alternative simpler approach is to use the Monte Carlo Wald test procedure which is conducted as follows:

Step 1. Estimate θ which is the vector comprising all the parameter involved in the model (3.1) by the modified maximum Whittle likelihood method and evaluate the vector $G(\hat{\theta}, \lambda)$.

Step 2. Generate the data series $\{x(t)^{\dagger}, y(t)^{\dagger}, z(t)^{\dagger}; t = 1, \cdots, T\}$ by the model (3.1) using the parameter estimate $\hat{\theta}$ obtained in Step 1 and a set of simulated independently normally distributed random vectors $\{\varepsilon(t)\}$ with mean 0 and the estimated variance-covariance matrix $\hat{\Sigma}^{\dagger}$ in Step 1.

Step 3. Estimate the parameter θ by employing the simulated series $\{x(t)^{\dagger}, y(t)^{\dagger}, z(t)^{\dagger};$ $t = 1, \cdots, T\}$ and set the estimate of $G(\theta, \lambda)$ by $G(\theta^{\dagger}, \lambda)$.

Step 4. Iterate Steps 2 and 3 N times, and produce $G(\theta_n^{\dagger}, \lambda); n = 1, \cdots, N$, and estimate the covariance matrix $H(\theta, \lambda)$, denoted as $\bar{H}(\hat{\theta}, \lambda)$, as the Monte Carlo sample covariance matrix of $G(\theta_n^{\dagger}, \lambda)$; namely,

$$\bar{H}(\hat{\theta}, \lambda) = \frac{T}{N} \sum_{n=1}^{N} \left(G(\theta_n^{\dagger}, \lambda) - \bar{G}(\theta^{\dagger}, \lambda) \right) \left(G(\theta_n^{\dagger}, \lambda) - \bar{G}(\theta^{\dagger}, \lambda) \right)^{*}, \tag{3.7}$$

where

$$\bar{G}(\theta^{\dagger}, \lambda) = \frac{1}{N} \sum_{n=1}^{N} G(\theta_n^{\dagger}, \lambda).$$

Remark 3.1. It is difficult to give a general rule to relate the size of N to the observation size T, but in practice it is not difficult to determine an appropriate size N by inspecting how the calculated covariance matrices are numerically stabilized as the number N increases by means of a Monte-Carlo simulation. □

As alluded above, the foregoing approach is not suited for testing non-causality, and so we must look for other statistics. For the latter test, Breitung and Candelon (2006), based on bivariate stationary as well as cointegrated VAR models, propose an F-test for a set of linear restriction hypotheses on certain distributed lag parameters in their AR-DR model. To deal with a more wider class such as the VARMA models, however, we need a more general approach.

The Breitung and Candelon test uses the standard asymptotic theory of the stationary time-series regression estimation and testing, whereas our Monte Carlo Wald test uses the standard asymptotic theory of the Whittle quasi-likelihood inference for the stationary multivariate ARMA processes. Namely, what we assume in the following arguments is that the consistency and the asymptotic normality of the subject-matter statistics $G(\hat{\theta}, \lambda)$ used in the Wald tests and the consistency of the Monte Carlo estimate of the covariance matrix of the statistics based on sufficient number of Monte Carlo iterations. Although the Gaussian pseudo random number series is the most convenient choice to simulate observation series, for more sophisticated approach, we may as well apply some other time-series bootstrap methods.

Since the formula (2.12) implies that the measure $PM_{y\to x:z}(\lambda)$ is not determined by $\Gamma^{\dagger}_{12}(e^{-i\lambda})$ alone, but is determined in terms of the ratio $\Gamma^{\dagger}_{11}(e^{-i\lambda})^{-1}\Gamma^{\dagger}_{12}(e^{-i\lambda})$, we may conduct the test of the null hypothesis of v not causing u at frequency λ by testing

$$\Gamma^{\dagger}_{11}(e^{-i\lambda})^{-1}\Gamma^{\dagger}_{12}(e^{-i\lambda}) = 0,$$

rather than testing the hypothesis $\Gamma^{\dagger}_{12}(e^{-i\lambda}) = 0$, where $\Gamma^{\dagger}_{kl}(e^{-i\lambda}) \equiv \sum_{j=0}^{\bar{a}} \Gamma^{\dagger}_{kl}[j]e^{-ij\lambda}$ and $\Gamma^{\dagger}_{kl}[j]$ is the j-th coefficient matrix of the polynomial $\Gamma^{\dagger}_{kl}(z)$, where $k, l = 1, 2$ and $\bar{a} \equiv a(p_1 + p_2 + p_3 - 1) + b$. Define

$$\psi(\theta, \lambda) \equiv \mathrm{vec}\{Re\Gamma^{\dagger}_{11}(e^{-i\lambda})^{-1}\Gamma^{\dagger}_{12}(e^{-i\lambda}), Im\Gamma^{\dagger}_{11}(e^{-i\lambda})^{-1}\Gamma^{\dagger}_{12}(e^{-i\lambda})\}.$$

