MANTEL-HAENSZEL TYPE ESTIMATORS FOR THE COUNTER-MATCHED SAMPLING DESIGN IN NESTED CASE-CONTROL STUDY

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MANTEL-HAENSZEL TYPE ESTIMATORS FOR THE COUNTER-MATCHED SAMPLING DESIGN IN NESTED CASE-CONTROL STUDY

By

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Abstract

We are concerned with a counter-matched nested case-control study. Assuming the proportional hazards model, the Mantel-Haenszel estimators of hazard rates are presented in two situations. The proposed estimators can be calculated without estimating the nuisance parameter. Consistent estimators of the variance of the proposed hazard rate estimators are also developed. We compare these estimators to the maximum partial likelihood estimators in the asymptotic variance. The methods are illustrated using the Colorado Plateau uranium miner cohort data.

Key Words and Phrases: proportional hazard model, conditional distribution, nuisance parameter.

1. Introduction

We often encounter the situations that a cohort has already been enumerated and more information for the cohort members is needed. Since it is often expensive to collect such additional information for all cohort members, the nested case-control designs have been developed. In the nested case-control designs the information is given from the selected controls of the cohort at risk at each event time. The sampling methods of controls are discussed by many articles for example, see Langholz and Thomas (1991). Langholz and Borgan (1995) proposed the counter-matched sampling designs in the nested case-control study. They show that it is more efficient than random sampling designs especially in the cases that the exposure or a surrogate measure of the exposure is available for all cohort members. Langholz and Borgan (1995) also developed the inference based on the maximum partial likelihood estimator(PL estimator) assuming the proportional hazards model. Borgan et al. (1995) derived the asymptotic properties of the estimator using process and martingale theory.

However the proportional hazards model has many assumptions, for example the multiplicity relationship between the baseline hazard function and the regression function of covariates. The violation of these assumptions may have adverse effects on the statistical inference. For checking the model assumptions, it is important to have another inference procedure. The purpose of this article is to develop the Mantel-Haenszel

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type estimators for the counter-matched sampling design. In the estimation for the common odds ratio in several 2x2 tables, the Mantel-Haenszel estimator is known to be not only simple but also having many preferable properties. Especially the unbiasedness of the estimating function provides the robustness for the assumption of probability models. In this article we consider the case that exposure variable and another covariate are dichotomous. In Section 2, we give a class of unbiased estimating functions and propose the Mantel-Haenszel type estimators in two situations. Consistent estimators of the variance of the proposed estimators are also given. In Section 3 we compare the proposed estimator with the PL estimator in the asymptotic relative efficiency and an example is presented in Section 4. Some discussions are made in Section 5.

2. Model and Estimation

2.1. Formulation

We consider the cohort study that consists of n individuals, who have a failure time T and a censoring time C. We can observe $\tilde{T} = min(T,C)$ and $\Delta = I_{\{T \leq C\}}$. We assume the Cox's(1972) proportional hazards model so that the hazard function for the subject with a vector of covariates Z is given by

 $\lambda_0(t)exp\{\beta'Z\}$

where $\lambda_0(t)$ is a baseline hazard function. We also assume that the censoring time C is independent on the failure time T. In this paper we consider the case that the covariate vector consists of two time independent covariates Z_1 and Z_2 which take values zero or one. Z_1 is available for all cohort members and Z_2 is for cases and sampled controls in the counter-matched sampling design.

Let $t_1 < t_2 < \ldots < t_K$ be the ordered failure time and assuming no tied failure i_k be the index of the subject failed at t_k . $R(t_k)$, the risk set at t_k , is defined as the set of individuals at risk just prior to t_k . For the case i_k , controls are drawn from the risk set $R(t_k)$ as follows. The risk set $R(t_k)$ is partitioned into two strata accoding to the value of Z_1 and the controls are drawn from each stratum such that the sum of the numbers of the case and controls with $Z_1 = j$ is m_j .