Then the Wald statistic for the null hypothesis that $y(t)$ does not cause $x(t)$ in the presence of $z(t)$ is given by

$$W^{(n)}(\lambda) = T(\psi(\hat{\theta}, \lambda))^{*}H(\hat{\theta}, \lambda)^{-1}\psi(\hat{\theta}, \lambda),$$

where $H(\theta, \lambda)$ is the asymptotic covariance matrix of $\sqrt{T}(\psi(\hat{\theta}, \lambda) - \psi(\theta, \lambda))$. The statistic $W^{(n)}(\lambda)$ is, under the null hypothesis, asymptotically distributed as χ^2 distribution with degrees of freedom which is equal to the dimension of the vector $\psi(\theta, \lambda)$. One way to evaluate $H(\theta, \lambda)$ is to use the stochastic expansion of $\psi(\hat{\theta}, \lambda)$

$$\sqrt{T}(\psi(\hat{\theta}, \lambda) - \psi(\theta, \lambda)) = (D_{\theta}\psi(\theta, \lambda))\sqrt{T}(\hat{\theta} - \theta) + o_p(1), \tag{3.8}$$

where $D_{\theta}\psi(\theta, \lambda)$ denotes the Jacobian matrix of $\psi(\theta, \lambda)$. The expansion (3.8) implies the asymptotic distributional relationship:

$$\sqrt{T}(\psi(\hat{\theta}, \lambda) - \psi(\theta, \lambda)) \overset{a}{\sim} N(0, H(\theta, \lambda)),$$

where $H(\theta, \lambda) = D_{\theta}\psi(\theta, \lambda)^{*}\Psi(\theta)D_{\theta}\psi(\theta, \lambda)$. The approach by the stochastic expansion (3.8) is useful in case $\Psi(\theta)$ is numerically tractable from the asymptotic theory. Another approach to evaluate the covariance matrix $H(\theta, \lambda)$ without relying on the stochastic expansion is to apply the four-step Monte Carlo procedure given in the paragraph preceding to Remark 3.1 directly to $\psi(\hat{\theta}, \lambda)$.

Lastly consider testing the null hypothesis of the overall measure of one-way effect $M_{v \to u} = 0$ namely $\{v(t)\}$ not causing $\{u(t)\}$. The component $\Gamma^{\dagger}_{12}(z)$ in (2.11) has a finite-order MA expression such that $\Gamma^{\dagger}_{12}(z) = \sum_{j=0}^{\bar{a}} \Gamma^{\dagger}_{12}[j] z^j$ where the coefficients $\Gamma^{\dagger}_{12}[j]$ are in general nonlinear functions of θ; namely, $\Gamma^{\dagger}_{12}[j] = \Gamma^{\dagger}_{12}[j, \theta], j = 0, \cdots, \bar{a}$ There are several approaches to the test. One way of testing the hypothesis $M_{v \to u} = 0$ is to test $vec(\Gamma^{\dagger}_{12}[j, \theta], j = 0, \cdots, \bar{a}) = 0$, which does not constitute a boundary-value test. In case of $p_1 > 1$, another method to test the null OMO is to test $\Gamma^{\dagger}_{11}(z)^{-1}\Gamma^{\dagger}_{12}(z) = 0$. The test is reduced to the test of $vec(\Delta[j, \theta], j = 0, \cdots, \bar{a}p_1) = 0$, where $\Delta[j, \theta]$ is determined by the equality

$$\Gamma^{\dagger}_{11}(z, \theta)^{\sharp}\Gamma_{12}(z, \theta) = \sum_{j=0}^{\bar{a}p_1} \Delta[j, \theta] z^j.$$

For those tests, we can apply the Wald test approach, using the Whittle estimator $\hat{\theta}$ and its relevant covariance matrix estimate. A third candidate would be to test

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} PM_{y \to x:z}(\lambda) d\lambda = 0,$$

where the integrand is defined in (2.12). Although we can evaluate the integral numerically for estimated $\hat{\theta}$, the test constitutes a boundary value test and standard large-sample test techniques do not apply.

Remark 3.2. Suppose that $u(t)$ and $v(t)$ are scalar-valued and generated by the bivariate AR process:

$$u(t) = \sum_{j=1}^{a} \alpha_{1j} u(t-j) + \sum_{k=1}^{a} \beta_{1k} v(t-k) + \varepsilon_1(t),$$
$$v(t) = \sum_{j=1}^{a} \alpha_{2j} u(t-j) + \sum_{k=1}^{a} \beta_{2k} v(t-k) + \varepsilon_2(t).$$

For such a model, Breitung and Candelon (2006) notes that the test of $\Gamma^{\dagger}_{12}(e^{-i\lambda}) = 0$ in (2.12) for the purpose of testing of $\{v(t)\}$ not simply causing $\{u(t)\}$ is equivalent to testing

$$\Gamma^{\dagger}_{12}(e^{-i\lambda})^{\sharp} = 0, \tag{3.9}$$

where $\Gamma^{\dagger}_{12}(e^{-i\lambda})^{\sharp}$ is the (1,2) component of the 2×2 adjugate matrix $\Gamma^{\dagger}(e^{-i\lambda})^{\sharp}$, but the test (3.9) is reduced to testing

$$\sum_{k=1}^{a} \beta_{1k} e^{-ik\lambda} = 0. \tag{3.10}$$

Testing the hypothesis (3.10) can be dealt with by an F-test, since it imposes linear restrictions on the distributed-lag coefficients. But, this method does not extend to a higher-dimensional $\Gamma^{\dagger}(e^{-i\lambda})$, because (3.9) imposes non-linear restrictions on the model parameters. Accordingly a certain version of either the likelihood ratio test or the Wald test of non-linear restrictions rather than the F-test is required for more general case. □

Remark 3.3. To deal with the third-series presence problem, Breitung and Candelon (2006, p.369) propose a way to eliminate a third series effect by a time-domain regression. Specifically, they propose to fit a single-equation autoregressive-distributed lag (AR-DL) model

$$x(t) = \sum_{j=1}^{a} \alpha_j x(t-j) + \sum_{k=1}^{a} \beta_k y(t-k) + \sum_{l=1}^{a} \gamma_l v(t-l) + \varepsilon(t) \tag{3.11}$$

and to test the null hypothesis $\sum_{k=1}^{a} \beta_k e^{-ik\lambda} = 0$ by an F-statistic, where $v(t)$ is equal to either $z(t)$ or the residual obtained by regressing $z(t)$ on $x(t), y(t)$ and $w(t-1), \cdots, w(t-p)$ where $w = (x, y, z)^*$. Breitung and Candelon (2006, p.375) present an empirical analysis of the one-way effect of the yield spread to the growth rate of real GDP in the U.S. by eliminating the effect of the real balances in the time domain, and conclude that the test results do not seem to depend on the choice of $v(t)$ in (3.11). Furthermore, based on their test approach, Gronwald (2009) provides an empirical analysis on the partial one-way causality running from oil price series to macro and financial time-series in Germany. Even though their method suggests a way to avoid the spectral canonical factorization problem, it would not produce the same test results as this paper proposes. The approach we propose have the following merits:

- The MA part can be included in the basic model so that the partial causal analysis can be extended to the ARMA model as shown in the the foregoing arguments.

- The dimensions p_1, p_2, p_3 can be greater than 1.

- Even without assuming such a specific parametric model as the ARMA model, measures of interdependency are able to be constructed as long as a canonical factorization of partial spectral density is available.

- Partial measures of reciprocity and association can be dealt with. □

Remark 3.4. Breitung and Candelon (2006) gave a characterization of local power of the F-type test of the null hypothesis of the frequency-wise absence of the simple one-way effect by focusing on the Gegenbauer-polynomial frequency-response function. As regards the determination of the number of the different λ values for the FMO to be estimated or tested, Breitung and Candelon's local power analysis provides a useful indication of how finer resolution being able to be attained in relation to the observation size T. Although our test is not directly comparable with theirs in general, for a possible comparable case, such as the one where the third-series one-way effect is absent and need not be eliminated (and hence the partial and the corresponding simple measures are expected equal), the first-order asymptotic local powers of ours and theirs would be equal under common regularity conditions, since the comparison is nothing but the comparison between the Wald test and the LR test under the standard conditions. But the power would be different in general in the presence of a third series. For such a case, in conventional terms of econometrics, the Breitung and Candelon test is a single-equation limited information likelihood-ratio (LR) test whereas our tests are a kind of Wald test based on the full-information maximum-likelihood (ML) estimation. If the complete system of the VARMA model is true, the power of tests using only a limited information is expected inferior. But well designed numerical comparison between the Breitung and Candelon test and ours in a comparable set-up remains to be investigated. □

4 Concluding remarks

By means of the cointegrated VAR model fitted to Japanese macroeconomic data, Hosoya (1997a), Yao and Hosoya (2000) and Hosoya, Yao and Takimoto (2005) investigated the empirical one-way effect structure for a variety of pairs of variables. But the studies were limited to the simple one-way effects. The new contribution of the present paper is the presentation of a numerically practicable method which enables estimating and testing the partial measures introduced in Hosoya (2001). The latter paper covers only theoretical representation and did not go into the numerical evaluation and inference problems. In contrast to the simple measures of interdependency, the numerical construction of the corresponding partial measures needs an explicit knowledge of canonical factor of a spectral density matrix involved. By implementing the numerical factorization procedure of Hosoya and Takimoto (2010), which is an improved version of the Rozanov (1967)'s factorization method, this paper suggested a numerical procedure to evaluate the partial measures of interdependency for stationary VARMA model. This paper showed that the evaluation of the measures is reducible to the one for a finite-order MA spectral density matrix, and presented a parametric statistical inference approach which consists of estimation based on the Whittle likelihood asymptotic theory and testing and confidence-set construction relied on the standard limiting theory of the Wald statistics. Although all the measures of interdependency were defined for vector second-order stationary processes in this paper, they are extensible to non-stationary cointegrated processes with the aid of the reproducibility assumption introduced in Hosoya (1997a, 2001).

There remain some open issues. First of all, the paper has not examined the numerical performance of the proposed theory; the authors' research is in progress on the issues of simulation and empirical performance. To improve the performance of the Wald test in small-sample circumstances and the feasibility in application, employment of a certain time-series bootstrap method for probability evaluation and/or introduction of nonlinear transformation as proposed by Hosoya and Terasaka (2009) might be useful. Although Section 3 focuses on the stationary ARMA process mainly for the sake of expositional simplicity, extension to a wider class of processes is necessary for applications to empirical economic analyses. By utilizing the asymptotic covariance-matrix formula provided by Hosoya (1997b), our statistical inference procedure can be extended to more general time-series models in which the disturbance series is possibly non-Gaussian. Hosoya, Yao and Takimoto (2005) took trend-breaks explicitly into account for testing the simple one-way effect measures in a cointegrated VAR set-up. The extension of the partial measures of interdependency in that direction as well as the extension to nonlinear processes might be also important. But the most important open issue above all would be to develop a testing theory of the Granger causality which is more conformable to out-sample prediction, and thus to find a way to identify predictors equipped with substantial out-sample prediction ability; see Granger (1999) who emphasized the importance of this kind of research.