Let $n_{i+1}^k, X_{i,l}^k$ and $Y_{i,l}^k$ be defined as follows:

$$\begin{split} n_{j+1}^{k} &= \#\{i \in R(t_{k}) | Z_{i,1} = j\} \\ X_{j,l}^{k} &= \#\{i \in R(t_{k}) | \tilde{T}_{i} = t_{k}, \Delta_{j} = 1, Z_{i,1} = j \text{ and } Z_{i,2} = l\} \\ Y_{j,l}^{k} &= \#\{i \in R(t_{k}) | \tilde{T}_{i} > t_{k}, Z_{i,1} = j \text{ and } Z_{i,2} = l\} \end{split}$$

where $Z_{i,r}$ denotes the value of Z_r of the individual *i*. We also define $W_{j,l}^k$ and $S_{j,l}^k$ for sampled subjects as Table 1.

The conditional distribution of X^k_{jh} given $S^k = (S^k_{11}, S^k_{10}, S^k_{01}, S^k_{00})^T$ is

$$P(X_{jh}^{k} = 1|S^{k}) = \frac{S_{jh}^{k}e^{\beta_{1}j+\beta_{2}h}\frac{n_{j+}^{k}}{m_{j}}}{(S_{11}^{k}e^{\beta_{1}+\beta_{2}} + S_{10}^{k}e^{\beta_{1}})\frac{n_{1+}^{k}}{m_{1}} + (S_{01}^{k}e^{\beta_{2}} + S_{00}^{k})\frac{n_{0+}^{k}}{m_{0}}}.$$
 (1)

Note that the formula is similar to the non-central hypergeometric distribution except for the coefficients n_{0+}^k/m_0 or n_{1+}^k/m_1 exists. So we can construct the Mantel-Haenszel type estimator for β in the following two situations.

	Z_1	= 1	Z_1	= 0	
	$Z_2 = 1$	$Z_2 = 0$	$Z_2 = 1$	$Z_2 = 0$	
case	X_{11}^{k}	X_{10}^k	X_{01}^k	X_{00}^k	1
control	W_{11}^k	W_{10}^k	W_{01}^k	W_{00}^k	$m_1 + m_0 - 1$
	S_{11}^{k}	S_{10}^k	S_{01}^{k}	S_{00}^{k}	$m_1 + m_0$

Table 1: Sampled risk sets

2.2. Counter-matching on a surrogate measure of exposure

When the exposure of interest is expensive to collect but there is an inexpensive surrogate measure, the true exposure is evaluated for the sampled subjects countermatching on the surrogate measure. In this case Z_1 is the surrogate measure for exposure and Z_2 is the true exposure measurement. Z_1 is assumed to be dependent on the failure time only through the true exposure, i.e. $\beta_1 = 0$. Put

$$A^{k} = S_{11}^{k} \frac{n_{1+}^{k}}{m_{1}} + S_{01}^{k} \frac{n_{0+}^{k}}{m_{0}} \quad and \quad B^{k} = S_{10}^{k} \frac{n_{1+}^{k}}{m_{1}} + S_{00}^{k} \frac{n_{0+}^{k}}{m_{0}}$$

then it is easily shown from (1) that $X_{\pm 1}^k B^k - exp(\beta_2) X_{\pm 0}^k A^k$ is an unbiased estimating function for β_2 for each k where the subscript + denotes the sum over all possible subscript. We construct the following family of estimating functions

$$g(\beta_2) = \sum_{k=1}^{K} w_k \{ X_{+1}^k B^k - exp(\beta_2) X_{+0}^k A^k \}$$

where w_k is positive constant. To yield optimum weights we apply the theory of estimating functions (Godambe, 1991). It requires the minimization of the following criteria

$$\frac{Var(g)}{E[\frac{\partial}{\partial\beta_2}g]^2}$$

THEOREM 2.1. Suppose w_k is non-random variable conditioned by S^k and n^k , an optimal weight is given by

$$w_k = \frac{1}{A^k exp(\beta_2) + B^k} \tag{2}$$

PROOF. We calculate Var(g) and $E[\frac{\partial}{\partial \beta_2}g]$ respectively. We use the conditional independence of the estimating functions between the failure times and the double expectation theorem. We have following identity:

$$Var(g) = E\left[\left(\sum_{k=1}^{K} w_k \{X_{+1}^k B^k - exp(\beta_2) X_{+0}^k A^k\}\right)^2\right]$$

= $E\left[\sum_{k=1}^{K} w_k^2 \{X_{+1}^k (B^k)^2 + exp(2\beta_2) X_{+0}^k (A^k)^2\}\right]$