An enormous amount of empirical economic studies has dealt with predictive ability of the term structure and other asset-price characteristics for the future growth rate of economic activities and inflation rates. Stock and Watson (2003) and Wheelock and Wohar (2009) respectively give wide-ranging reviews of the literature; see also Hamilton and Kim (2002) and Assenmacher-Wesche, Gerlach and Sekine (2008) for example. A common understanding seems to be that the prediction ability of the term structure has fallen since the middle of 1980's in the U.S. economy and also that the predictive content of the original as well as the Freedman version of the Phillips curve is rather meager; see Staiger, Stock and Watson (1997) for the latter aspect.

Stock and Watson (2003) claim that in- sample tests of significance for Granger causality are, in general, poor guides for identification of potent predictors, providing little assurance that the identified predictive relations are stable. Although focusing not on the causality issue itself but on predictability, Wheelock and Wohar (2009) also note considerable variation of prediction ability of the term spread across countries and over time, as far as prediction of a variety of economic-activity changes is concerned.

To be more specific, Stock and Watson (2003) argue the problem of prediction ability, relying mainly on the single equation autoregressive distributive-lag model of the form

$$x(t) = \sum_{j=1}^{a} \alpha_j x(t-j) + \sum_{k=1}^{b} \beta_j y(t-k) + \varepsilon(t). \tag{4.1}$$

The Granger test result in itself does not bring into question how much the prediction is improved by inclusion of the sum $\sum_{k=1}^{b} \beta_j y(t-k)$ in case the null hypothesis $(\beta_1, \cdots, \beta_b) = 0$ is rejected. The problem, however, is not indigenous to the Granger non-causality test. If a relation changes over time, it is natural to expect that in-sample observation attributes are not extrapolated for out-sample prediction. Characteristically, while giving negative assessment to the Granger causality test in respect of prediction ability, Stock and Watson does not question directly the use of the Granger test itself when the stability of the relation (4.1) extends over a certain out-sample range; namely, they do not ask whether the rejection of non-causality indicates the usefulness of the corresponding variable over such a time interval of relative stability.

In case dependency relation is stable over time, the relation between statistical and practical significance is reduced to the general dictum that a significant test result does not measure the practical significance. Even if the estimates of the β_j are small in magnitude in (4.1), they can be well significant when the corresponding standard errors are small and the Granger non-causality hypothesis is rejected, but it does not necessarily imply the notable prediction improvement by inclusion of those predictors. In contrast to statistical significance, confidence statements seem fit to represent the strength of effects. The one-way effect measures proposed in the paper are a way of quantifying prediction improvement and the suggested confidence sets would provide predictive information the Granger causality test does not cover; see also Yao and Hosoya (2000) who exemplified an approach to confidence-set construction of the OMO.

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A Appendix

A.1 Glossary on partial measures of interdependency

This subsection collects basic definitions and equations related to partial interdependencies. Denote by H the Hilbert space defined over the real-number field which is the closure of the linear hull of the union $\{x_j(t);\ t \in \mathbb{Z},\ j = 1, \cdots, p_1\} \cup \{y_k(t);\ t \in \mathbb{Z},\ k = 1, \cdots, p_2\} \cup \{z_l(t);\ t \in \mathbb{Z},\ l = 1, \cdots, p_3\}$, where $x_j(t)$ denotes the j-th element of the vector $x(t)$ and all the $x(t)$, $y(t)$ and $z(t)$ have finite second-order moments. For brevity, $H\{x(t_1 - j), y(t_2 - j), z(t_3 - j);\ j \in \mathbb{Z}^{0+}\}$ is written as $H\{x(t_1), y(t_2), z(t_3)\}$ and $H\{x(j); j \in \mathbb{Z}\}$ is written as $H\{x(\infty)\}$. For a random vector $x(t)$ indexed by t, $\{x(t)\}$ denotes the process $\{x(t); t \in \mathbb{Z}\}$ unless otherwise specified.

The projection of a random vector $w = \{w_j;\ j = 1, \cdots, s\}$ to a closed subspace $H(\cdot)$ of H implies the element-wise orthogonal projection. Namely, if $\bar{w}_j$ is the orthogonal projection of w_j onto $H(\cdot)$, the projection implies the vector $\bar{w}$ whose j-th element is $\bar{w}_j$. In the system of three series $\{x(t), y(t), z(t)\}$, the one-way effect component of $z(t)$ implies the projection residual (the perpendicular) of $z(t)$ when it is projected onto the closed linear subspace $H\{x(t), y(t), z(t-1)\}$ and the residual is denoted by $z_{0,0,-1}(t)$. The vectors $u(t)$ and $v(t)$ are respectively the projection residuals of $x(t)$ and $y(t)$ onto $H\{z_{0,0,-1}(t); j \in \mathbb{Z}\}$

D1. The partial overall measure of one-way effect (OMO) from $\{y(t)\}$ to $\{x(t)\}$ is the simple OMO from $\{v(t)\}$ to $\{u(t)\}$ and defined by

$$PM_{y\to x:z} \equiv M_{v\to u} = \log \frac{\det Cov\{u_{-1,\cdot}(t)\}}{\det Cov\{u'_{-1,-1}(t)\}},$$

where $u_{-1,\cdot}(t)$ and $u'_{-1,-1}(t)$ are the projection residuals of $u(t)$ onto $H\{u(t-1)\}$ and onto $H\{u(t-1), v_{0,1}(t-1)\}$ respectively.

D2. The partial frequency-wise measure of one-way effect (FMO), in terms of the frequency-response function $\Gamma^\dagger(e^{-i\lambda})$ as given in (2.11), is defined by

$$\begin{aligned} PM_{y\to x:z}(\lambda) &\equiv M_{v\to u}(\lambda) \\ &= \log \frac{\det\{\Gamma^\dagger_{11}(e^{-i\lambda})\Gamma^\dagger_{11}(e^{-i\lambda})^* + \Gamma^\dagger_{12}(e^{-i\lambda})\Gamma^\dagger_{12}(e^{-i\lambda})^*\}}{\det\{\Gamma^\dagger_{11}(e^{-i\lambda})\Gamma^\dagger_{11}(e^{-i\lambda})^*\}} \end{aligned} \tag{A.1}$$

where the $\Gamma^\dagger_{ij}(e^{-i\lambda})$ are defined in (2.11). The partial FMO $PM_{x\to y:z}(\lambda)$ is given in a similar way; see Hosoya (2001) for a different representation.

D3. The partial measure of reciprocity at frequency λ and the corresponding overall measure between $x(t)$ and $y(t)$ are defined respectively by:

$$PM_{x.y:z}(\lambda) \equiv M_{u.v}(\lambda) = \log\left[\frac{\det \ddot{h}_{11}(\lambda)\det \ddot{h}_{22}(\lambda)}{\det \ddot{h}(\lambda)}\right]; \quad PM_{x.y:z} \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} PM_{x.y:x}(\lambda)d\lambda$$

where a representation of $\ddot{h}_{ij}(\lambda)$ is given in (A.4) below and another expression is given in Theorem A in section A.3.

D4. The partial measure of association at frequency λ and the corresponding overall measure between $x(t)$ and $y(t)$ are defined respectively by:

$$PM_{x,y:z}(\lambda) \equiv M_{u,v}(\lambda) \equiv \log\left[\frac{\det h_{11}(\lambda)\det h_{22}(\lambda)}{\det \ddot{h}(\lambda)}\right],$$

$$PM_{x,y:z} \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} PM_{x,y:z}(\lambda)d\lambda.$$

where $h(\lambda)$ is the spectral density matrix of the joint process $\{u(t),\, v(t)\}$.

E1. The following equality holds between the partial OMO and FMO:

$$PM_{y\to x:z} = \frac{1}{2\pi}\int_{-\pi}^{\pi} PM_{y\to x:z}(\lambda)d\lambda;$$

see for the proof Hosoya (1991, p.433).

E2. It follows from the definitions of the respective measures and the equality E1 and the corresponding equality for $PM_{x\to y:z}$ that:

$$\begin{aligned} PM_{x,y:z}(\lambda) &= PM_{x\to y:z}(\lambda) + PM_{x.y:z}(\lambda) + PM_{y\to x:z}(\lambda), \\ PM_{x,y:z} &= PM_{x\to y:z} + PM_{x.y:z} + PM_{y\to x:z}. \end{aligned}$$

A.2 Proofs of Theorems

Proof of Theorem 2.1. It follows from the representation (2.11) that the reciprocal component $\ddot{u}_{\cdot,\infty}(t)$ of $u(t)$ is given by

$$\ddot{u}_{\cdot,\infty}(t) = \Gamma_{11}^{\dagger}(L)\Sigma_{11}^{-1/2}\epsilon_1(t). \tag{A.2}$$

Similarly, setting

$$\Psi = \begin{bmatrix} \Sigma_{11:2} & 0 \\ 0 & \Sigma_{22}^{-1/2} \end{bmatrix} \begin{bmatrix} I_{p_1} & -\Sigma_{12}\Sigma_{22}^{-1} \\ 0 & I_{p_2} \end{bmatrix} \quad \text{and} \quad \xi(t) = \Psi\epsilon(t),$$

we have

$$\begin{aligned} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} &= \Gamma(L)\Gamma(0)^{-1}\Psi^{-1}\Psi\epsilon(t) \\ &\equiv \check{\Gamma}(L)\xi(t) \\ &\equiv \begin{bmatrix} \check{\Gamma}_{11}(L) & \check{\Gamma}_{12}(L) \\ \check{\Gamma}_{21}(L) & \check{\Gamma}_{22}(L) \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix}. \end{aligned} \tag{A.3}$$

In view of the construction, $\{\xi_1(t)\}$ is the one-way effect componet process of $\{u(t)\}$ to $\{v(t)\}$. It follows from the representations (A.2) and (A.3), the reciprocal components of $u(t)$ and $v(t)$ are respectively given by

$$\ddot{u}_{\cdot,\infty}(t) = \Gamma_{11}^{\dagger}(L)\Sigma_{11}^{-1/2}\epsilon_1(t) \quad \text{and} \quad \ddot{v}_{\infty,\cdot}(t) = \check{\Gamma}_{22}(L)\Sigma_{22}^{-1/2}\epsilon_2(t).$$