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$$= E[\sum_{k=1}^{K} w_k^2 E[X_{+1}^k (B^k)^2 + exp(2\beta_2) X_{+0}^k (A^k)^2 | n^k, S^k]]$$

$$= E[\sum_{k=1}^{K} w_k^2 \frac{A^k (B^k)^2 exp(\beta_2) + (A^k)^2 B^k exp(2\beta_2)}{A^k exp(\beta_2) + B^k}]$$

$$= E[\sum_{k=1}^{K} w_k^2 A^k B^k exp(\beta_2)].$$

On the other hand

$$E[\frac{\partial}{\partial\beta_2}g] = -E[\sum_{k=1}^{K} w_k exp(\beta_2) X_{+0}^k A^k]$$

=
$$-E[\sum_{k=1}^{K} w_k exp(\beta_2) \frac{A^k B^k exp(\beta_2)}{A^k exp(\beta_2) + B^k}].$$

We have

$$\frac{Var(g)}{E[\frac{\partial}{\partial\beta_2}g]^2} = \frac{E[\sum_{k=1}^K w_k^2 A^k B^k exp(\beta_2)]}{E[\sum_{k=1}^K w_k exp(\beta_2) \frac{A^k B^k exp(\beta_2)}{A^k exp(\beta_2) + B^k}]^2}$$

Using Cauchy-Schwartz inequality, we may show that $w_k = \frac{1}{A^{k}exp(\beta_2)+B^{k}}$ provides the minimum of the above criteria.

The estimator with above optimal weights is equivalent to the PL estimator in this situation. Yanagimoto (1990) shows that the simplicity of the Mantel-Haenszel estimator results from using the locally optimum weights. In this case by using the optimum weights at $\beta = 0$, we have the following estimating equation:

$$\sum_{k=1}^{K} \frac{1}{n_{++}^{k}} \{ X_{+1}^{k} B^{k} - exp(\beta_{2}) X_{+0}^{k} A^{k} \} = 0.$$
(3)

The solution of the estimating equation is called the Mantel-Haenszel type estimator in counter-matching on surrogate measure. The result in the proof of the Theorem 2.1 leads to an estimator of variance of the Mantel-Haenszel type (MH) estimator as follows

$$\frac{\sum_{k=1}^{K} A_k B_k / (n_{++}^k)^2}{e^{\beta_2} \sum_{k=1}^{K} A_k B_k / (n_{++}^k) (A_k + e^{\beta_2} B_k)}$$
(4)

2.3. Counter-matching on exposure

The second situation is when the exposure of interest is known for the full cohort and a confounder variable is collected for the sampled subjects counter-matching on the exposure. In this case β_1 is an interesting parameter and β_2 is nuisance. At each failure time t_k , we have two unbiased estimating functions

$$X_{1j}^{k}S_{0j}^{k}\frac{n_{0+}^{k}}{m_{0}} - exp(\beta_{1})X_{0j}^{k}S_{1j}^{k}\frac{n_{1+}^{k}}{m_{1}} \qquad j = 0, 1$$

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by considering the conditional distribution X_{jk}^k given n_{i+}^k s and S_{ij}^k s. We have the following theorem using the similar method of Theorem 1.

THEOREM 2.2. Among the weighted estimating functions

$$\sum_{i=1}^{K} \sum_{j=0}^{1} w_{jk} \{ X_{1j}^{k} S_{0j}^{k} \frac{n_{0+}^{k}}{m_{0}} - exp(\beta_{1}) X_{0j}^{k} S_{1j}^{k} \frac{n_{1+}^{k}}{m_{1}} \},$$

an optimal weight is given by

$$w_{jk} = \frac{1}{S_{0j}^k \frac{n_{0+}^k}{m_0} + exp(\beta_1) S_{1j}^k \frac{n_{1+}^k}{m_1}}$$

We also call the Mantel-Haenszel type (MH) estimator for the solution of the estimating equation with above optimal weight in $\beta_1 = 0$. Because this estimating equation is not dependent on β_2 , we don't have to estimate the nuisance parameter β_2 . This property may lead to the robustness for the modeling for the effect of Z_2 . We also proposed the following estimator of the variance of the MH estimator

$$\frac{\sum_{k=1}^{K}\sum_{j=0}^{1}X_{+j}^{k}S_{0j}^{k}S_{1j}^{k}\frac{n_{0+}^{k}}{m_{0}}\frac{n_{1+}^{k}}{m_{1}}/\left(V_{0}^{k}\right)^{2}}{e^{\hat{\beta}_{1}}\left\{\sum_{k=1}^{K}\sum_{j=0}^{1}X_{+j}^{k}S_{0j}^{k}S_{1j}^{k}\frac{n_{0+}^{k}}{m_{0}}\frac{n_{1+}^{k}}{m_{1}}/\left(V_{0}^{k}V_{1}^{k}\right)\right\}^{2}}$$

where

$$V_0^k = S_{0j}^k rac{n_{0+}^k}{m_0} + S_{1j}^k rac{n_{1+}^k}{m_1}$$
 $V_1^k = S_{0j}^k rac{n_{0+}^k}{m_0} + e^{eta_1} S_{1j}^k rac{n_{1+}^k}{m_1}$

3. Comparison with partial likelihood method

In this section we compare the Mantel-Haenszel type estimators to the maximum partial likelihood estimators. One of the advantages of proposed estimators is the robustness for the assumptions of the distribution of the failure time because of not assuming the proportionality of hazard rate for Z_2 . However the MH estimator losses some efficiency in exchange. The characteristic of the poposed estimator may be remarkable in counter-matching on exposure. So we investigate the asymptotic relative efficiency in the case of counter-matching on exposure.

First we compare the asymptotic relative efficiency. We use the same situation in Langholz and Borgan (1995). We assume that the joint distribution of Z_1 and Z_2 for individuals at risk remains constant over time with

$$\pi_{ij} = pr\{Z_1 = i, Z_2 = j\}, \ \pi_{i+} = pr\{Z_1 = i\}, \ \pi_{+i} = pr\{Z_2 = i\}$$

We measure the correlation between Z_1 and Z_2 by the odds ratio $\theta = (\pi_{11}\pi_{00})/(\pi_{10}\pi_{01})$. The asymptotic information matrix of the PL estimator is given in the Appendix of Langholz and Borgan (1995). The similar method leads to the asymptotic variance of the MH estimator of β_1 . With $t_{01} = m_0 - t_{00}$ and $t_{11} = m_1 - t_{10}$, it is given by

$$\frac{\sum_{t_{00}=0}^{m_0} \sum_{t_{10}=0}^{m_1} C(t) E(t)}{e^{\beta_1} \left(\sum_{t_{00}=0}^{m_0} \sum_{t_{10}=0}^{m_1} D(t) E(t) \right)^2}$$

where

$$\begin{split} C(t) &= \sum_{j=0}^{1} \frac{t_{0j} t_{1j} \frac{\pi_{0+}}{m_{0}} \frac{\pi_{1+}}{m_{1}}}{\left(t_{0j} \frac{\pi_{0+}}{m_{0}} + t_{1j} \frac{\pi_{1+}}{m_{1}}\right)^{2}} F_{j}(t), \\ D(t) &= \sum_{j=0}^{1} \frac{t_{0j} t_{1j} \frac{\pi_{0+}}{m_{0}} \frac{\pi_{1+}}{m_{1}}}{\left(t_{0j} \frac{\pi_{0+}}{m_{0}} + t_{1j} \frac{\pi_{1+}}{m_{1}}\right) \left(t_{0j} \frac{\pi_{0+}}{m_{0}} + e^{\beta_{1}} t_{1j} \frac{\pi_{1+}}{m_{1}}\right)} F_{j}(t), \\ E(t) &= \sum_{i=0}^{1} \sum_{j=0}^{1} t_{ij} e^{\beta_{1}i+\beta_{2}j} \frac{\pi_{i+}}{m_{i}} \left(\frac{m_{0}}{t_{00}}\right) \left(\frac{m_{1}}{t_{10}}\right) \prod_{i,j} \left(\frac{\pi_{ij}}{\pi_{i+}}\right)^{t_{ij}}, \\ F_{j}(t) &= \frac{\sum_{i=0}^{1} t_{ij} \frac{\pi_{i+}}{m_{i}} e^{\beta_{1}i+\beta_{2}j}}{\sum_{i=0}^{1} \sum_{j=0}^{1} t_{ij} \frac{\pi_{i+}}{m_{i}} e^{\beta_{1}i+\beta_{2}j}}. \end{split}$$