Consequently, the joint spectral density matrix $\ddot{h}(\lambda)$ of the process $\{\ddot{u}_{\cdot,\infty}(t), \ddot{v}_{\infty,\cdot}(t)\}$ is given by

$$
\begin{aligned}
\ddot{h}(\lambda) &= \frac{1}{2\pi}\left[\begin{array}{cc} \Gamma_{11}^{\dagger}(e^{-i\lambda})\Sigma_{11}^{-1/2} & 0 \\ 0 & \check{\Gamma}_{22}(e^{-i\lambda})\Sigma_{22}^{-1/2} \end{array}\right]\left[\begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{array}\right] \\
&\quad \cdot \left[\begin{array}{cc} \{\Gamma_{11}^{\dagger}(e^{-i\lambda})\Sigma_{11}^{-1/2}\}^{*} & 0 \\ 0 & \{\check{\Gamma}_{22}(e^{-i\lambda})\Sigma_{22}^{-1/2}\}^{*} \end{array}\right] \\
&= \frac{1}{2\pi}\left[\begin{array}{cc} \Gamma_{11}^{\dagger}(e^{-i\lambda})\Gamma_{11}^{\dagger}(e^{-i\lambda})^{*} & \Gamma_{11}^{\dagger}(e^{-i\lambda})\Sigma_{11}^{-1/2}\Sigma_{12}\Sigma_{22}^{-1/2}\check{\Gamma}_{22}(e^{-i\lambda})^{*} \\ \check{\Gamma}_{22}(e^{-i\lambda})\Sigma_{22}^{-1/2}\Sigma_{21}\Sigma_{11}^{-1/2}\Gamma_{11}^{\dagger}(e^{-i\lambda})^{*} & \check{\Gamma}_{22}(e^{-i\lambda})\check{\Gamma}_{22}(e^{-i\lambda})^{*} \end{array}\right] \\
&= \frac{1}{2\pi}\left[\begin{array}{cc} \Gamma_{11}^{\dagger}(e^{-i\lambda}) & 0 \\ 0 & \check{\Gamma}_{22}(e^{i\lambda}) \end{array}\right]\left[\begin{array}{cc} I_{p_1} & \Sigma_{11}^{-1/2}\Sigma_{12}\Sigma_{22}^{-1/2} \\ \Sigma_{22}^{-1/2}\Sigma_{21}\Sigma_{11}^{-1/2} & I_{p_2} \end{array}\right] \\
&\quad \cdot \left[\begin{array}{cc} \Gamma_{11}^{\dagger}(e^{-i\lambda})^{*} & 0 \\ 0 & \check{\Gamma}_{22}(e^{-i\lambda})^{*} \end{array}\right]. \qquad (A.4)
\end{aligned}
$$

Then, in view of the definition of $\ddot{\sigma}^2$ in Theorem 2.1, it follows straightforwardly from (A.4) that

$$\det \ddot{h}_{11}(\lambda) \det \ddot{h}_{22}(\lambda) / \det \ddot{h}(\lambda) = \ddot{\sigma}^2,$$

since the determinant of the second matrix on the right-hand side of (A.4) is nothing but

$$\ddot{\sigma}^2 \quad = \quad \det \Sigma_{11} \det \Sigma_{22} / \det \Sigma. \quad \square$$

Proof of Theorem 2.2. Let $\epsilon_i^{\dagger}(t)$ and $\Gamma_{ij}^{\dagger}(L), i, j = 1, 2$, be defined as in (2.11) based on the factorization $h(\lambda) = \frac{1}{2\pi}\Gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})^{*}$. If the spectral density $k(\lambda)$ has the canonical factorization (2.16), in parallel to (2.9), we have the time domain representation

$$
\begin{aligned}
\left[\begin{array}{c} u(t) \\ v(t) \end{array}\right] &= \gamma(L)\Gamma(L)\Gamma(0)^{-1}\left[\begin{array}{c} \epsilon_1(t) \\ \epsilon_2(t) \end{array}\right] \\
&= \left[\begin{array}{cc} \Gamma_{11}^{\dagger\dagger}(L) & \Gamma_{12}^{\dagger\dagger}(L) \\ \Gamma_{21}^{\dagger\dagger}(L) & \Gamma_{22}^{\dagger\dagger}(L) \end{array}\right]\left[\begin{array}{c} \epsilon_1^{\dagger}(t) \\ \epsilon_2^{\dagger}(t) \end{array}\right] \qquad (A.5)
\end{aligned}
$$

where

$$\Gamma_{ij}^{\dagger\dagger} = \gamma(L)\Gamma_{ij}^{\dagger}(L), i = 1, 2.$$

Hence we have

$$
\begin{aligned}
M_{v\to u}(\lambda) &\equiv \log \frac{\det\{\Gamma_{11}^{\dagger\dagger}(e^{-i\lambda})\Gamma_{11}^{\dagger\dagger}(e^{-i\lambda})^{*} + \Gamma_{12}^{\dagger\dagger}(e^{-i\lambda})\Gamma_{12}^{\dagger\dagger}(e^{-i\lambda})^{*}\}}{\det\{\Gamma_{11}^{\dagger\dagger}(e^{-i\lambda})\Gamma_{11}^{\dagger\dagger}(e^{-i\lambda})^{*}\}} \\
&\equiv \log \frac{\det\{\Gamma_{11}^{\dagger}(e^{-i\lambda})\Gamma_{11}^{\dagger}(e^{-i\lambda})^{*} + \Gamma_{12}^{\dagger}(e^{-i\lambda})\Gamma_{12}^{\dagger}(e^{-i\lambda})^{*}\}}{\det\{\Gamma_{11}^{\dagger}(e^{-i\lambda})\Gamma_{11}^{\dagger}(e^{-i\lambda})^{*}\}}. \qquad (A.6)
\end{aligned}
$$