Table 2 gives asymptotic relative efficiencies of the PL estimator and the MH estimator in the counter-matched design relative to the maximum partial likelihood estimator in simple nested case-control sampling. The result of the asymptiotic relative efficiencies comparing the simple random sampling shows that in every case the effeciency of the Mantel-Haenszel type estimator is smaller than the maximum partial likelihood estimator, but it is still larger than 1. The counter-matching design has the gain in efficiency even if we use the MH estimator. We also calculate the asymptotic relative efficiencies of the MH estimator versus the PL estimator. They are almost 50-70% except for the case both β_2 and θ are small. In such cases the probability that the case and controls have different Z_2 is high and the MH estimator does not use the information of those cases. The asymptotic relative efficiency increases with increasing the number of the sampled control. It is notable that there are little differences in the asymptotic relative efficiency comparing the PL estimator enen if the locally optimal weight is employed in the Mantel-Haenszel estimator.

4. Example

Consider the Colorado Plateau uranium miner cohort data. The data set has been described in earlier publications(see Langholz and Goldstein (1996) and references therein). The cohort consists of 3347 male miners with 258 lung cancer deaths. The association of radon exposure with lung cancer mortality rate is of interest. We dichotomize the exposure by the total cumulative radon exposure level measured in working level months(WLM). Set Exposure=1 for the total cumulative radon exposure level less than 1200 WLM and Exposure=0 otherwise.

First, we consider a situation that the Exposure is unknown and the surrogate measure is given for the full cohort. The exposed period is used for a surrogate measure and it is dichotomized by 60 months. We draw controls for 1:1 counter-matching and 1:3 counter-matching and then compute the MH estimates $\hat{\beta}_{MH}$ and the proposed estimated

Table 2: Asymptotic relative efficiencies of the partial likelihood estimator and the Mantel-Haenszel estimator in counter-matched sampling versus partial likelihood estimator in simple nested case-control sampling when the exposure relative risk $exp(\beta_1)$ is 2 or 4, for $m_0 = m_1$, $\pi_{1+} = 0.05$ and $\pi_{+1} = 0.30$

(a)
$$exp(\beta_1) = 2$$

		1:1 matching			1:3 matching			
e^{eta_2}	θ	\mathbf{PL}	MH	MH:PL	\mathbf{PL}	$\mathbf{M}\mathbf{H}$	MH:PL	
0.20	0.20	2.65	2.47	0.93	1.73	1.61	0.94	
0.20	0.50	2.75	2.36	0.86	1.72	1.58	0.91	
0.20	1.00	2.83	2.22	0.78	1.71	1.52	0.89	
0.20	2.00	2.86	2.01	0.70	1.68	1.44	0.86	
0.20	5.00	2.63	1.66	0.63	1.59	1.29	0.81	
1.00	0.20	2.46	1.91	0.78	1.58	1.38	0.88	
1.00	0.50	2.75	1.79	0.65	1.60	1.32	0.82	
1.00	1.00	2.86	1.66	0.58	1.61	1.25	0.77	
1.00	2.00	2.72	1.50	0.55	1.60	1.16	0.73	
1.00	5.00	2.23	1.28	0.58	1.54	1.05	0.68	
5.00	0.20	2.56	1.55	0.61	1.55	1.21	0.78	
5.00	0.50	2.85	1.58	0.55	1.71	1.22	0.72	
5.00	1.00	2.80	1.61	0.57	1.80	1.24	0.69	
5.00	2.00	2.58	1.64	0.63	1.82	1.24	0.68	
5.00	5.00	2.24	1.68	0.75	1.78	1.25	0.71	