Namely the right-hand side member of (A.6) implies that the FMO based on $k(\lambda)$ is equal to the FMO for the spectral density $h(\lambda) = \frac{1}{2\pi}\Gamma(e^{-i\lambda})\Gamma(e^{-i\lambda})^*$. In the same way, for the process given by (A.5), the joint spectral density matrix $\ddot{k}(\lambda)$ of the reciprocal-component process $\{\ddot{u}_{\cdot,\infty}(t), \ddot{v}_{\infty,\cdot}(t)\}$ is equal to $|\gamma(e^{-i\lambda})|^2\ddot{h}(\lambda)$ where $\ddot{h}(\lambda)$ the density given by (A.4). Therefore the frequency-wise measure of reciprocity is given by

$$\begin{aligned} M_{u.v}(\lambda) &= \log[\det\{|\gamma(e^{-i\lambda})|^2\ddot{h}_{11}(\lambda)\}\det\{|\gamma(e^{-i\lambda})|^2\ddot{h}_{22}(\lambda)\}/\det\{|\gamma(e^{-i\lambda})|^2\ddot{h}(\lambda)\} \\ &= \log\ddot{\sigma}^2. \quad \square \end{aligned}$$

A.3 The joint spectral density of the reciprocal components

The representation (A.4) of the joint spectral density of the reciprocal components is new, whereas another representation was provided by Hosoya (1991). The representation of the 1991 paper, however, contains some errata, and we present a corrected version in Theorm A. Suppose that the joint process $\{u(t), v(t)\}$ introduced in Section 2 has the spectral representation with respect to a random spectral measure:

$$r(t) \equiv \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \int_{-\pi}^{\pi} e^{i\lambda t} \begin{bmatrix} \Phi_u(d\lambda) \\ \Phi_v(d\lambda) \end{bmatrix} \equiv \int_{-\pi}^{\pi} e^{i\lambda t}\Phi_r(d\lambda) \quad t \in \mathbb{Z}.$$

Denote by $h(\lambda)$ the spectral density matrix of the process $\{r(t)\}$. Let $\tilde{h}$ and $\breve{h}$ be respectively the joint spectral densities of the pairs of processes $\{u(t), v_{0,-1}(t)\}$ and $\{u_{-1,0}(t), v(t)\}$ and let the partitions of them be given by

$$\tilde{h}(\lambda) = \begin{pmatrix} \tilde{h}_{11}(\lambda) & \tilde{h}_{12}(\lambda) \\ \tilde{h}_{21}(\lambda) & \tilde{h}_{22}(\lambda) \end{pmatrix} \quad \text{and} \quad \breve{h}(\lambda) = \begin{pmatrix} \breve{h}_{11}(\lambda) & \breve{h}_{12}(\lambda) \\ \breve{h}_{21}(\lambda) & \breve{h}_{22}(\lambda) \end{pmatrix}.$$

Also denote the partition of the spectral density matrix $\ddot{h}(\lambda)$ of the joint reciprocal component process $\{\ddot{u}_{\cdot,\infty}(t), \ddot{v}_{\infty,\cdot}(t)\}$ by

$$\ddot{h}(\lambda) = \begin{bmatrix} \ddot{h}_{11}(\lambda) & \ddot{h}_{12}(\lambda) \\ \ddot{h}_{21}(\lambda) & \ddot{h}_{22}(\lambda) \end{bmatrix}.$$

Set $A = (-\Sigma_{21}\Sigma_{11}^{-1}, I_{p_2})$; $\quad B = (I_{p_1}, -\Sigma_{12}\Sigma_{22}^{-1})$, where

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

is the covariance of the one-step ahead prediction error of the process $\{u(t), v(t)\}$.

Theorem A. The spectral density $\ddot{h}(\lambda)$ is represented as follows:

$$\begin{aligned}
\ddot{h}_{11}(\lambda) &= h_{11}(\lambda) - 2\pi h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^* A^*\Sigma_{22:1}^{-1}A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 1}(\lambda), &\text{(A.7)}\\
\ddot{h}_{22}(\lambda) &= h_{22}(\lambda) - 2\pi h_{2\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^* B^*\Sigma_{11:2}^{-1}B\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda), &\text{(A.8)}\\
\ddot{h}_{12}(\lambda) &= h_{12}(\lambda) - 2\pi h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*(A^*\Sigma_{22:1}^{-1}A + B^*\Sigma_{11:2}^{-1}B)\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda)\\
&\quad +2\pi\tilde{h}_{12}(\lambda)\Sigma_{22:1}^{-1}(-\Sigma_{21} + \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})\Sigma_{11:2}^{-1}\breve{h}_{12}(\lambda), &\text{(A.9)}
\end{aligned}$$

where $\Sigma_{11:2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$ and $\Sigma_{22:1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$.

Proof. Since the proofs of the three components proceed in parallel ways, only the proof for (A.9) is given below. It follows from the definitions of $\ddot{u}_{\cdot,\infty}(t)$ and $\ddot{v}_{\infty,\cdot}(t)$ which are given in the paragraph containing (2.13) in Section 2 that

$$\begin{aligned}
\Phi_{\ddot{u}_{\cdot,\infty}}(d\lambda) &= \Phi_u(d\lambda) - \tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)\Phi_{v_{0,-1}}(d\lambda),\\
\Phi_{\ddot{v}_{\infty,\cdot}}(d\lambda) &= \Phi_v(d\lambda) - \breve{h}_{21}(\lambda)\breve{h}_{11}^{-1}(\lambda)\Phi_{u_{-1,0}}(d\lambda). &\text{(A.10)}
\end{aligned}$$

On the other hand, it follows from the definition of the one-way effect components that

$$\begin{aligned}
\Phi_{v_{0,-1}}(d\lambda) &= A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}\Phi_r(d\lambda),\\
\Phi_{u_{-1,0}}(d\lambda) &= B\Gamma(0)\Gamma(e^{-i\lambda})^{-1}\Phi_r(d\lambda). &\text{(A.11)}
\end{aligned}$$

Now in view of (A.10) the submatrix $\ddot{h}_{12}(\lambda)$ is given by

$$\begin{aligned}
\ddot{h}_{12}(\lambda) &= E\{\Phi_u(d\lambda) - \tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)\Phi_{v_{0,-1}}(d\lambda)\}\{\Phi_v^*(d\lambda) - \Phi_{u_{-1,0}}^*(d\lambda)\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\}\\
&= E\{\Phi_u(d\lambda)\Phi_v^*(d\lambda)\} - \tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)E\{\Phi_{v_{0,-1}}(\lambda)\Phi_v^*(d\lambda)\}\\
&\quad -E\{\Phi_u(d\lambda)\Phi_{u_{-1,0}}^*(d\lambda)\}\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\
&\quad +\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)E\{\Phi_{v_{0,-1}}(\lambda)\Phi_{u_{-1,0}}^*(d\lambda)\}\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\
&= h_{12}(\lambda) - \tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda)\\
&\quad -h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*B^*\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\
&\quad +\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*B^*\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\
&\equiv C1 - C2 - C3 + C4. &\text{(A.12)}
\end{aligned}$$

Since $h(\lambda)$ has the canonical factorization

$$h(\lambda) = \frac{1}{2\pi}\Gamma(e^{-i\lambda})\Gamma^*(e^{-i\lambda}),$$

the last member C4 on the right hand side of (A.12) is equal to

$$\begin{aligned}&\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)A\frac{1}{2\pi}\Gamma(0)\Gamma(0)^*B^*\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\&=\frac{1}{2\pi}\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)A\Sigma B^*\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda).\end{aligned}\tag{A.13}$$

Furthermore, since

$$\begin{aligned}A\Sigma B^* &= (-\Sigma_{21}\Sigma_{11}^{-1}, I_{p_2})\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}\begin{bmatrix}I_{p_1}\\ -\Sigma_{22}^{-1}\Sigma_{21}\end{bmatrix}\\ &= \left[0, -\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}+\Sigma_{22}\right]\begin{bmatrix}I_{p_1}\\ -\Sigma_{22}^{-1}\Sigma_{21}\end{bmatrix}\\ &= \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}-\Sigma_{21},\end{aligned}$$

the last member in (A.13) is expressed as

$$\begin{aligned}&\frac{1}{2\pi}\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)(-\Sigma_{21}+\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\&=2\pi\tilde{h}_{12}(\lambda)\Sigma_{22:1}^{-1}(-\Sigma_{21}+\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})\Sigma_{11:2}^{-1}\breve{h}_{12}(\lambda).\end{aligned}\tag{A.14}$$

Hence the expression of the third member on the right-hand side of (A.9) is derived. On the other hand, it follows from (A.11) that

$$\begin{aligned}\tilde{h}_{12}(\lambda) &= E[\Phi_u(d\lambda)\Phi_{v_{0,-1}}^*(\lambda)] = h_{1\cdot}\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*A^*,\\ \breve{h}_{12}(\lambda) &= E[\Phi_{u_{-1,0}}(d\lambda)\Phi_v^*(\lambda)] = B\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda).\end{aligned}\tag{A.15}$$

Also, it follows from (A.15) that the second member C2 on the right-hand side of (A.12) is expressed as

$$\begin{aligned}&\tilde{h}_{12}(\lambda)\tilde{h}_{22}^{-1}(\lambda)A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda)\\&=2\pi h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*A^*\Sigma_{22:1}^{-1}A\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda)\end{aligned}\tag{A.16}$$

whereas the third member C3 is given by

$$\begin{aligned}&h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*B^*\breve{h}_{11}^{-1}(\lambda)\breve{h}_{12}(\lambda)\\&=2\pi h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*B^*\Sigma_{11:2}^{-1}B\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda).\end{aligned}$$

Therefore the sum of the second and third members C2, C3 is equal to

$$2\pi h_{1\cdot}(\lambda)\Gamma(e^{-i\lambda})^{-1*}\Gamma(0)^*(A^*\Sigma_{22:1}^{-1}A+B^*\Sigma_{11:2}^{-1}B)\Gamma(0)\Gamma(e^{-i\lambda})^{-1}h_{\cdot 2}(\lambda).\tag{A.17}$$

Hence (A.9) follows from (A.14) and (A.17). It provides a representation of the off-diagonal (1, 2)-th block of the joint spectral density of $\{\ddot{u}_{\cdot,\infty}(t), \ddot{v}_{\infty,\cdot}(t)\}$. By means of parallel arguments, we have (A.7) and (A.8). $\square$

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