(b) $exp(\beta_1) = 4$

		1:1 matching			1	1:3 matching		
$e^{oldsymbol{eta}_2}$	θ	\mathbf{PL}	MH	MH:PL	\mathbf{PL}	MH	MH:PL	
0.20	0.20	4.06	3.77	0.93	2.23	2.11	0.94	
0.20	0.50	4.15	3.55	0.86	2.23	2.05	0.92	
0.20	1.00	4.20	3.30	0.78	2.21	1.97	0.89	
0.20	2.00	4.13	2.95	0.71	2.16	1.86	0.86	
0.20	5.00	3.63	2.37	0.65	2.02	1.65	0.82	
1.00	0.20	3.75	2.91	0.78	2.01	1.76	0.87	
1.00	0.50	4.19	2.73	0.65	2.07	1.68	0.81	
1.00	1.00	4.35	2.52	0.58	2.09	1.59	0.76	
1.00	2.00	4.15	2.28	0.55	2.06	1.50	0.73	
1.00	5.00	3.41	1.96	0.58	1.93	1.36	0.70	
5.00	0.20	3.46	2.18	0.63	1.89	1.47	0.78	
5.00	0.50	3.99	2.25	0.56	2.13	1.53	0.72	
5.00	1.00	4.07	2.33	0.57	2.26	1.59	0.70	
5.00	2.00	3.88	2.43	0.63	2.29	1.64	0.72	
5.00	5.00	3.47	2.57	0.74	2.23	1.70	0.76	

		MH	PL		
Sampling	$\hat{\beta}_{MH}$	variance	$\hat{\beta}_{PL}$	variance	
1:1	1.617	0.07385	1.577	0.07110	
1:3	1.414	0.06419	1.467	0.06505	
Full	1.309	0.01579	1.338	0.01574	

Table 3: The estimates in counter matching on the surrogate measure for Colorado Plateau uranium miners cohort

Table 4: The estimates in counter matching on the exposure for Colorado Plateau uranium miners cohort

	MH		Optimal		PL	
Sampling	$\hat{\beta}_{MH}$	variance	$\hat{\beta}_{O_P}$	variance	$\hat{\beta}_{PL}$	variance
1:1	1.372	0.02790	1.400	0.02780	1.333	0.01698
1:3	1.382	0.02185	1.371	0.02139	1.292	0.01600
Full	1.344	0.01609	1.366	0.01597	1.309	0.01575

variance of $\hat{\beta}_{MH}$. For reference, we also compute the PL estimates $\hat{\beta}_{PL}$ and its estimated variance with the same nested samples. For full cohort data, the PL estimate and the MH estimate proposed by Zhang and Yanagawa (2001) and the variance estimates of these estimates are also calculated. The result are shown in Table 3.

Table 3 shows that the MH estimates are close to the PL estimate. The estimated variances of these estimator are also close, but it is strange that the estimated variance of the MH estimate is less than that of PL estimates for 1:3 counter matching design. We try to draw controls in several times but the same phenomenon occurs. Further investigations are needed for the model fitting and others.

We consider the second situation that the Exposure is available for the full cohort and the confounding factor is measured for cases and sampled controls. We use the smoking level for a confounding variable and dichotomize the smoking level by 120 cumulative packs. We draw controls by 1:1 and 1:3 counter-matching on Exposure level. We compute the MH estimates and the optimal weight estimates and the estimated variance of these estimates. For reference the PL estimates and its estimated variance are calculated for the same data. The results are shown in Table 4.

Table 4 shows that the MH estimates and the optimal weight estimates are slightly large than the PL estimates for each sampling design. It may provide the possibility of the lack of model fitting, but the effect of it is small. On the other hand, the differences between the MH estimates and the optimal weight estimates are small. The results of two situations lead to the usefullness of the MH estimates because of slight difference to the optimal estimates.

5. Discussion

In this article, we developed the theory in the proportional hazards model, but the assumption of the exponentiality of the regression form is not needed. Along the Langholz and Goldstein (1996), the MH estimator can be extended when the hazard function is given by

$$\lambda(t|Z) = \lambda_0(t)r(\beta^T Z)$$

where r is a positive function on $(-\infty,\infty)$. However the multiplicity of the relationship between the baseline hazard function and the regression function of covariate is needed. If we don't assume it, the effect of covariate is difficult to understand and we need some over-all measure of the effect.

The proposed estimator in the counter matching on the exposure is interpreted as adjusting the confounding factor by stratification. Of course we can extend it when Z_2 has several categories but decreasing the efficiency of the proposed estimator with increasing the number of the category. In such a case, it might be preferable to model the effect of confounding factor like the proportional hazards model and we must check the model assumption by other methods.

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